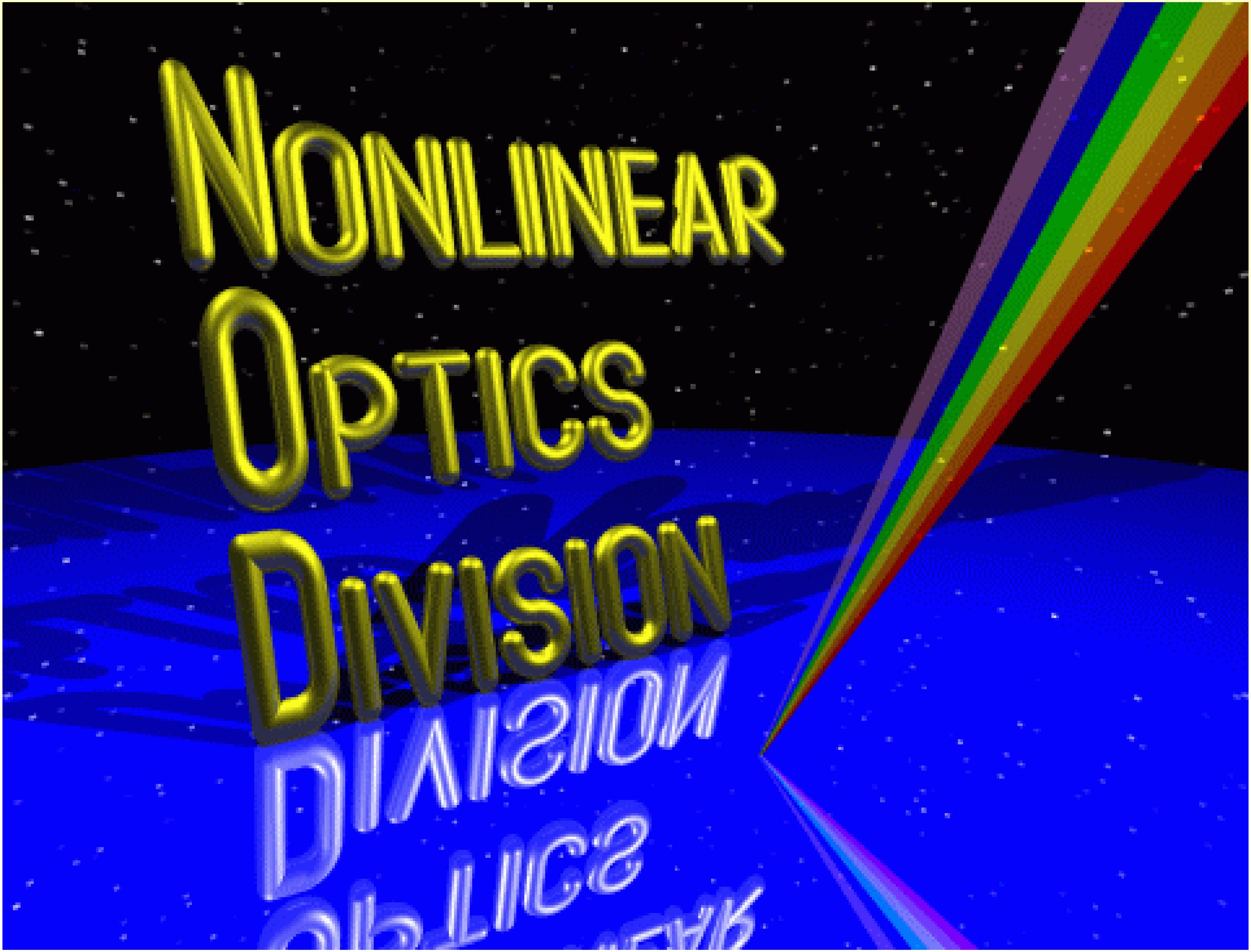
NONLINEAR
OPTICS
DIVISION

Seminarium

z informatyki kwantowej

Splątanie dwóch atomów indukowane ściśniętą próżnią

Ryszard Tanaś

http://zon8.physd.amu.edu.pl/~tanas

5 kwietnia 2005

Spis treści

1 Miary splątania

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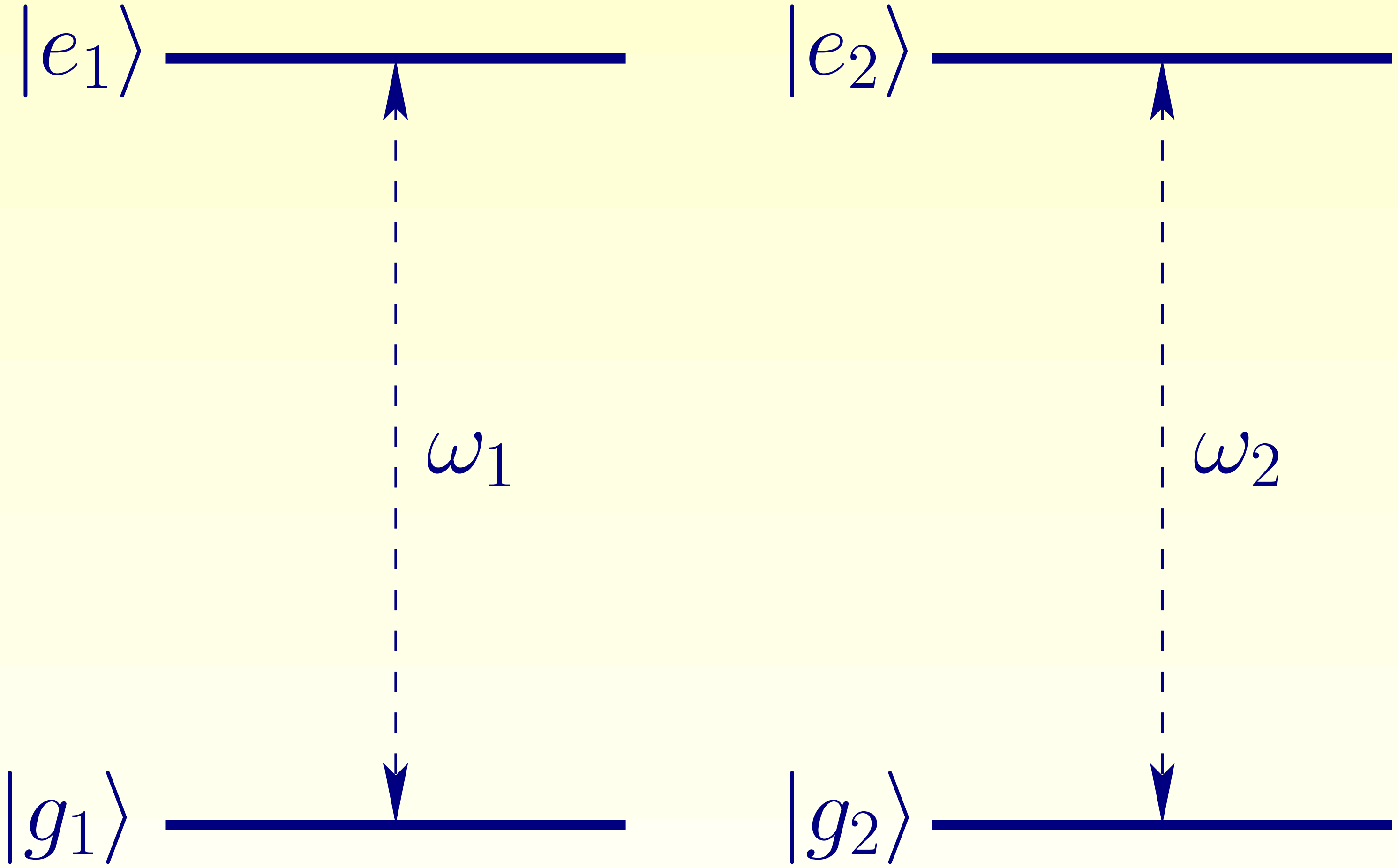
2 Macierz gęstości



Stany kwantowe dwóch atomów

2.1 Baza obliczeniowa

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle$$

2.1 Baza obliczeniowa

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$

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$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle$$

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & 0 & 0 \\ \rho_{21} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{34} \\ 0 & 0 & \rho_{43} & \rho_{44} \end{pmatrix}$$

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$$R = \rho\tilde{\rho}$$

$$\tilde{\rho} = \sigma_y \otimes \sigma_y \, \rho^* \, \sigma_y \otimes \sigma_y$$

$$\tilde{\rho} = \begin{pmatrix} \rho_{22} & \rho_{12} & 0 & 0 \\ \rho_{21} & \rho_{11} & 0 & 0 \\ 0 & 0 & \rho_{44} & \rho_{34} \\ 0 & 0 & \rho_{43} & \rho_{33} \end{pmatrix}$$

$$\left\{\sqrt{\lambda_i}\right\} = \{\sqrt{\rho_{11}\rho_{22}} - |\rho_{12}|, \sqrt{\rho_{11}\rho_{22}} + |\rho_{12}|, \\ \sqrt{\rho_{33}\rho_{44}} - |\rho_{34}|, \sqrt{\rho_{33}\rho_{44}} + |\rho_{34}|\}$$

$$\mathcal{C} = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

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$$\mathcal{C} = \max\{0,\, \mathcal{C}_1,\, \mathcal{C}_2\}$$

$$\mathcal{C} = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

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$$\sqrt{\rho_{33}\rho_{44}} - |\rho_{34}|, \sqrt{\rho_{33}\rho_{44}} + |\rho_{34}|\,\}$$

$$\mathcal{C} = \max\{0, \mathcal{C}_1, \mathcal{C}_2\}$$

$$\mathcal{C}_1 = 2\left(|\rho_{12}| - \sqrt{\rho_{33}\rho_{44}}\,\right)$$
$$\mathcal{C}_2 = 2\left(|\rho_{34}| - \sqrt{\rho_{11}\rho_{22}}\,\right)$$

2.1.2 Negativity

$$\rho^{\mathrm{T}_1} = \begin{pmatrix} \rho_{11} & \rho_{43} & 0 & 0 \\ \rho_{34} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{21} \\ 0 & 0 & \rho_{12} & \rho_{44} \end{pmatrix}$$

2.1.2 Negativity

$$\rho^{\mathrm{T}_1} = \begin{pmatrix} \rho_{11} & \rho_{43} & 0 & 0 \\ \rho_{34} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{21} \\ 0 & 0 & \rho_{12} & \rho_{44} \end{pmatrix}$$

$$\{\nu_i\} = \left\{ \frac{1}{2}\left(\rho_{11} + \rho_{22} \pm \sqrt{(\rho_{11} + \rho_{22})^2 + 4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right)}\right), \right.$$
$$\left. \frac{1}{2}\left(\rho_{33} + \rho_{44} \pm \sqrt{(\rho_{33} + \rho_{44})^2 + 4\left(|\rho_{12}|^2 - \rho_{33}\rho_{44}\right)}\right) \right\}$$

$$\mathcal{N} = \max\Big\{0,\ \sqrt{4\left(|\rho_{12}|^2 - \rho_{33}\rho_{44}\right) + \left(\rho_{33} + \rho_{44}\right)^2} - \left(\rho_{33} + \rho_{44}\right),$$
$$\sqrt{4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right) + \left(\rho_{11} + \rho_{22}\right)^2} - \left(\rho_{11} + \rho_{22}\right)\Big\}$$

$$
\begin{aligned}
\mathcal{N} = \max\Big\{0,\ & \sqrt{4\,(|\rho_{12}|^2 - \rho_{33}\rho_{44}) + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44}), \\
& \sqrt{4\,(|\rho_{34}|^2 - \rho_{11}\rho_{22}) + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\} \\
= \max\Big\{0, & \sqrt{\mathcal{C}_1\,\mathcal{C}_1^{+} + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44})\ , \\
& \sqrt{\mathcal{C}_2\,\mathcal{C}_2^{+} + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N} = \max\Big\{0,\ & \sqrt{4\left(|\rho_{12}|^2 - \rho_{33}\rho_{44}\right) + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44}), \\
& \sqrt{4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right) + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\} \\
= \max\Big\{0, & \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44}), \\
& \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}
\end{aligned}
$$

$$
\mathcal{C}_1^+ = 2\left(|\rho_{12}| + \sqrt{\rho_{33}\rho_{44}}\right)
$$

$$
\mathcal{C}_2^+ = 2\left(|\rho_{34}| + \sqrt{\rho_{11}\rho_{22}}\right)
$$

2.2 Baza Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |e_2\rangle + |g_1\rangle \otimes |g_2\rangle\right) = |1'\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle + |1\rangle\right)$$

2.2 Baza Bella

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$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |e_2\rangle - |g_1\rangle \otimes |g_2\rangle\right) = |2'\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - |1\rangle\right)$$

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$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |g_2\rangle + |g_1\rangle \otimes |e_2\rangle\right) = |3'\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle + |3\rangle\right)$$

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2.2.1 Transformacja do bazy Bella

$$\rho' = U \rho U^{+}$$

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$$\rho' = U \rho U^{+}$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

2.2.2 Macierz gęstości w bazie Bella

$$
\begin{aligned}
\rho_{1'1'} &= \quad \frac{1}{2}\left[\rho_{11}+\rho_{22}+(\rho_{12}+\rho_{21})\right] \\
\rho_{2'2'} &= \quad \frac{1}{2}\left[\rho_{11}+\rho_{22}-(\rho_{12}+\rho_{21})\right] \\
\rho_{1'2'} &= -\frac{1}{2}\left[\rho_{11}-\rho_{22}+(\rho_{12}-\rho_{21})\right] \\
\rho_{2'1'} &= -\frac{1}{2}\left[\rho_{11}-\rho_{22}-(\rho_{12}-\rho_{21})\right]
\end{aligned}
$$

$$
\begin{aligned}
\rho_{3'3'} &= \quad \frac{1}{2}\left[\rho_{33}+\rho_{44}+(\rho_{34}+\rho_{43})\right] \\
\rho_{4'4'} &= \quad \frac{1}{2}\left[\rho_{33}+\rho_{44}-(\rho_{34}+\rho_{43})\right] \\
\rho_{3'4'} &= -\frac{1}{2}\left[\rho_{33}-\rho_{44}+(\rho_{34}-\rho_{43})\right] \\
\rho_{4'3'} &= -\frac{1}{2}\left[\rho_{33}-\rho_{44}-(\rho_{34}-\rho_{43})\right]
\end{aligned}
$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$- \sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$- \sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2 = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2}$$
$$- \sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2} - \sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2 = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2} - \sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

$$\mathcal{C} = \max\{0, \mathcal{C}_1, \mathcal{C}_2\}$$

2.2.4 Negativity

$$\mathcal{N} = \max\left\{0, \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ \, + (\rho_{3'3'} + \rho_{4'4'})^2} - (\rho_{3'3'} + \rho_{4'4'}) \, , \right.$$
$$\left. \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{1'1'} + \rho_{2'2'})^2} - (\rho_{1'1'} + \rho_{2'2'}) \right\}$$

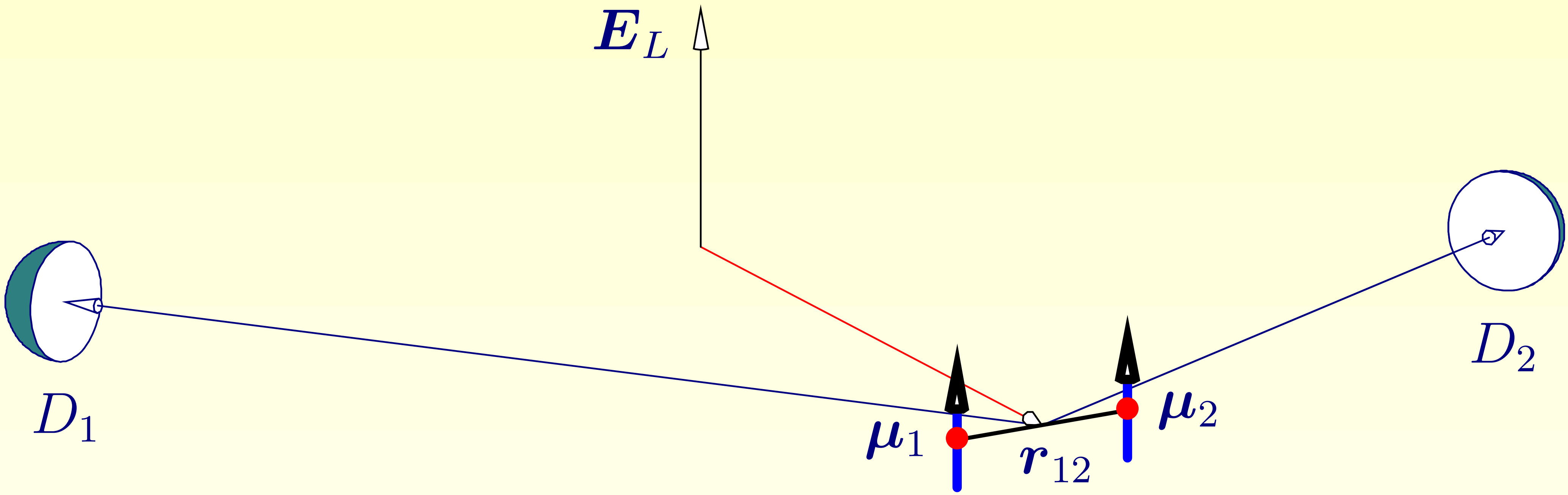
2.2.4 Negativity

$$\mathcal{N} = \max\left\{0, \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{3'3'} + \rho_{4'4'})^2} - (\rho_{3'3'} + \rho_{4'4'})\,, \right.$$
$$\left. \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{1'1'} + \rho_{2'2'})^2} - (\rho_{1'1'} + \rho_{2'2'})\right\}$$

$$\mathcal{C}_1^+ = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$+\sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2^+ = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2}$$
$$+\sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

3 Ewolucja dwóch atomów



Geometria układu: $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$ — momenty dipolowe przejść atomowych, $\boldsymbol{r}_{12}$ — odległość między atomami, $\boldsymbol{E}_L$ — pole laserowe, D_1, D_2 — detektory

3.1 Równanie „master" (ściśnięta próżnia)

$$\frac{\partial \rho}{\partial t} = -\, i \sum_{i=1}^{2} \omega_i \left[S_i^z, \rho\right] - i \sum_{i \neq j}^{2} \Omega_{ij} \left[S_i^+ S_j^-, \rho\right]$$

$$- \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} \left(1 \qquad\right) \left(\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2 S_j^- \rho S_i^+\right)$$

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$$- \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} \left(1 + N\right) \left(\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2 S_j^- \rho S_i^+\right)$$

$$- \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} N \left(\rho S_i^- S_j^+ + S_i^- S_j^+ \rho - 2 S_j^+ \rho S_i^-\right)$$

3.1 Równanie „master” (ściśnięta próżnia)

$$\frac{\partial\rho}{\partial t} = -\,i\sum_{i=1}^{2}\omega_i\left[S_i^z,\rho\right] - i\sum_{i\neq j}^{2}\Omega_{ij}\left[S_i^+S_j^-,\rho\right]$$

$$-\,\frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}\left(1+N\right)\left(\rho S_i^+S_j^- + S_i^+S_j^-\rho - 2S_j^-\rho S_i^+\right)$$

$$-\frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}N\left(\rho S_i^-S_j^+ + S_i^-S_j^+\rho - 2S_j^+\rho S_i^-\right)$$

$$+\,\frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}M\left(\rho S_i^+S_j^+ + S_i^+S_j^+\rho - 2S_j^+\rho S_i^+\right)e^{-2i\omega_s t}$$

$$+\,\frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}M^*\left(\rho S_i^-S_j^- + S_i^-S_j^-\rho - 2S_j^-\rho S_i^-\right)e^{2i\omega_s t}$$

3.2 Parametry kolektywne

Tłumienie kolektywne:

$$\Gamma_{ij} = \Gamma_{ji} = \frac{3}{2}\sqrt{\Gamma_i\Gamma_j}\left\{\left[1 - \left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right]\frac{\sin\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1 - 3\left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right]\left[\frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2} - \frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right]\right\}$$

3.2 Parametry kolektywne

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Dla $k_0 r_{ij} \to 0$ mamy:

$$\Gamma_{ij} = \Gamma_{ji} = \sqrt{\Gamma_i \Gamma_j}$$

$$\Gamma_i \equiv \Gamma_{ii} = \frac{\omega_i^3 \mu_i^2}{3\pi\varepsilon_0 \hbar c^3}$$

Kolektywne przesunięcie poziomów

(oddziaływanie dipol-dipol):

$$\Omega_{ij} = \frac{3}{4}\Gamma \left\{ -\left[1 - \left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \frac{\cos\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1 - 3\left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \left[\frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2} + \frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right] \right\}$$

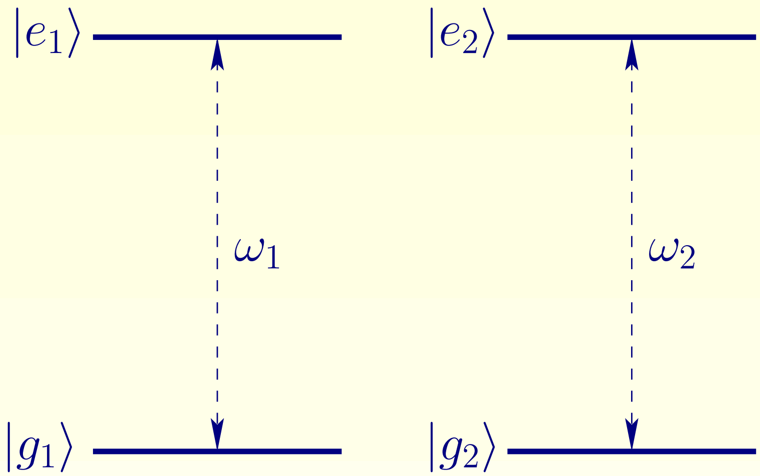
Kolektywne przesunięcie poziomów

(oddziaływanie dipol-dipol):

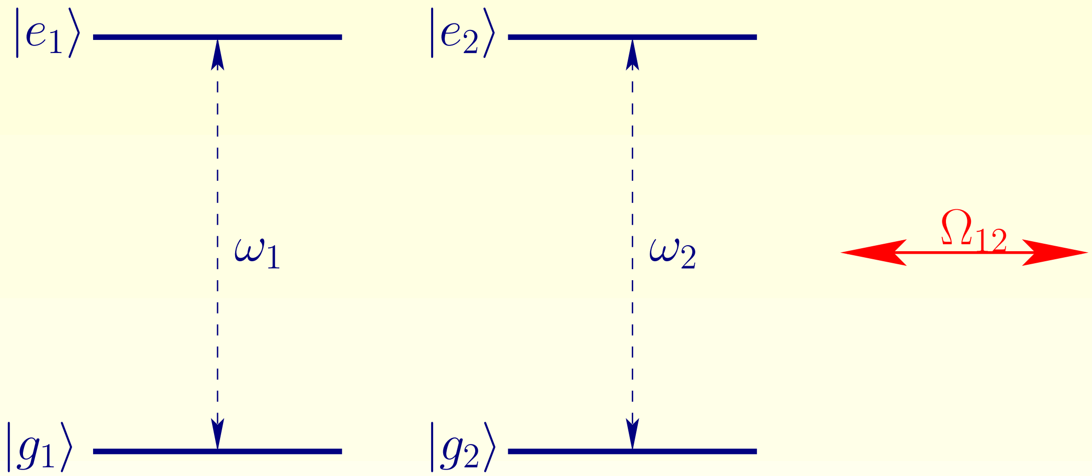
$$\Omega_{ij} = \frac{3}{4}\Gamma\left\{ -\left[1-\left(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij}\right)^2\right]\frac{\cos\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1-3\left(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij}\right)^2\right]\left[\frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2}+\frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right]\right\}$$

Dla $k_0 r_{ij} \ll 1$ mamy:

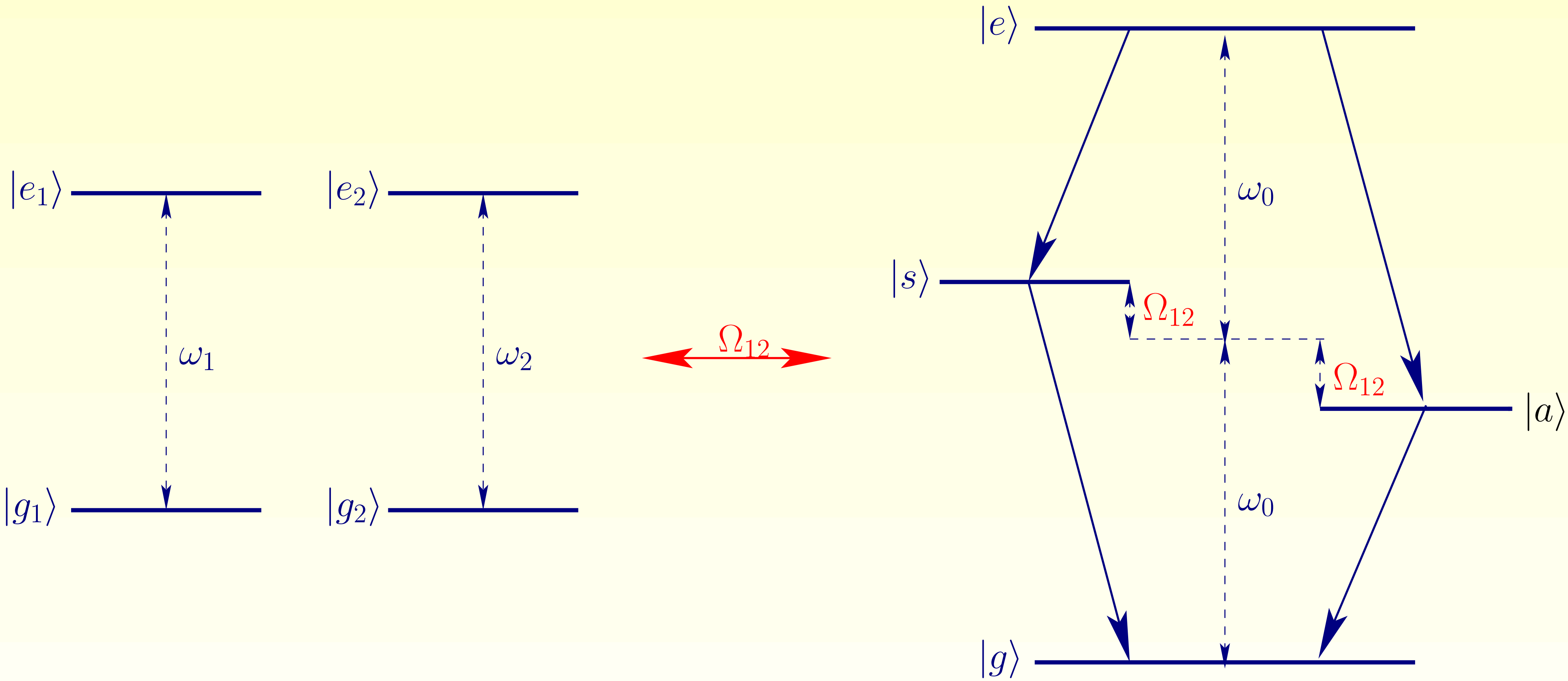
$$\Omega_{ij} \approx \frac{3}{4}\frac{\sqrt{\Gamma_i \Gamma_j}}{(k_0 r_{ij})^3}\left[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij})^2\right]$$



Stany kolektywne dwóch atomów



Stany kolektywne dwóch atomów



Stany kolektywne dwóch atomów

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$

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$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle$$

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle \qquad |g\rangle = |1\rangle$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle \qquad |e\rangle = |2\rangle$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle \qquad |s\rangle = \frac{1}{\sqrt{2}}\big(|3\rangle + |4\rangle\big)$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\big(|4\rangle - |3\rangle\big)$$

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle \qquad |g\rangle = |1\rangle \qquad E_g = -\hbar\omega_0$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle \qquad |e\rangle = |2\rangle \qquad E_e = \hbar\omega_0$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle \qquad |s\rangle = \frac{1}{\sqrt{2}}\big(|3\rangle + |4\rangle\big) \qquad E_s = \hbar\Omega_{12}$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\big(|4\rangle - |3\rangle\big) \qquad E_a = -\hbar\Omega_{12}$$

3.3 Stany kolektywne

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$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\big(|4\rangle - |3\rangle\big) \qquad E_a = -\hbar\Omega_{12}$$

$$\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$$

$$\Delta = \frac{1}{2}(\omega_2 - \omega_1)$$

4 Ewolucja w stanach kolektywnych

4.1 Identyczne atomy ($\Delta = 0,\ \omega_1 = \omega_2$)

$$
\begin{aligned}
\dot{\rho}_{ee} =& -2\Gamma\left(N+1\right)\rho_{ee} + N\left[\left(\Gamma+\Gamma_{12}\right)\rho_{ss} + \left(\Gamma-\Gamma_{12}\right)\rho_{aa}\right] \\
&+ \Gamma_{12}|M|\rho_u \\
\dot{\rho}_{ss} =& \left(\Gamma+\Gamma_{12}\right)\left\{N - \left(3N+1\right)\rho_{ss} - N\rho_{aa} + \rho_{ee} - |M|\rho_u\right\} \\
\dot{\rho}_{aa} =& \left(\Gamma-\Gamma_{12}\right)\left\{N - \left(3N+1\right)\rho_{aa} - N\rho_{ss} + \rho_{ee} + |M|\rho_u\right\} \\
\dot{\rho}_{u} =& 2\Gamma_{12}|M| - \left(2N+1\right)\Gamma\rho_u - 2|M|\left[\left(\Gamma+2\Gamma_{12}\right)\rho_{ss}\right. \\
&\left. - \left(\Gamma-2\Gamma_{12}\right)\rho_{aa}\right]
\end{aligned}
$$

4 Ewolucja w stanach kolektywnych

4.1 Identyczne atomy ($\Delta = 0,\ \omega_1 = \omega_2$)

$$\begin{aligned}
\dot{\rho}_{ee} =& -2\Gamma\left(N+1\right)\rho_{ee} + N\left[\left(\Gamma+\Gamma_{12}\right)\rho_{ss} + \left(\Gamma-\Gamma_{12}\right)\rho_{aa}\right] \\
&+ \Gamma_{12}|M|\rho_u \\
\dot{\rho}_{ss} =& \left(\Gamma+\Gamma_{12}\right)\left\{N - \left(3N+1\right)\rho_{ss} - N\rho_{aa} + \rho_{ee} - |M|\rho_u\right\} \\
\dot{\rho}_{aa} =& \left(\Gamma-\Gamma_{12}\right)\left\{N - \left(3N+1\right)\rho_{aa} - N\rho_{ss} + \rho_{ee} + |M|\rho_u\right\} \\
\dot{\rho}_u =& 2\Gamma_{12}|M| - \left(2N+1\right)\Gamma\rho_u - 2|M|\left[\left(\Gamma+2\Gamma_{12}\right)\rho_{ss}\right. \\
&\left. - \left(\Gamma-2\Gamma_{12}\right)\rho_{aa}\right]
\end{aligned}$$

$$\begin{aligned}
\rho_u =& \rho_{eg}\exp(-i\phi_s) + \rho_{ge}\exp(i\phi_s) \\
M =& |M|\exp\left(i\phi_s\right)
\end{aligned}$$

4.1.1 Rozwiązania stacjonarne ($\Gamma_{12} \neq \Gamma$)

$$\rho_{ee} = \frac{N^2\left[(2N+1)^2 - 4|M|^2\right] + |M|^2\gamma_{12}^2}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]}$$

$$\rho_{ss} = \frac{N(N+1)\left[(2N+1)^2 - 4|M|^2\right] + |M|^2\gamma_{12}(\gamma_{12} - 2)}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]}$$

$$\rho_{aa} = \frac{N(N+1)\left[(2N+1)^2 - 4|M|^2\right] + |M|^2\gamma_{12}(\gamma_{12} + 2)}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]}$$

$$\rho_{u} = \frac{2\,(2N+1)\,|M|\gamma_{12}}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]}$$

$$\gamma_{12} = \Gamma_{12}/\Gamma$$

4.1.2 Maksymalny „squeezing” ($|M| = \sqrt{N(N+1)}$)

$$\rho_{ee} = \frac{N^2 + N(N+1)\,\gamma_{12}^2}{1 + 4\,N(N+1)(1+\gamma_{12}^2)}$$

$$\rho_{ss} = \frac{N(N+1)(1-\gamma_{12})^2}{1 + 4\,N(N+1)(1+\gamma_{12}^2)}$$

$$\rho_{aa} = \frac{N(N+1)(1+\gamma_{12})^2}{1 + 4\,N(N+1)(1+\gamma_{12}^2)}\,,$$

$$\rho_{u} = \frac{2\,\sqrt{N(N+1)}\,(2N+1)\,\gamma_{12}}{1 + 4\,N(N+1)(1+\gamma_{12}^2)}$$

4.2 Nieidentyczne atomy ($\Delta \gg \Gamma,\ \Gamma_1 = \Gamma_2 = \Gamma$)

Przybliżenie sekularne:

$$\dot{\rho}_{ee} = -\,2\Gamma\,(N+1)\,\rho_{ee} + N\Gamma\,(\rho_{ss} + \rho_{aa}) + \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_{ss} = \Gamma\left[N - (3N+1)\,\rho_{ss} - N\rho_{aa} + \rho_{ee}\right] - \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_{aa} = \Gamma\left[N - (3N+1)\,\rho_{aa} - N\rho_{ss} + \rho_{ee}\right] - \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_u = 2\Gamma_{12}|M| - (2N+1)\,\Gamma\rho_u - 4\Gamma_{12}|M|\,(\rho_{ss} + \rho_{aa})$$

4.2 Nieidentyczne atomy ($\Delta \gg \Gamma,\ \Gamma_1 = \Gamma_2 = \Gamma$)

Przybliżenie sekularne:

$$\dot{\rho}_{ee} = -2\Gamma\left(N+1\right)\rho_{ee} + N\Gamma\left(\rho_{ss}+\rho_{aa}\right) + \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_{ss} = \Gamma\left[N - \left(3N+1\right)\rho_{ss} - N\rho_{aa} + \rho_{ee}\right] - \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_{aa} = \Gamma\left[N - \left(3N+1\right)\rho_{aa} - N\rho_{ss} + \rho_{ee}\right] - \Gamma_{12}|M|\rho_u$$

$$\dot{\rho}_u = 2\Gamma_{12}|M| - \left(2N+1\right)\Gamma\rho_u - 4\Gamma_{12}|M|\left(\rho_{ss}+\rho_{aa}\right)$$

$$\rho_u = \rho_{eg}\exp(-i\phi_s) + \rho_{ge}\exp(i\phi_s)$$

$$M = |M|\exp\left(i\phi_s\right)$$

4.2.1 Rozwiązania stacjonarne

$$\rho_{ee} = \frac{1}{4}\left\{\frac{(2N-1)}{2N+1} + \frac{1}{\left[(2N+1)^2 - 4|M|^2\,\gamma_{12}^2\right]}\right\}$$

$$\rho_{ss} = \rho_{aa} = \frac{1}{4}\left\{1 - \frac{1}{\left[(2N+1)^2 - 4|M|^2\,\gamma_{12}^2\right]}\right\}$$

$$\rho_u = \frac{2|M|\,\gamma_{12}}{(2N+1)\left[(2N+1)^2 - 4|M|^2\,\gamma_{12}^2\right]}$$

4.2.2 Maksymalny „squeezing” ($|M| = \sqrt{N(N+1)}$)

$$\rho_{ee} = \frac{1}{4}\left\{\frac{2N-1}{2N+1} + \frac{1}{1+4N(N+1)(1-\gamma_{12}^2)}\right\}$$

$$\rho_{ss} = \rho_{aa} = \frac{N(N+1)(1-\gamma_{12}^2)}{1+4N(N+1)(1-\gamma_{12}^2)}$$

$$\rho_u = \frac{2\sqrt{N(N+1)}\,\gamma_{12}}{(2N+1)[1+4N(N+1)(1-\gamma_{12}^2)]}$$

5 Stacjonarne splątanie dwóch atomów

5.1 Identyczne atomy ($\Delta = 0, \ \omega_1 = \omega_2$)

$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_1(t)\right\}$$

5 Stacjonarne splątanie dwóch atomów

5.1 Identyczne atomy ($\Delta = 0,\ \omega_1 = \omega_2$)

$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_1(t)\right\}$$

$$\begin{aligned}
\mathcal{C}_1 =& |\rho_u| - (\rho_{ss} + \rho_{aa}) \\
=& 2\left\{ \frac{(2N+1)|M|\,\gamma_{12} - |M|^2\gamma_{12}^2}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]} \right. \\
& \left. - \frac{N(N+1)\left[(2N+1)^2 - 4|M|^2\right]}{(2N+1)^4 - 4|M|^2\left[(2N+1)^2 - \gamma_{12}^2\right]} \right\}
\end{aligned}$$

5.1.1 Maksymalny „squeezing” ($|M| = \sqrt{N(N+1)}$)

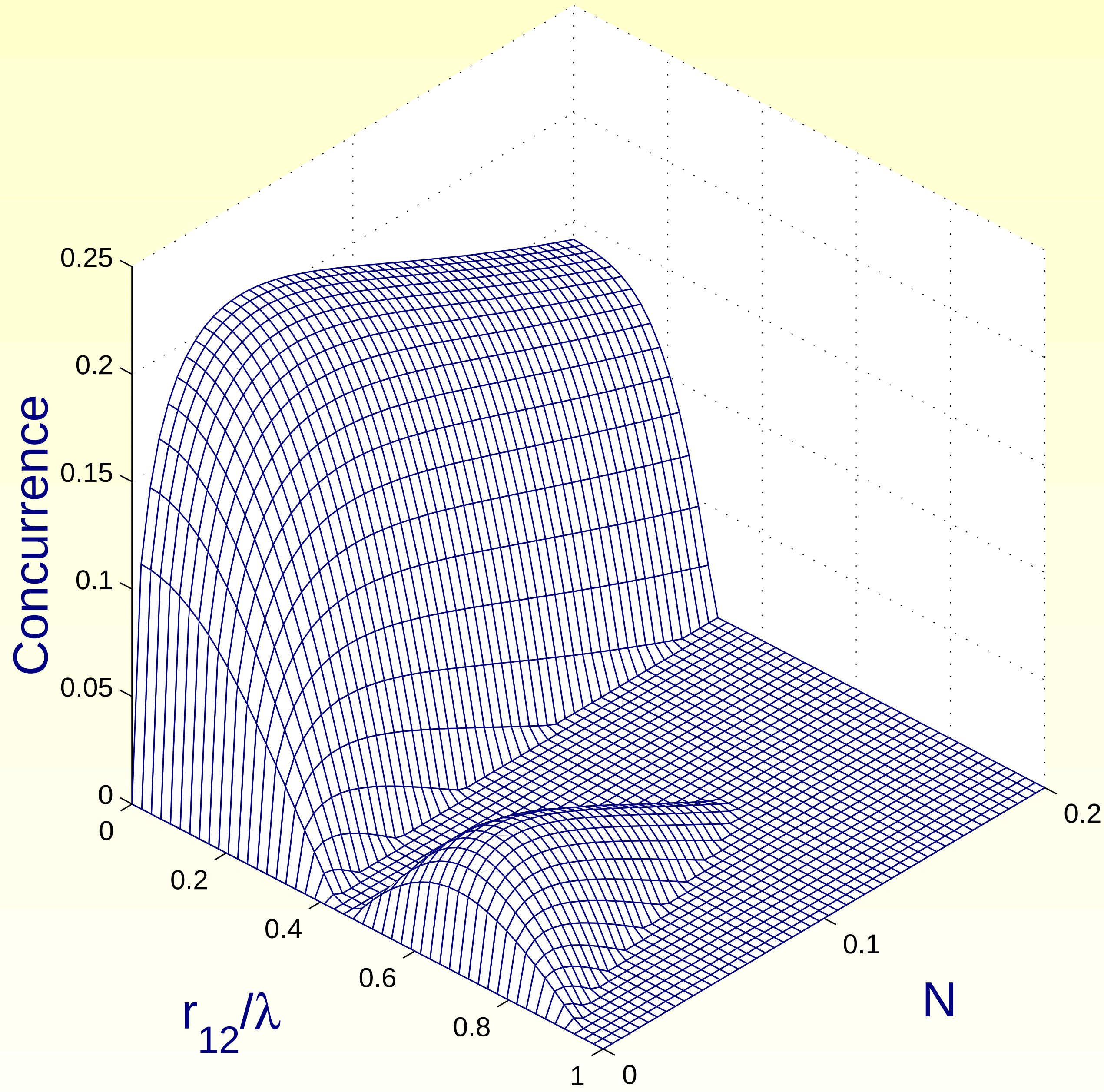
$$\mathcal{C}_1 = 2\sqrt{N(N+1)}\,\frac{(2N+1)\,\gamma_{12} - \sqrt{N(N+1)}\,(1+\gamma_{12}^2)}{1+4\,N(N+1)(1+\gamma_{12}^2)}$$

5.1.1 Maksymalny „squeezing” ($|M| = \sqrt{N(N+1)}$)

$$\mathcal{C}_1 = 2\sqrt{N(N+1)}\,\frac{(2N+1)\,\gamma_{12} - \sqrt{N(N+1)}\,(1+\gamma_{12}^2)}{1 + 4\,N(N+1)(1+\gamma_{12}^2)}$$

$$\gamma_{12} = \gamma_{21} = \frac{3}{2}\left\{\left[1 - (\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{12})^2\right]\frac{\sin(k_0 r_{12})}{k_0 r_{12}} + \left[1 - 3\,(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{12})^2\right]\left[\frac{\cos(k_0 r_{12})}{(k_0 r_{12})^2} - \frac{\sin(k_0 r_{12})}{(k_0 r_{12})^3}\right]\right\}$$

$\gamma_{12} \to 1$ dla $r_{12} \ll \lambda$



Rys. 1: $\mathcal{C}_1$ jako funkcja r_{12}/λ i N dla $|M| = \sqrt{N(N+1)}$

5.2 Nieidentyczne atomy ($\Delta \gg \Gamma,\ \Gamma_1 = \Gamma_2 = \Gamma$)

$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_1(t)\right\}$$

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$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_1(t)\right\}$$

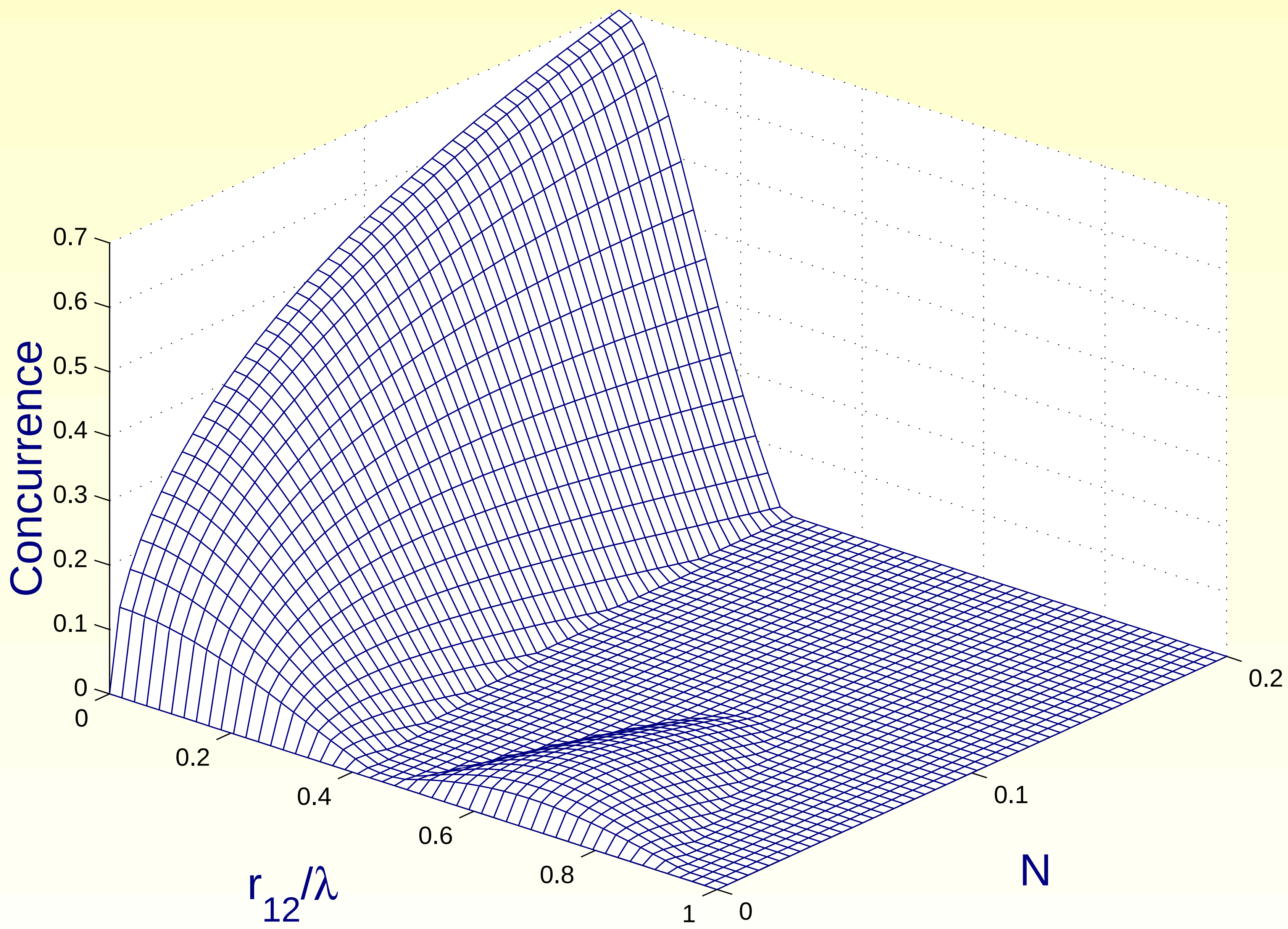
$$\begin{aligned}
\mathcal{C}_1 =& |\rho_u| - (\rho_{ss} + \rho_{aa}) \\
=& \frac{2|M||\gamma_{12}|}{(2N+1)\left[(2N+1)^2 - 4|M|^2\,\gamma_{12}^2\right]} \\
& - \frac{1}{2}\left\{1 - \frac{1}{(2N+1)^2 - 4|M|^2\,\gamma_{12}^2}\right\}
\end{aligned}$$

5.2.1 Maksymalny „squeezing” ($|M| = \sqrt{N(N+1)}$)

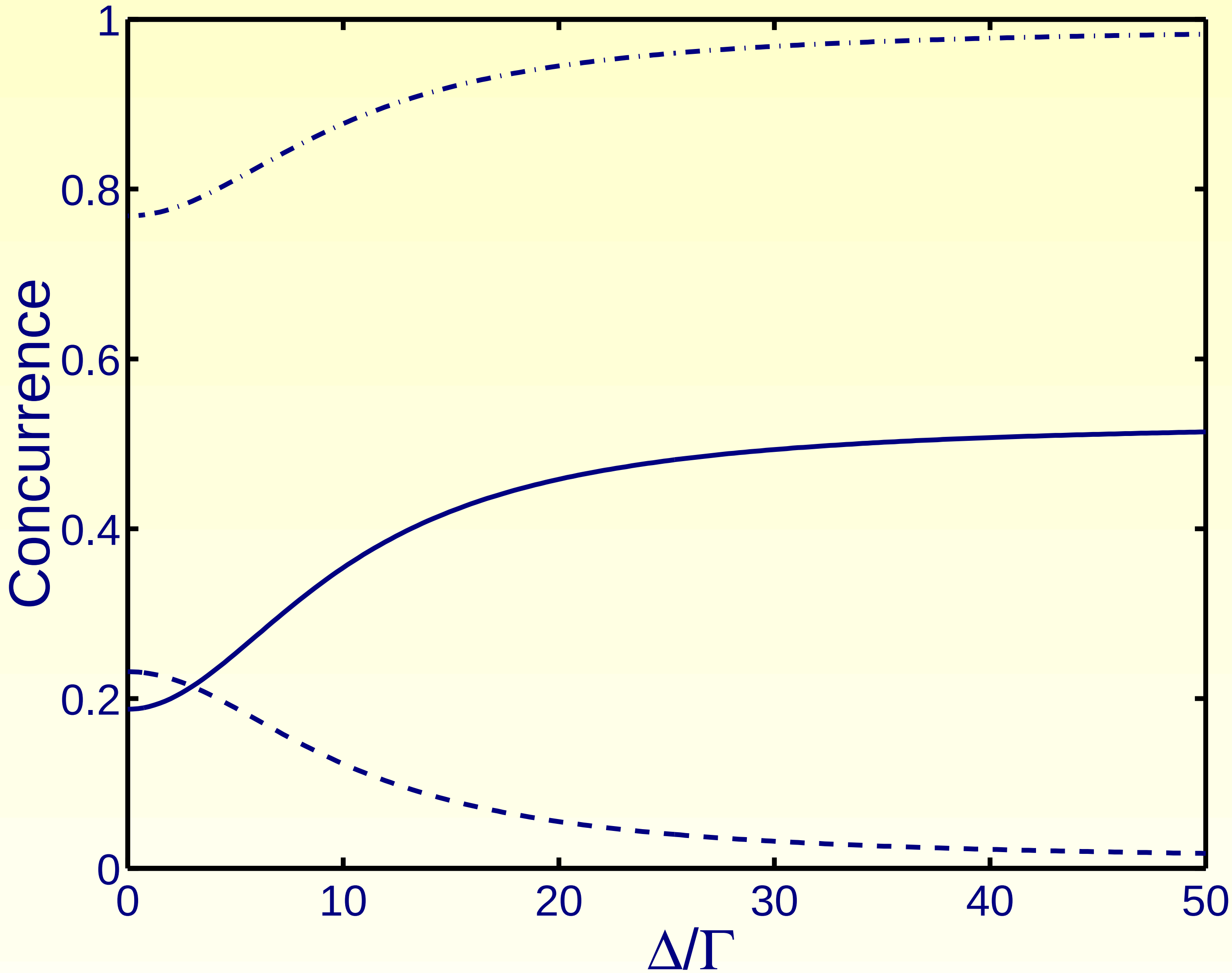
$$\mathcal{C}_1 = 2\sqrt{N(N+1)}\,\frac{\gamma_{12} - (2N+1)\sqrt{N(N+1)}\,(1-\gamma_{12}^2)}{(2N+1)[1+4N(N+1)(1-\gamma_{12}^2)]}$$

$$r_{12}/\lambda \ll 1 \Rightarrow \gamma_{12} \approx 1$$

$$\mathcal{C}_1 = \frac{2\sqrt{N(N+1)}}{2N+1}$$



Rys. 2: $\mathcal{C}_1$ jako funkcja r_{12}/λ i N dla $|M| = \sqrt{N(N+1)}$



Rys. 3: $\mathcal{C}_1$ (ciągła), populacje $\rho_{ss} + \rho_{aa}$ (kreski), i populacje $\rho_{gg} + \rho_{ee}$ (kreski-kropki) jako funkcja Δ dla $r_{12}/\lambda = 0.05$ i $N = 0.1$ przy $|M| = \sqrt{N(N+1)}$

6 Jak blisko stanów Bella?

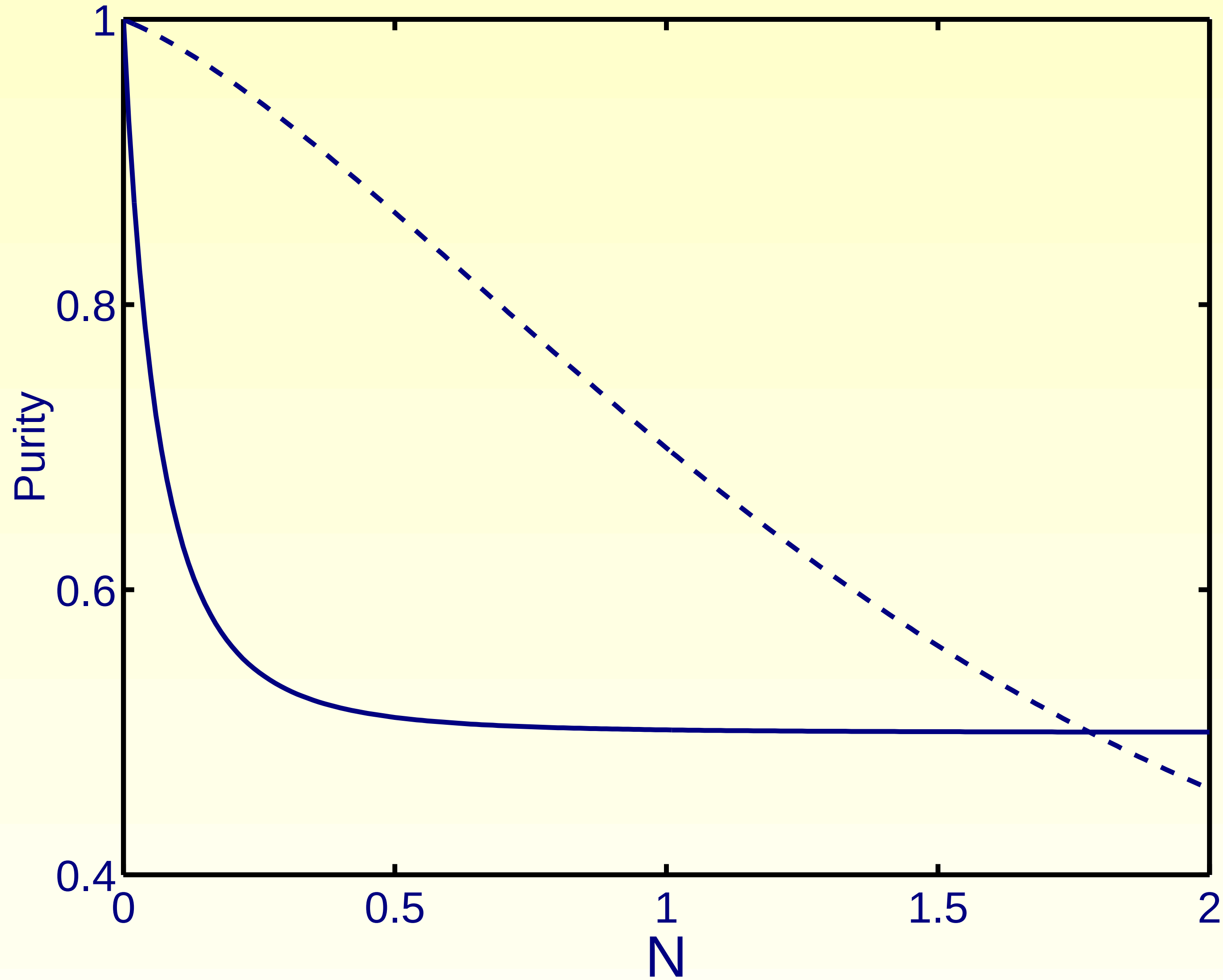
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right)$$

$$|\Phi^-\rangle = -\frac{1}{\sqrt{2}}\left(|g\rangle - |e\rangle\right)$$

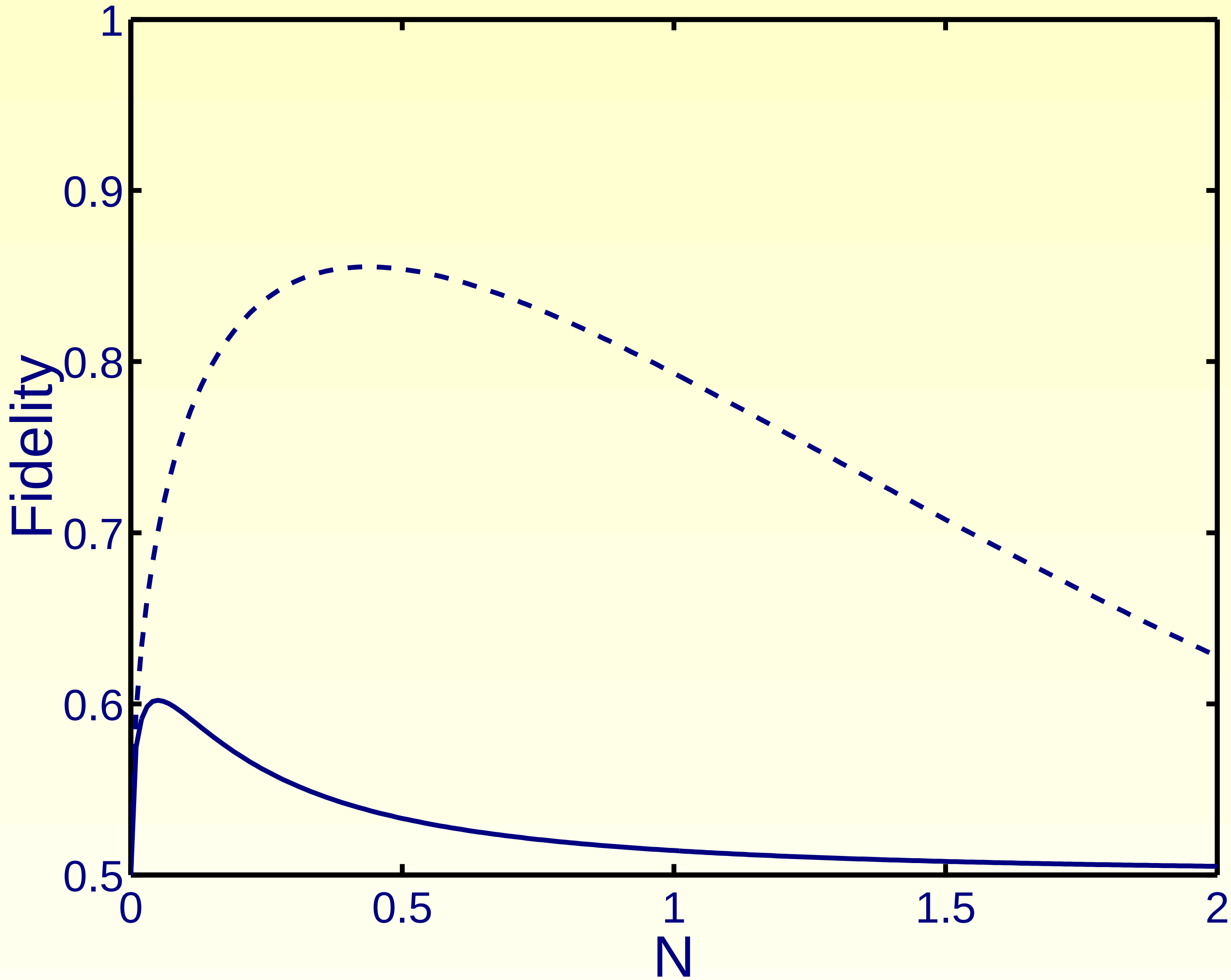
6 Jak blisko stanów Bella?

$$
\begin{aligned}
|\Phi^+\rangle &= \;\; \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right) \\
|\Phi^-\rangle &= -\frac{1}{\sqrt{2}}\left(|g\rangle - |e\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
F_+ =& \langle\Phi^+|\rho|\Phi^+\rangle = \frac{1}{2}\left(\rho_{gg} + \rho_{ee} + \rho_u\right) \\
F_- =& \langle\Phi^-|\rho|\Phi^-\rangle = \frac{1}{2}\left(\rho_{gg} + \rho_{ee} - \rho_u\right)
\end{aligned}
$$



Rys. 4: Purity $P = \mathrm{Tr}(\rho^2)$ jako funkcja N dla $r_{12}/\lambda = 0.05$ przy $|M| = \sqrt{N(N+1)}$: identyczne atomy (ciągła), nieidentyczne atomy (kreski)



Rys. 5: Fidelity F_+ jako funcja N dla $r_{12}/\lambda = 0.05$ przy $|M| = \sqrt{N(N+1)}$: identyczne atomy (ciągla), nieidentyczne atomy (kreski)

7 Nasze prace

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  in Modern Nonlinear Optics, Part I, ed. M. Evans (Wiley, New York, 2001) vol 119 of Advances in Chemical Physics, pp. 215-266
- Z. Ficek, R. Tanaś
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  Physics Reports **372**, 369 (2002)
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  Entanglement induced by spontaneous emission in spatially extended two-atom systems
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  J. Opt. B **6**, S610 (2004)

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