Seminarium

z informatyki kwantowej

Splątanie dwóch atomów

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28 maja 2004

Spis treści

1 Miary splątania

1.1 Concurrence

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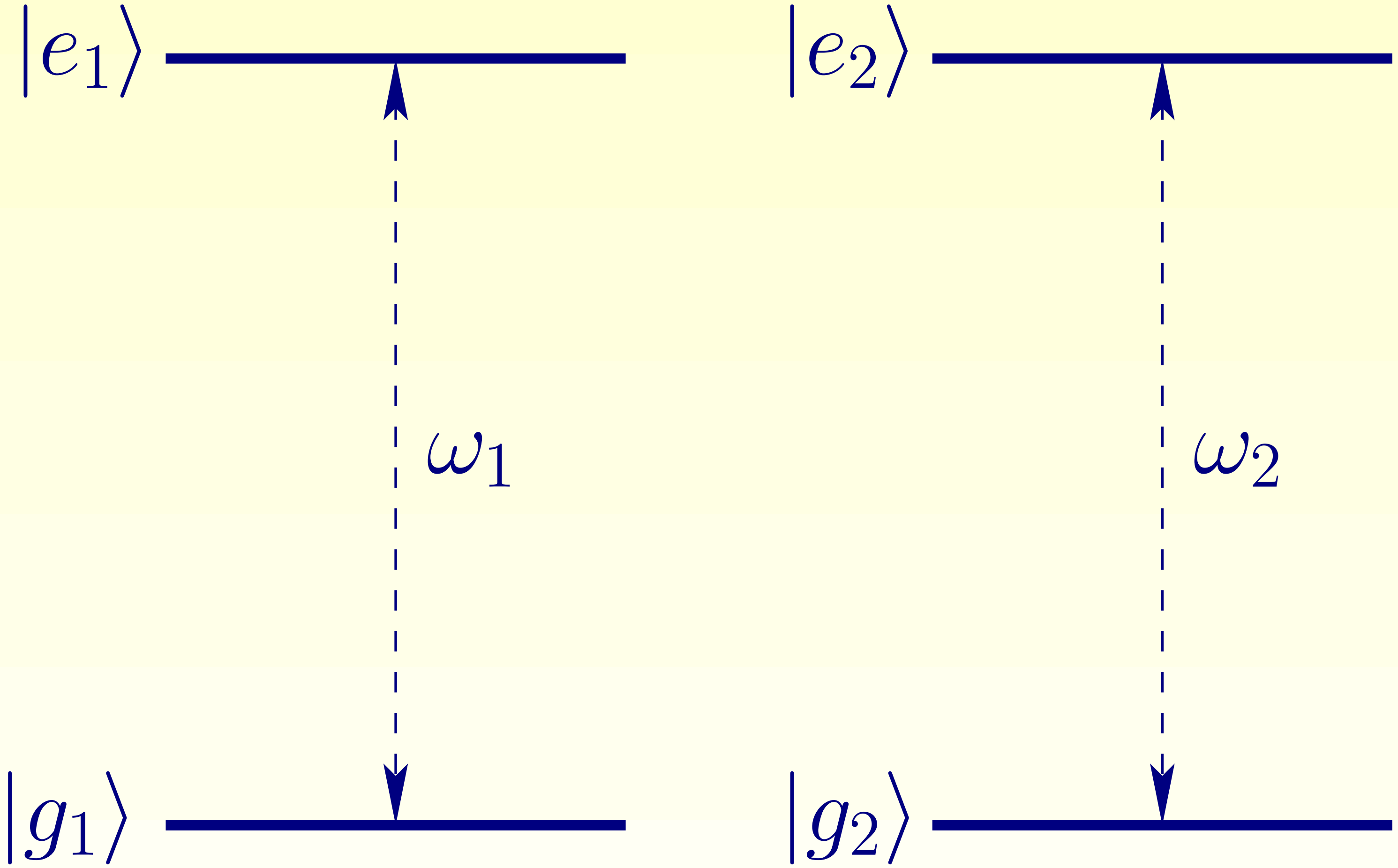
2 Macierz gęstości



Stany kwantowe dwóch atomów

2.1 Baza obliczeniowa

$$
\begin{aligned}
|1\rangle &= |g_1\rangle \otimes |g_2\rangle \\
|2\rangle &= |e_1\rangle \otimes |e_2\rangle \\
|3\rangle &= |g_1\rangle \otimes |e_2\rangle \\
|4\rangle &= |e_1\rangle \otimes |g_2\rangle
\end{aligned}
$$

2.1 Baza obliczeniowa

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle$$

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & 0 & 0 \\ \rho_{21} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{34} \\ 0 & 0 & \rho_{43} & \rho_{44} \end{pmatrix}$$

2.1.1 Concurrence

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$$R = \rho\tilde{\rho}$$

$$\tilde{\rho} = \sigma_y \otimes \sigma_y\, \rho^*\, \sigma_y \otimes \sigma_y$$

$$\tilde{\rho} = \begin{pmatrix} \rho_{22} & \rho_{12} & 0 & 0 \\ \rho_{21} & \rho_{11} & 0 & 0 \\ 0 & 0 & \rho_{44} & \rho_{34} \\ 0 & 0 & \rho_{43} & \rho_{33} \end{pmatrix}$$

$$\left\{\sqrt{\lambda_i}\right\} = \{\sqrt{\rho_{11}\rho_{22}} - |\rho_{12}|, \sqrt{\rho_{11}\rho_{22}} + |\rho_{12}|, \sqrt{\rho_{33}\rho_{44}} - |\rho_{34}|, \sqrt{\rho_{33}\rho_{44}} + |\rho_{34}|\,\}$$

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$$\mathcal{C} = \max\{0, \mathcal{C}_1, \mathcal{C}_2\}$$

$$\mathcal{C}_1 = 2\left(|\rho_{12}| - \sqrt{\rho_{33}\rho_{44}}\right)$$

$$\mathcal{C}_2 = 2\left(|\rho_{34}| - \sqrt{\rho_{11}\rho_{22}}\right)$$

2.1.2 Negativity

$$\rho^{\mathrm{T}_1} = \begin{pmatrix} \rho_{11} & \rho_{43} & 0 & 0 \\ \rho_{34} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{21} \\ 0 & 0 & \rho_{12} & \rho_{44} \end{pmatrix}$$

2.1.2 Negativity

$$\rho^{\mathrm{T}_1} = \begin{pmatrix} \rho_{11} & \rho_{43} & 0 & 0 \\ \rho_{34} & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & \rho_{21} \\ 0 & 0 & \rho_{12} & \rho_{44} \end{pmatrix}$$

$$\{\nu_i\} = \left\{ \frac{1}{2}\left(\rho_{11} + \rho_{22} \pm \sqrt{(\rho_{11}+\rho_{22})^2 + 4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right)}\right), \right.$$
$$\left. \frac{1}{2}\left(\rho_{33} + \rho_{44} \pm \sqrt{(\rho_{33}+\rho_{44})^2 + 4\left(|\rho_{12}|^2 - \rho_{33}\rho_{44}\right)}\right) \right\}$$

$$\mathcal{N} = \max\Big\{0,\ \sqrt{4\left(|\rho_{12}|^2 - \rho_{33}\rho_{44}\right) + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44}),\\ \sqrt{4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right) + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}$$

$$\mathcal{N} = \max\Big\{0,\ \sqrt{4\,(|\rho_{12}|^2 - \rho_{33}\rho_{44}) + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44}),$$

$$\sqrt{4\,(|\rho_{34}|^2 - \rho_{11}\rho_{22}) + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}$$

$$= \max\Big\{0, \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44})\ ,$$

$$\sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}$$

$$
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& \sqrt{4\left(|\rho_{34}|^2 - \rho_{11}\rho_{22}\right) + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\} \\
= \max\Big\{0, & \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{33}+\rho_{44})^2} - (\rho_{33}+\rho_{44})\ , \\
& \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{11}+\rho_{22})^2} - (\rho_{11}+\rho_{22})\Big\}
\end{aligned}
$$

$$
\mathcal{C}_1^+ = 2\left(|\rho_{12}| + \sqrt{\rho_{33}\rho_{44}}\,\right)
$$

$$
\mathcal{C}_2^+ = 2\left(|\rho_{34}| + \sqrt{\rho_{11}\rho_{22}}\,\right)
$$

2.2 Baza Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |e_2\rangle + |g_1\rangle \otimes |g_2\rangle\right) = |1'\rangle \ = \frac{1}{\sqrt{2}}\left(|2\rangle + |1\rangle\right)$$

2.2 Baza Bella

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |e_2\rangle + |g_1\rangle \otimes |g_2\rangle\right) = |1'\rangle \; = \frac{1}{\sqrt{2}}\left(|2\rangle + |1\rangle\right)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |e_2\rangle - |g_1\rangle \otimes |g_2\rangle\right) = |2'\rangle \; = \frac{1}{\sqrt{2}}\left(|2\rangle - |1\rangle\right)$$

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$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |g_2\rangle + |g_1\rangle \otimes |e_2\rangle\right) = |3'\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle + |3\rangle\right)$$

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$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\big(|e_1\rangle \otimes |e_2\rangle + |g_1\rangle \otimes |g_2\rangle\big) = |1'\rangle \; = \frac{1}{\sqrt{2}}\big(|2\rangle + |1\rangle\big)$$

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2.2.1 Transformacja do bazy Bella

$$\rho' = U\rho U^{+}$$

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$$\rho' = U\rho U^{+}$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

2.2.2 Macierz gęstości w bazie Bella

$$\begin{aligned}
\rho_{1'1'} &= \frac{1}{2}\left[\rho_{11} + \rho_{22} + (\rho_{12} + \rho_{21})\right] \\
\rho_{2'2'} &= \frac{1}{2}\left[\rho_{11} + \rho_{22} - (\rho_{12} + \rho_{21})\right] \\
\rho_{1'2'} &= -\frac{1}{2}\left[\rho_{11} - \rho_{22} + (\rho_{12} - \rho_{21})\right] \\
\rho_{2'1'} &= -\frac{1}{2}\left[\rho_{11} - \rho_{22} - (\rho_{12} - \rho_{21})\right]
\end{aligned}$$

$$\begin{aligned}
\rho_{3'3'} &= \frac{1}{2}\left[\rho_{33} + \rho_{44} + (\rho_{34} + \rho_{43})\right] \\
\rho_{4'4'} &= \frac{1}{2}\left[\rho_{33} + \rho_{44} - (\rho_{34} + \rho_{43})\right] \\
\rho_{3'4'} &= -\frac{1}{2}\left[\rho_{33} - \rho_{44} + (\rho_{34} - \rho_{43})\right] \\
\rho_{4'3'} &= -\frac{1}{2}\left[\rho_{33} - \rho_{44} - (\rho_{34} - \rho_{43})\right]
\end{aligned}$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2} - \sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$-\sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2 = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2}$$
$$-\sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

2.2.3 Concurrence

$$\mathcal{C}_1 = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$-\sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2 = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2}$$
$$-\sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

$$\mathcal{C} = \max\{0,\, \mathcal{C}_1,\, \mathcal{C}_2\}$$

2.2.4 Negativity

$$\mathcal{N} = \max\left\{0, \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{3'3'} + \rho_{4'4'})^2} - (\rho_{3'3'} + \rho_{4'4'})\,, \right.$$
$$\left. \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{1'1'} + \rho_{2'2'})^2} - (\rho_{1'1'} + \rho_{2'2'})\right\}$$

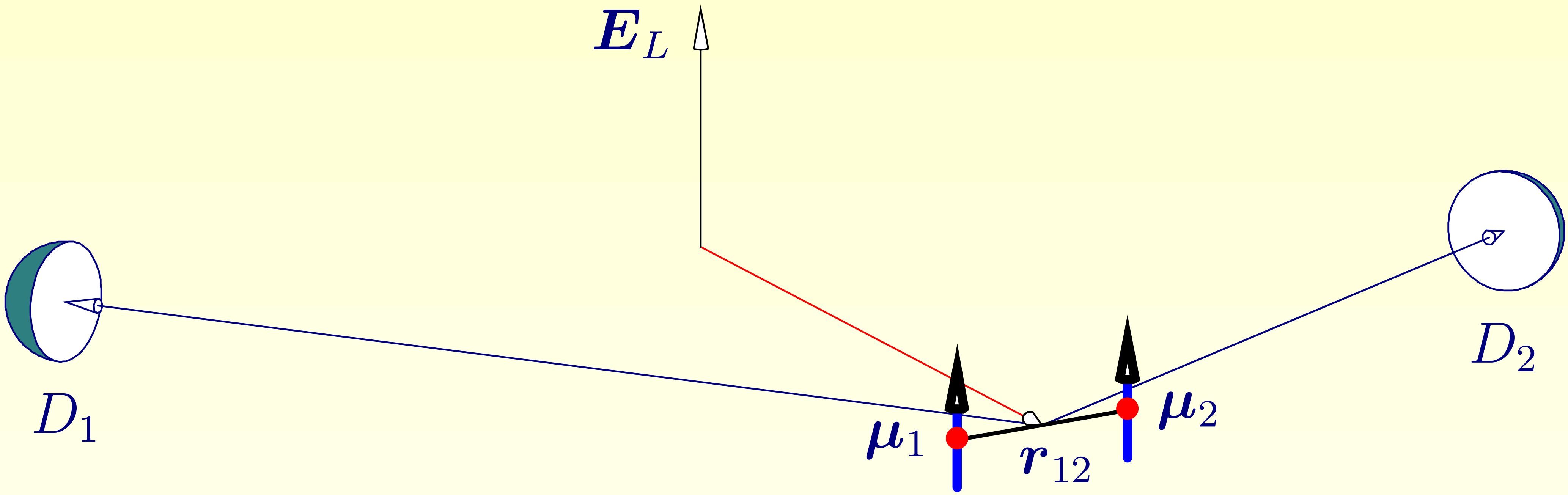
2.2.4 Negativity

$$\mathcal{N} = \max\left\{0, \sqrt{\mathcal{C}_1\,\mathcal{C}_1^+ + (\rho_{3'3'} + \rho_{4'4'})^2} - (\rho_{3'3'} + \rho_{4'4'})\,, \right.$$
$$\left. \sqrt{\mathcal{C}_2\,\mathcal{C}_2^+ + (\rho_{1'1'} + \rho_{2'2'})^2} - (\rho_{1'1'} + \rho_{2'2'})\right\}$$

$$\mathcal{C}_1^+ = \sqrt{(\rho_{1'1'} - \rho_{2'2'})^2 - (\rho_{1'2'} - \rho_{2'1'})^2}$$
$$+\sqrt{(\rho_{3'3'} + \rho_{4'4'})^2 - (\rho_{3'4'} + \rho_{4'3'})^2}$$

$$\mathcal{C}_2^+ = \sqrt{(\rho_{3'3'} - \rho_{4'4'})^2 - (\rho_{3'4'} - \rho_{4'3'})^2}$$
$$+\sqrt{(\rho_{1'1'} + \rho_{2'2'})^2 - (\rho_{1'2'} + \rho_{2'1'})^2}$$

3 Ewolucja dwóch atomów



Geometria układu: $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$ — momenty dipolowe przejść atomowych, $\boldsymbol{r}_{12}$ — odległość między atomami, $\boldsymbol{E}_L$ — pole laserowe, D_1, D_2 — detektory

3.1 Równanie „master”

Dwa atomy w próżni

$$\frac{\partial \hat{\rho}}{\partial t} = -\, i \sum_{i=1}^{2} \omega_i \left[S_i^z, \hat{\rho}\right] - i \sum_{i \neq j}^{2} \Omega_{ij} \left[S_i^+ S_j^-, \hat{\rho}\right]$$

$$-\frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} \left(\hat{\rho} S_i^+ S_j^- + S_i^+ S_j^- \hat{\rho} - 2 S_j^- \hat{\rho} S_i^+\right)$$

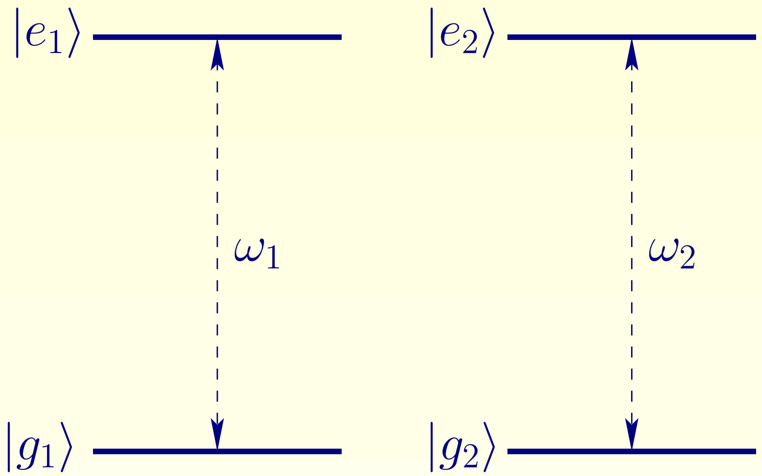
3.2 Parametry kolektywne

$$\Gamma_{ij} = \Gamma_{ji} = \frac{3}{2}\Gamma \left\{ \left[1 - \left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \frac{\sin\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1 - 3\left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \left[\frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2} - \frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right] \right\}$$

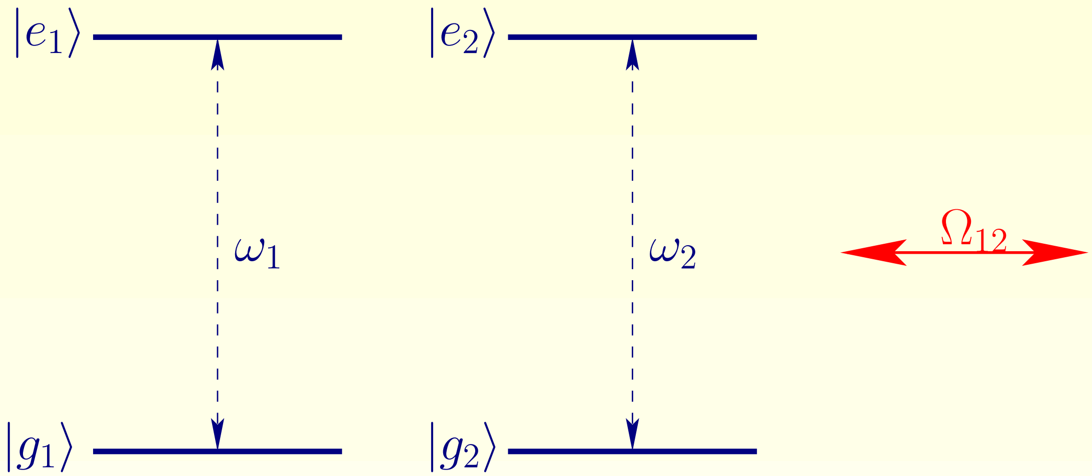
3.2 Parametry kolektywne

$$\Gamma_{ij} = \Gamma_{ji} = \frac{3}{2}\Gamma \left\{ \left[1 - \left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \frac{\sin\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1 - 3\left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \left[\frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2} - \frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right] \right\}$$

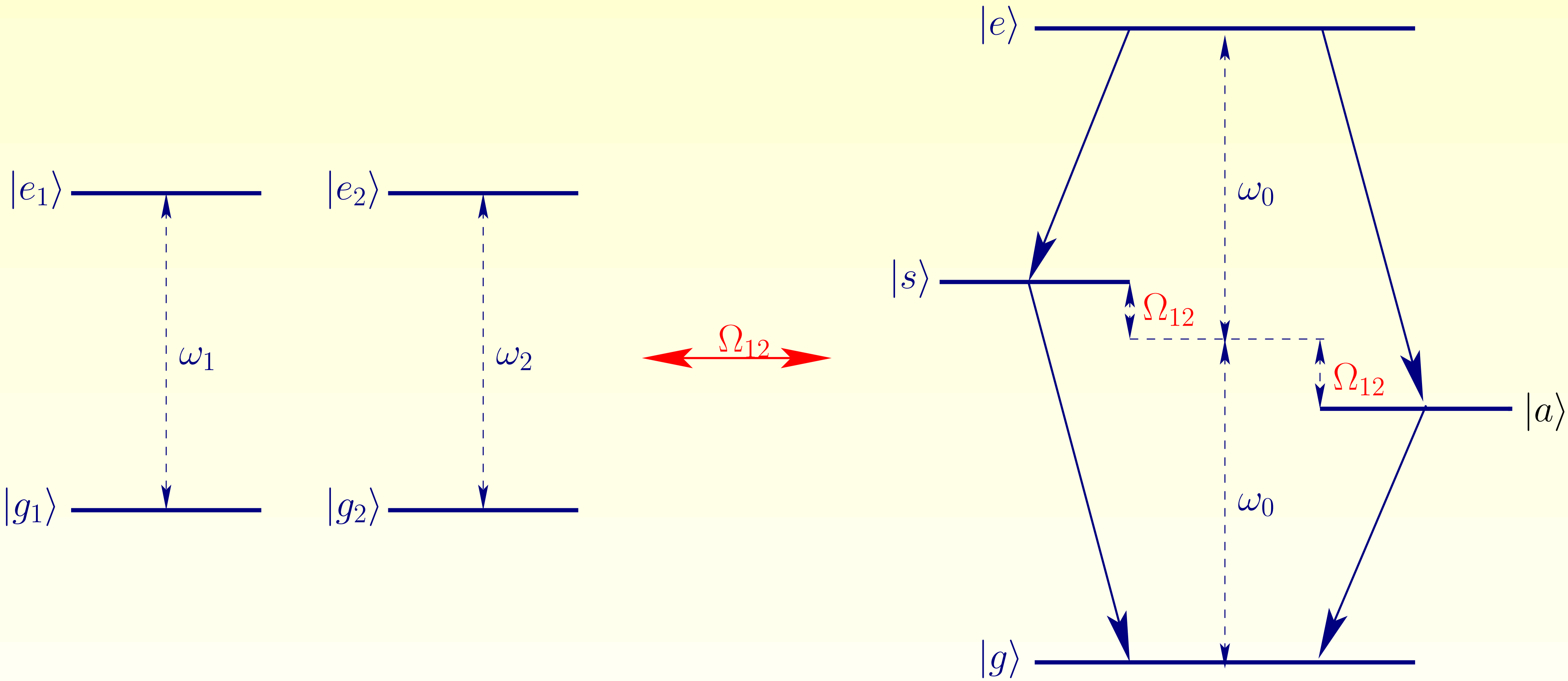
$$\Omega_{ij} = \frac{3}{4}\Gamma \left\{ - \left[1 - \left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \frac{\cos\left(k_0 r_{ij}\right)}{k_0 r_{ij}} + \left[1 - 3\left(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{ij}\right)^2\right] \left[\frac{\sin\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^2} + \frac{\cos\left(k_0 r_{ij}\right)}{\left(k_0 r_{ij}\right)^3}\right] \right\}$$



Stany kolektywne dwóch atomów



Stany kolektywne dwóch atomów



Stany kolektywne dwóch atomów

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle$$

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle \qquad |g\rangle = |1\rangle$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle \qquad |e\rangle = |2\rangle$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle \qquad |s\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle + |3\rangle\right)$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle - |3\rangle\right)$$

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle \qquad |g\rangle = |1\rangle \qquad E_g = -\hbar\omega_0$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle \qquad |e\rangle = |2\rangle \qquad E_e = \hbar\omega_0$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle \qquad |s\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle + |3\rangle\right) \qquad E_s = \hbar\Omega_{12}$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\left(|4\rangle - |3\rangle\right) \qquad E_a = -\hbar\Omega_{12}$$

3.3 Stany kolektywne

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle \qquad |g\rangle = |1\rangle \qquad E_g = -\hbar\omega_0$$

$$|2\rangle = |e_1\rangle \otimes |e_2\rangle \qquad |e\rangle = |2\rangle \qquad E_e = \hbar\omega_0$$

$$|3\rangle = |g_1\rangle \otimes |e_2\rangle \qquad |s\rangle = \frac{1}{\sqrt{2}}\big(|4\rangle + |3\rangle\big) \quad E_s = \hbar\Omega_{12}$$

$$|4\rangle = |e_1\rangle \otimes |g_2\rangle \qquad |a\rangle = \frac{1}{\sqrt{2}}\big(|4\rangle - |3\rangle\big) \quad E_a = -\hbar\Omega_{12}$$

$$\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$$

$$\Delta = \frac{1}{2}(\omega_2 - \omega_1)$$

3.4 Ewolucja w stanach kolektywnych

$$\dot{\rho}_{ee} = -2\Gamma\rho_{ee}$$

$$\dot{\rho}_{eg} = -\left(\Gamma + 2i\omega_0\right)\rho_{eg}$$

$$\dot{\rho}_{ss} = -\left(\Gamma + \Gamma_{12}\right)\left(\rho_{ss} - \rho_{ee}\right) + i\Delta\left(\rho_{as} - \rho_{sa}\right)$$

$$\dot{\rho}_{aa} = -\left(\Gamma - \Gamma_{12}\right)\left(\rho_{aa} - \rho_{ee}\right) - i\Delta\left(\rho_{as} - \rho_{sa}\right)$$

$$\dot{\rho}_{as} = -\left(\Gamma + 2i\Omega_{12}\right)\rho_{as} + i\Delta\left(\rho_{ss} - \rho_{aa}\right)$$

3.4 Ewolucja w stanach kolektywnych

$$\dot{\rho}_{ee} = -\,2\Gamma\rho_{ee}$$

$$\dot{\rho}_{eg} = -\,(\Gamma + 2i\omega_0)\,\rho_{eg}$$

$$\dot{\rho}_{ss} = -\,(\Gamma + \Gamma_{12})\,(\rho_{ss} - \rho_{ee}) + i\Delta\,(\rho_{as} - \rho_{sa})$$

$$\dot{\rho}_{aa} = -\,(\Gamma - \Gamma_{12})\,(\rho_{aa} - \rho_{ee}) - i\Delta\,(\rho_{as} - \rho_{sa})$$

$$\dot{\rho}_{as} = -\,(\Gamma + 2i\Omega_{12})\,\rho_{as} + i\Delta\,(\rho_{ss} - \rho_{aa})$$

$$\rho_{ee}(t) = \rho_{ee}(0)\,\mathrm{e}^{-2\Gamma t}$$

$$\rho_{eg}(t) = \rho_{eg}(0)\,\mathrm{e}^{-(\Gamma+2i\omega_0)t}$$

3.4 Ewolucja w stanach kolektywnych

$$\dot{\rho}_{ee} = -\,2\Gamma\rho_{ee}$$

$$\dot{\rho}_{eg} = -\,(\Gamma + 2i\omega_0)\,\rho_{eg}$$

$$\dot{\rho}_{ss} = -\,(\Gamma + \Gamma_{12})\,(\rho_{ss} - \rho_{ee}) + i\Delta\,(\rho_{as} - \rho_{sa})$$

$$\dot{\rho}_{aa} = -\,(\Gamma - \Gamma_{12})\,(\rho_{aa} - \rho_{ee}) - i\Delta\,(\rho_{as} - \rho_{sa})$$

$$\dot{\rho}_{as} = -\,(\Gamma + 2i\Omega_{12})\,\rho_{as} + i\Delta\,(\rho_{ss} - \rho_{aa})$$

$$\rho_{ee}(t) = \rho_{ee}(0)\,\mathrm{e}^{-2\Gamma t}$$

$$\rho_{eg}(t) = \rho_{eg}(0)\,\mathrm{e}^{-(\Gamma + 2i\omega_0)t}$$

$$S^2\,(t) = 2[1 - \rho_{aa}\,(t)]$$

3.5 Identyczne atomy

$$\Delta = \frac{1}{2}(\omega_2 - \omega_1) = 0$$

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$$\rho_{ss}(t) = \rho_{ss}(0)\,\mathrm{e}^{-(\Gamma+\Gamma_{12})t} + \rho_{ee}(0)\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{aa}(t) = \rho_{aa}(0)\,\mathrm{e}^{-(\Gamma-\Gamma_{12})t} + \rho_{ee}(0)\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{as}(t) = \rho_{as}(0)\,\mathrm{e}^{-(\Gamma+2i\Omega_{12})t}$$

$$\rho_{ee}(t) = \rho_{ee}(0)\,\mathrm{e}^{-2\Gamma t}$$

$$\rho_{eg}(t) = \rho_{eg}(0)\,\mathrm{e}^{-(\Gamma+2i\omega_0)t}$$

$$\rho_{gg}(t) = 1 - \rho_{ee}(t) - \rho_{ss}(t) - \rho_{aa}(t)$$

4 Splątanie dwóch atomów

4.1 Concurrence

$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_2(t)\right\}$$

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$$\mathcal{C}(t) = \max\left\{0, \mathcal{C}_2(t)\right\}$$

$$\mathcal{C}_2(t) = \sqrt{\left[\rho_{ss}(t) - \rho_{aa}(t)\right]^2 - \left[\rho_{sa}(t) - \rho_{as}(t)\right]^2} \\ -2\sqrt{\rho_{ee}(t)\rho_{gg}(t)}$$

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$$\mathcal{C}_2^{+}(t) = \sqrt{\left[\rho_{ss}(t) - \rho_{aa}(t)\right]^2 - \left[\rho_{sa}(t) - \rho_{as}(t)\right]^2} \\ +2\sqrt{\rho_{ee}(t)\rho_{gg}(t)}$$

4.2 Negativity

$$\mathcal{N}(t) = \max\left\{0, \mathcal{N}_2(t)\right\}$$

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4.3 Jeden atom wzbudzony

$\rho_{44}(0) = 1$

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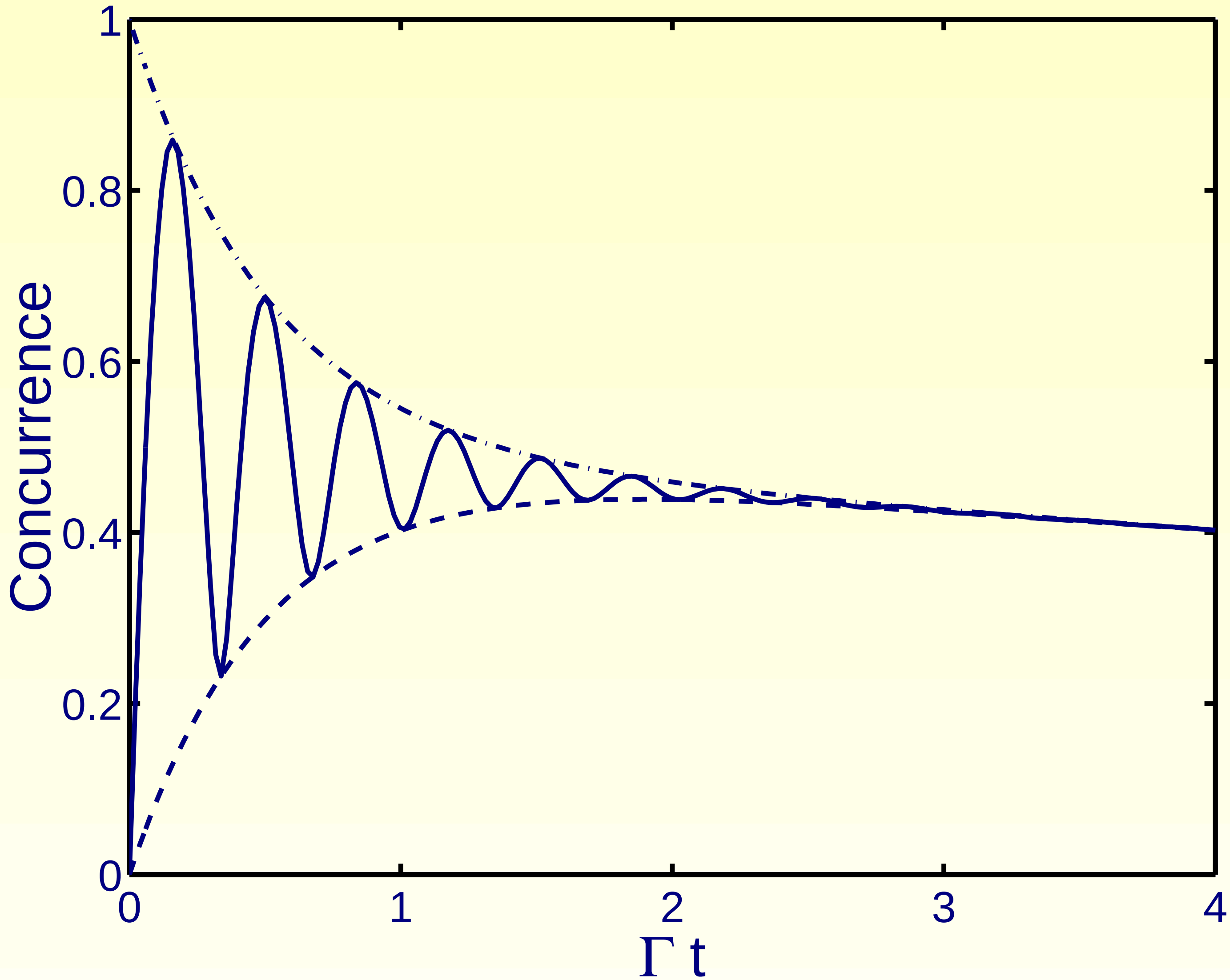
$\rho_{44}(0) = 1$

$\rho_{ss}(0) = \rho_{aa}(0) = \rho_{as}(0) = \rho_{sa}(0) = \frac{1}{2}$, $\rho_{ee}(0) = 0$

$$\mathcal{C}_2(t) = \frac{1}{2}\sqrt{\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-(\Gamma-\Gamma_{12})t}\right]^2 + \left[2\mathrm{e}^{-\Gamma t}\sin(2\Omega_{12}t)\right]^2}$$



Concurrence $\mathcal{C}(t)$ (ciągła), $\rho_{aa}(t) - \rho_{ss}(t)$ (kreski), $\rho_{aa}(t) + \rho_{ss}(t)$ (kreski-kropki);

$\rho_{44}(0) = 1$, $\hat{\boldsymbol{\mu}}, \perp \hat{\boldsymbol{r}}_{12}$, $r_{12} = \lambda/12$ ($\Gamma_{12} = 0.95\,\Gamma$, $2\Omega_{12} = 9.30\,\Gamma$)

$$\mathcal{N}_2(t) = \sqrt{\mathcal{C}_2^2(t) + \rho_{gg}^2(t)} - \rho_{gg}(t)$$

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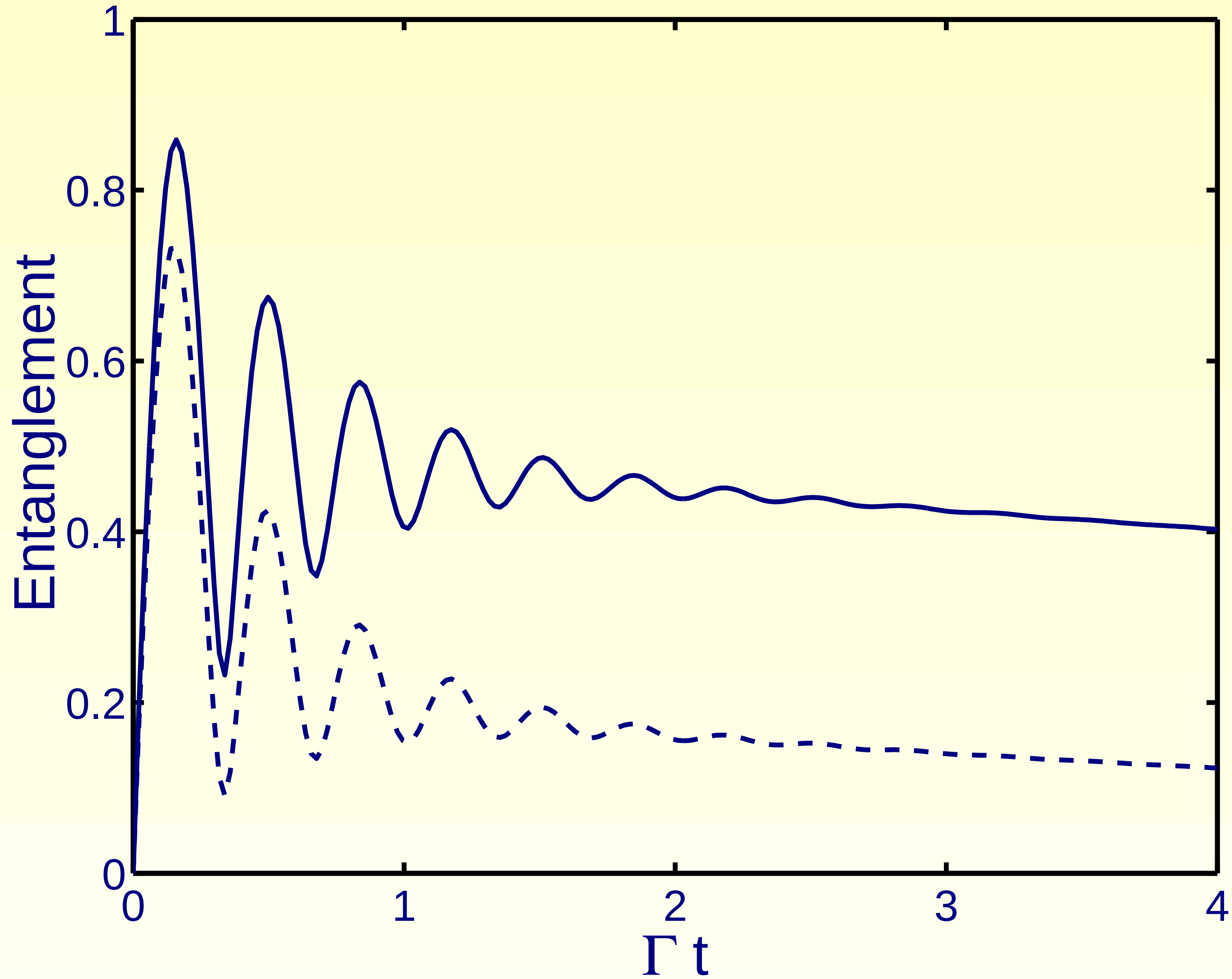
$$\rho_{gg}(t) = 1 - \frac{1}{2}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} + \mathrm{e}^{-(\Gamma-\Gamma_{12})t}\right]$$



Concurrence $\mathcal{C}(t)$ (ciągła) i negativity $\mathcal{N}(t)$ (kreski)

4.4 Dwa atomy wzbudzone

$\rho_{ee} = 1$

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$\rho_{ee} = 1$

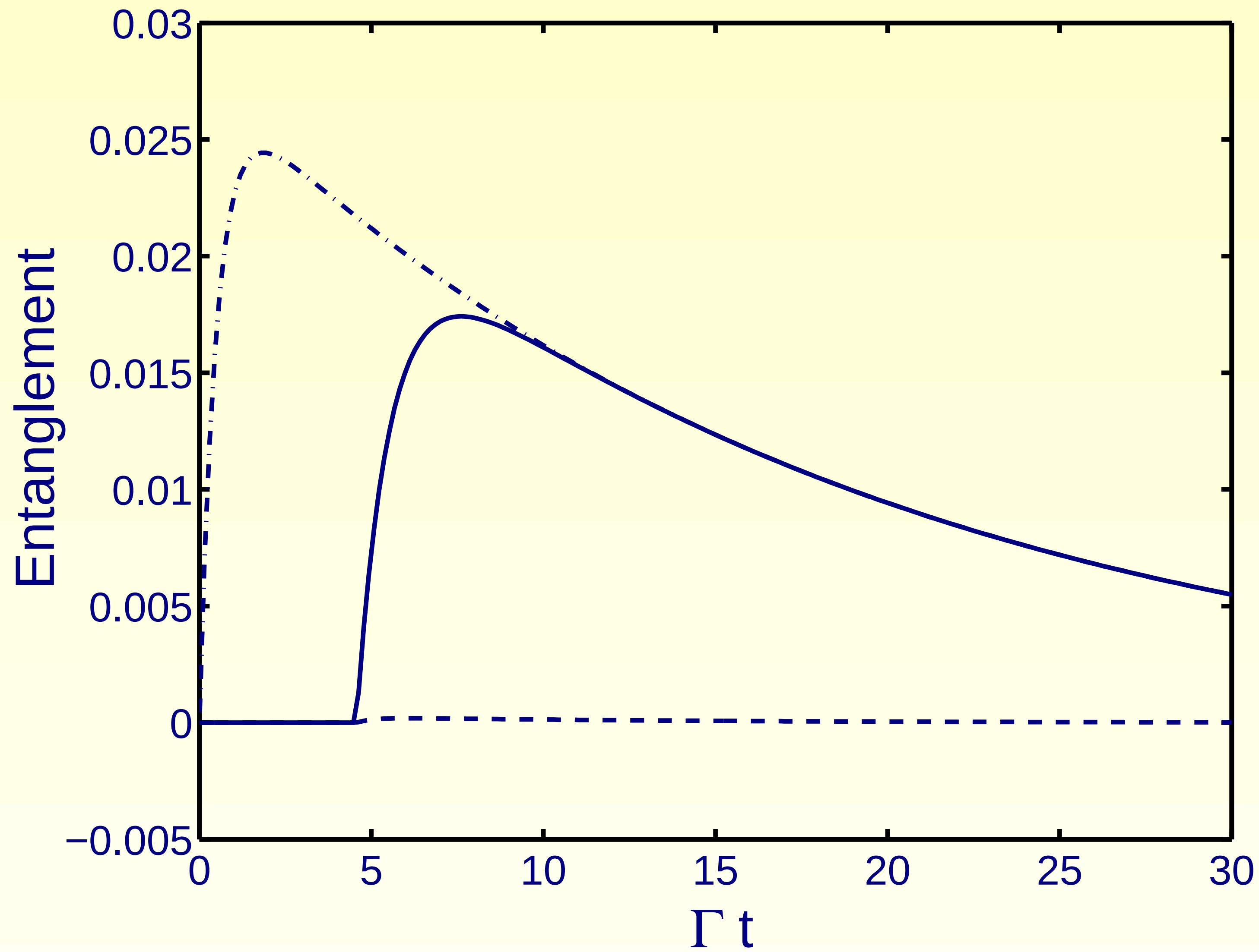
$$\mathcal{C}_2(t) = \left| \frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) \right.$$
$$\left. - \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) \right| - 2\mathrm{e}^{-\Gamma t} \sqrt{\rho_{gg}}$$

4.4 Dwa atomy wzbudzone

$\rho_{ee} = 1$

$$\mathcal{C}_2(t) = \left| \frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma + \Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) - \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma - \Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) \right| - 2\mathrm{e}^{-\Gamma t} \sqrt{\rho_{gg}}$$

$$\rho_{gg}(t) = 1 - \left[\frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma + \Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) + \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma - \Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) + \mathrm{e}^{-2\Gamma t} \right]$$

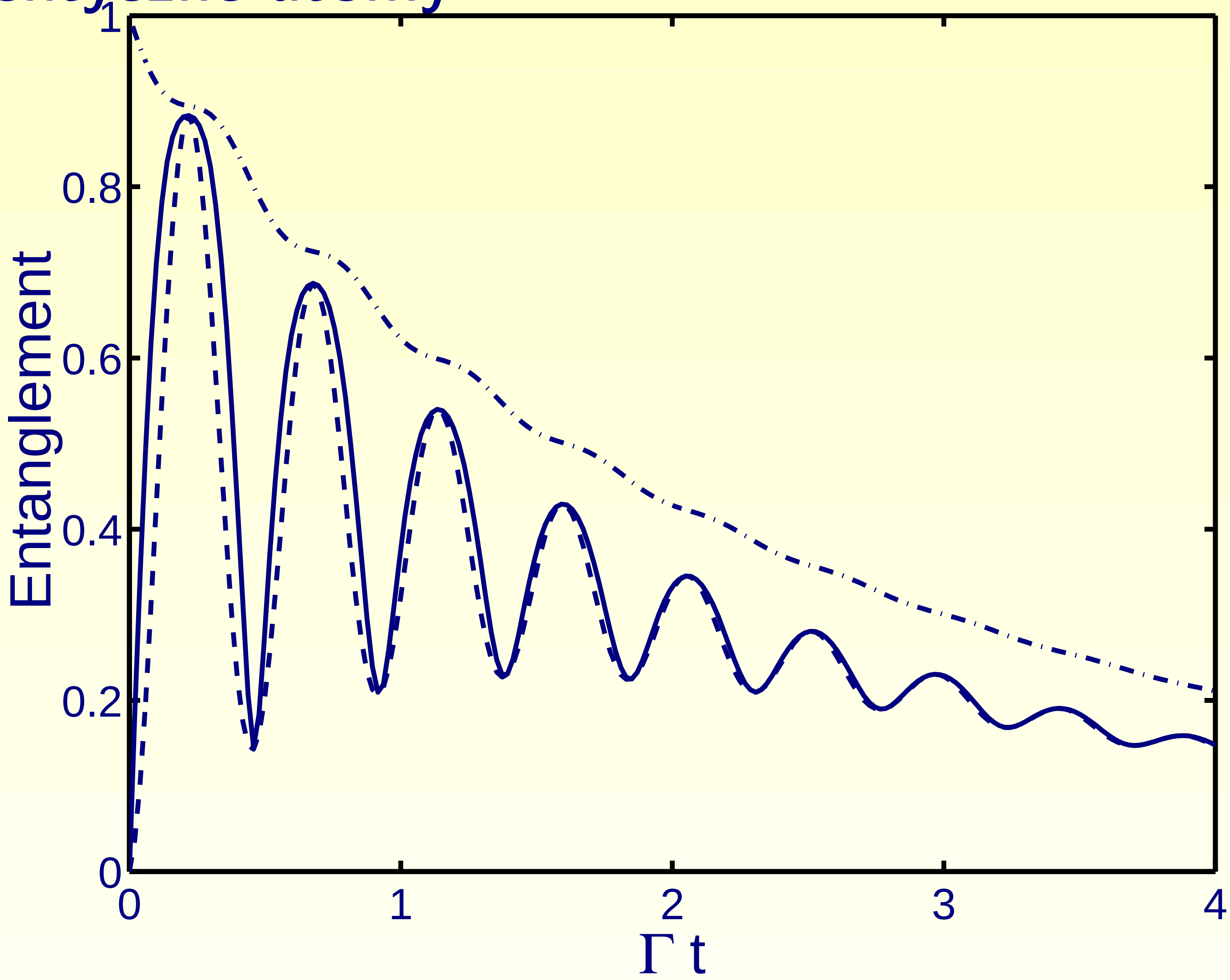


Concurrence $\mathcal{C}(t)$ (ciągła), negativity $\mathcal{N}(t)$ (kreski),

$\rho_{aa}(t)$ (kreski-kropki);

$\rho_{ee}(0) = 1$, $\hat{\boldsymbol{\mu}} \perp \hat{\boldsymbol{r}}_{12}$, $r_{12} = \lambda/12$ $(\Gamma_{12} = 0.95\,\Gamma,\ 2\Omega_{12} = 9.30\,\Gamma)$

4.5 Nieidentyczne atomy



Concurrence $\mathcal{C}(t)$ (ciągła), $\rho_{aa}(t) - \rho_{ss}(t)$ (kreski), $\rho_{aa}(t) + \rho_{ss}(t)$ (kreski-kropki);

nieidentyczne atomy: $\Delta = 10\,\Gamma$, $\rho_{44}(0) = 1$

5 Nasze prace

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  Correlated superposition states in two-atom systems
  in Modern Nonlinear Optics, Part I, ed. M. Evans (Wiley, New York, 2001) vol 119 of Advances in Chemical Physics, pp. 215-266

- Z. Ficek, R. Tanaś
  Entangled states and collective nonclassical effects in two-atom systems
  Physics Reports **372**, 369 (2002)

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  Entanglement induced by spontaneous emission in spatially extended two-atom systems
  J. Mod. Opt. **50**, 2765 (2003)

- R. Tanaś, Z. Ficek
  Entanglement of two atoms

Fortschr. Phys. **51**, 230 (2003)

- R. Tanaś, Z. Ficek
  Entangling two atoms via spontaneous emission
  J. Opt. B **6**, S90 (2004)
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  Stationary two-atom entanglement induced by nonclassical two-photon correlations
  J. Opt. B **6**, S610 (2004)

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