
QUANTUM OPTICS OF SINGLE OBJECTS

Master Equation Approach to the Problem of Two-Level Atom in a Squeezed Vacuum with Finite Bandwidth^{1, 2}

R. Tanas

Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
E-mail: tanas@kielich.amu.edu.pl

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Abstract—The master equation approach is used to describe the evolution of the two-level atom in a squeezed vacuum with finite bandwidth. The master equation is derived under the Born and Markov approximation that require the squeezed vacuum bandwidth to be much larger than the atomic linewidth, but not necessarily larger than the Rabi frequency of the driving field. It takes into account the detuning of the laser field from the atomic resonance. Examples of the fluorescence and absorption spectra are derived and compared to their equivalents for the broadband squeezing.

1. INTRODUCTION

In some circumstances, squeezed vacuums can assume the role of a reservoir to the atom, but properties of such reservoirs are quite different from ordinary vacuum. Squeezed vacuums are a reservoir with strong correlations between the field amplitudes at frequencies placed symmetrically with respect to a certain carrier frequency ω_s , and the evolution of a quantum system in such an unusual reservoir exhibits a number of new features. The first paper showing new features of such reservoirs was published by Gardiner [1], and later much work has been done to study the resonance fluorescence and probe absorption spectra of two- and three-level atoms in a squeezed vacuum [2–8]. Most of the studies dealing with the problem of a two-level atom in a squeezed vacuum assume that the squeezed vacuum is broadband; i.e., the bandwidth of the squeezed vacuum is much larger than the atomic linewidth and the Rabi frequency of the driving field. Experimental realizations of squeezed states [9–12], however, indicate that the bandwidth of the squeezed light is typically of the order of the atomic linewidth. The most popular schemes for generating squeezed light are those using a parametric oscillator operating below threshold, the output of which is a squeezed beam with a bandwidth of the order of the cavity bandwidth [13, 14].

Initial studies of the finite-bandwidth effects were performed by Gardiner *et al.* [13], Parkins and Gardiner [15] and Ritsch and Zoller [16]. The approaches were based on stochastic methods and numerical calculations and were applied to analyze the narrowing of the spontaneous emission and absorption lines. The fundamental effect of narrowing has been confirmed, but the effect of finite bandwidth was to degrade the narrowing

of the spectral lines rather than enhance it. Later, however, numerical simulations done by Parkins [17, 18] demonstrated that, for strong driving fields, a finite bandwidth of squeezing can have a positive effect on the narrowing of the Rabi sidebands.

Yeoman and Barnett [19] have recently proposed an analytical technique for investigating the behavior of a coherently driven atom damped by a squeezed vacuum with finite bandwidth. In the approach, they have derived a master equation and analytic expressions for the fluorescent spectrum for the simple case of a two-level atom exactly resonant with the frequencies of both the squeezed field and the driving field. Their analytical results agree with that of Parkins [17, 18] and explicitly show that the width of the central peak of the fluorescent spectrum depends solely on the squeezing present at the Rabi sideband frequencies. They have assumed that the atom is classically driven by a resonant laser field for which the Rabi frequency is much larger than the bandwidth of the squeezed vacuum, though this is still large compared to the natural linewidth. Unlike the conventional theory based on uncoupled states, it is possible to obtain a master equation consistent with the Born–Markov approximation by first including the interaction of the atom with the driving field exactly, and then considering the coupling of this combined dressed-atom system with the finite-bandwidth squeezed vacuum. The advantage of this dressed atom method over the more complex treatments based on adjoint equations or stochastic methods [17, 18, 20] is that simple analytical expressions for the spectra can be obtained. Within the secular approximation, the idea of Yeoman and Barnett has recently been extended by Ficek *et al.* [21, 22] to the case of a fully quantized dressed-atom model coupled to a finite bandwidth squeezed field inside an optical cavity.

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² This article was submitted by the authors in English.

Recently, Tanaš *et al.* [23] extended the Yeoman and Barnett [19] technique to include a nonzero detuning of the driving field from the atomic resonance and derived the master equation for a two-level atom driven by a classical laser field damped by a finite-bandwidth squeezed vacuum. Here, we use this master equation to study the fluorescence and probe absorption spectra for the two-level atom driven by a classical external field and damped by a squeezed vacuum with finite bandwidth produced by a degenerate parametric oscillator (DPO).

2. MASTER EQUATION

We consider a two-level atom driven by a detuned monochromatic laser field and damped by a squeezed vacuum with finite bandwidth. Applying the approach of paper [23], which is based on the idea of Yeoman and Barnett [19], being in turn an extension of the model proposed by Carmichael and Walls [24] and Cresser [25], we derive a master equation of the system which includes squeezing bandwidth effects. In this approach, we first perform the dressing transformation to include the interaction of the atom with the driving field, and then we couple the resulting dressed atom to the narrow bandwidth squeezed vacuum field. We derive the master equation under the Markov approximation which requires the squeezing bandwidth to be much greater than the atomic linewidth, but not necessarily greater than the Rabi frequency of the driving field and the detuning. For the sake of simplicity, we assume that the squeezing properties are symmetric about the central frequency of the squeezed field which, in turn, is exactly equal to the laser frequency. Our approach differs from that of Yeoman and Barnett in performing the Markov approximation in the time domain rather than the Laplace transform variable domain with pole approximation, and in adding a nonzero detuning.

The resulting master equation in the frame rotating with the laser frequency ω_L takes the form [23]

$$\begin{aligned} \dot{\rho} = & \frac{1}{2}i\gamma\delta[\sigma_z, \rho] \\ & + \frac{1}{2}\gamma\tilde{N}(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+) \\ & + \frac{1}{2}\gamma(\tilde{N}+1)(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \\ & - \gamma\tilde{M}\sigma_+\rho\sigma_+ - \gamma\tilde{M}^*\sigma_-\rho\sigma_- - \frac{1}{2}i\Omega[\sigma_+ + \sigma_-, \rho] \\ & + \frac{1}{4}i(\beta[\sigma_+, [\sigma_z, \rho]] - \beta^*[\sigma_-, [\sigma_z, \rho]]), \end{aligned} \quad (1)$$

where γ is the natural atomic linewidth, Ω is the Rabi frequency, $\Delta = \omega_L - \omega_A$ is the detuning of the laser field frequency from the atomic resonance,

$$\tilde{N} = N(\omega_L + \Omega') + \frac{1}{2}(1 - \tilde{\Delta}^2)\text{Re}\Gamma_-, \quad (2)$$

$$\tilde{M} = M(\omega_L + \Omega') - \frac{1}{2}(1 - \tilde{\Delta}^2)\Gamma_- + i\tilde{\Delta}\delta_M e^{i\phi}, \quad (3)$$

$$\delta = \frac{\Delta}{\gamma} - \frac{1}{2}(1 - \tilde{\Delta}^2)\text{Im}\Gamma_- + \tilde{\Delta}\delta_N, \quad (4)$$

$$\beta = \gamma\tilde{\Omega}[\delta_N + \delta_M e^{i\phi} - i\tilde{\Delta}\Gamma_-], \quad (5)$$

$$\Gamma_- = N(\omega_L) - N(\omega_L + \Omega') - [|M(\omega_L)| - |M(\omega_L + \Omega')|]e^{i\phi}, \quad (6)$$

$$\delta_N = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{N(x)}{x + \Omega'}dx, \quad \delta_M = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{|M(x)|}{x + \Omega'}dx, \quad (7)$$

$$\Omega' = \sqrt{\Omega^2 + \Delta^2}, \quad \tilde{\Omega} = \frac{\Omega}{\Omega'}, \quad \tilde{\Delta} = \frac{\Delta}{\Omega'}, \quad (8)$$

and ϕ is the phase of squeezing $M(\omega) = |M(\omega)|\exp(i\phi)$. In the derivation of equation (1), we have assumed that the phase ϕ does not depend on frequency [26], and we have included the divergent frequency shifts (the Lamb shift) with the redefinition of the atomic transition frequency [27]. Moreover, we have assumed that the squeezed vacuum is symmetric around the central frequency ω_L , so that $N(\omega_L - \Omega') = N(\omega_L + \Omega')$, and a similar relation holds for $M(\omega)$.

The master equation (1) has the standard form known from the broadband squeezing approaches with the new effective squeezing parameters \tilde{N} and \tilde{M} given by (2) and (3). There are also new terms proportional to β which are essentially narrow bandwidth modifications to the master equation. All the narrow bandwidth modifications are determined by the parameter Γ_- given by (6), which represents the difference between the squeezing values at the central line and the sidebands, and the shifts δ_N and δ_M defined in (7). They all become zero when the squeezing bandwidth goes to infinity.

The squeezing induced shifts δ_N and δ_M depend on the explicit form of $N(\omega)$ and $|M(\omega)|$. For a degenerate parametric oscillator (DPO), the squeezing properties are described by [13]

$$N(x) = \frac{\lambda^2 - \mu^2}{4} \left[\frac{1}{x^2 + \mu^2} - \frac{1}{x^2 + \lambda^2} \right], \quad (9)$$

$$|M(x)| = \frac{\lambda^2 - \mu^2}{4} \left[\frac{1}{x^2 + \mu^2} + \frac{1}{x^2 + \lambda^2} \right], \quad (10)$$

where $x = \omega - \omega_L$, and λ and μ are related to the cavity damping rate, γ_c , and the real amplification constant, ϵ , of the parametric oscillator according to

$$\lambda = \gamma_c + \epsilon, \quad \mu = \gamma_c - \epsilon.$$

The Cauchy principal values of the integrals (7) can be evaluated using the contour integration which gives

$$\delta_N = \delta_\mu - \delta_\lambda, \quad \delta_M = \delta_\mu + \delta_\lambda, \quad (11)$$

where the form of δ_μ and δ_λ for the degenerate parametric oscillator is given by

$$\begin{aligned} \delta_\mu &= \gamma\Omega \frac{\lambda^2 - \mu^2}{4} \frac{1}{\mu(\Omega'^2 + \mu^2)}, \\ \delta_\lambda &= \gamma\Omega \frac{\lambda^2 - \mu^2}{4} \frac{1}{\lambda(\Omega'^2 + \lambda^2)}. \end{aligned} \quad (12)$$

From the master equation (1), we easily derive the optical Bloch equations for the mean values of the atomic operators

$$\begin{aligned} \langle \dot{\sigma}_- \rangle &= -\gamma \left(\frac{1}{2} + \tilde{N} - i\delta \right) \langle \sigma_- \rangle - \gamma \tilde{M} \langle \sigma_+ \rangle + \frac{i}{2} \Omega \langle \sigma_z \rangle, \\ \langle \dot{\sigma}_z \rangle &= i(\Omega + \beta^*) \langle \sigma_- \rangle - i(\Omega + \beta) \langle \sigma_+ \rangle \\ &\quad - \gamma(1 + 2\tilde{N}) \langle \sigma_z \rangle - \gamma. \end{aligned} \quad (13)$$

The equation for $\langle \sigma_+ \rangle$ is obtained as a Hermitian conjugate of equation for $\langle \sigma_- \rangle$.

Defining the Hermitian operators σ_x and σ_y as

$$\sigma_x = \frac{1}{2}(\sigma_- + \sigma_+), \quad \sigma_y = \frac{1}{2i}(\sigma_- - \sigma_+), \quad (14)$$

we get from (13) the following equations of motion for the atomic polarization quadratures

$$\begin{aligned} \langle \dot{\sigma}_x \rangle &= -\gamma \left(\frac{1}{2} + \tilde{N} + \text{Re} \tilde{M} \right) \langle \sigma_x \rangle - \gamma (\text{Im} \tilde{M} + \delta) \langle \sigma_y \rangle, \\ \langle \dot{\sigma}_y \rangle &= -\gamma (\text{Im} \tilde{M} - \delta) \langle \sigma_x \rangle \\ &\quad - \gamma \left(\frac{1}{2} + \tilde{N} - \text{Re} \tilde{M} \right) \langle \sigma_y \rangle + \frac{1}{2} \Omega \langle \sigma_z \rangle, \\ \langle \dot{\sigma}_z \rangle &= 2 \text{Im} \beta \langle \sigma_x \rangle - 2(\Omega + \text{Re} \beta) \langle \sigma_y \rangle \\ &\quad - \gamma(1 + 2\tilde{N}) \langle \sigma_z \rangle - \gamma. \end{aligned} \quad (15)$$

The Bloch equations (15) clearly show the two different decay rates $\gamma_x = \gamma \left(\frac{1}{2} + \tilde{N} + \text{Re} \tilde{M} \right)$ and $\gamma_y = \gamma \left(\frac{1}{2} + \tilde{N} - \text{Re} \tilde{M} \right)$ for the two quadrature components of the atomic dipole $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$ which are already known from the Gardiner paper [1], but now the squeezing parameters \tilde{N} and \tilde{M} are more complicated. We can also see that the purely narrow-bandwidth features represented by $\text{Im} \tilde{M}$ and β introduce additional couplings between the components of the Bloch vector.

3. SPECTRA

The fluorescence and probe absorption spectra of a two-level atom are given by the Fourier transforms of the two-time atomic correlation functions

$$\mathcal{F}(\omega) = \frac{1}{\pi} \text{Re} \left\{ \int_0^\infty \langle \sigma_+(0) \sigma_-(\tau) \rangle_{ss} e^{i(\omega - \omega_L)\tau} d\tau \right\}, \quad (16)$$

$$\mathcal{A}(\omega) = \frac{1}{\pi} \text{Re} \left\{ \int_0^\infty \langle [\sigma_-(\tau), \sigma_+(0)] \rangle_{ss} e^{i(\omega - \omega_L)\tau} d\tau \right\}, \quad (17)$$

where Re denotes the real part of the integral. The Bloch equations (13), together with the quantum regression theorem [28], give the following equations of motion for the two-time atomic correlation functions

$$\begin{aligned} &\frac{\partial}{\partial \tau} \begin{pmatrix} \langle \sigma_+(0) \sigma_-(\tau) \rangle_{ss} \\ \langle \sigma_+(0) \sigma_+(\tau) \rangle_{ss} \\ \langle \sigma_+(0) \sigma_z(\tau) \rangle_{ss} \end{pmatrix} \\ &= \mathbf{B} \begin{pmatrix} \langle \sigma_+(0) \sigma_-(\tau) \rangle_{ss} \\ \langle \sigma_+(0) \sigma_+(\tau) \rangle_{ss} \\ \langle \sigma_+(0) \sigma_z(\tau) \rangle_{ss} \end{pmatrix} + \langle \sigma_+ \rangle_{ss} \begin{pmatrix} 0 \\ 0 \\ -\gamma \end{pmatrix}, \end{aligned} \quad (18)$$

where \mathbf{B} is the 3×3 matrix

$$\mathbf{B} = \begin{pmatrix} -\gamma \left(\frac{1}{2} + \tilde{N} - i\delta \right) & -\gamma \tilde{M} & \frac{i}{2} \Omega \\ -\gamma \tilde{M}^* & -\gamma \left(\frac{1}{2} + \tilde{N} + i\delta \right) & -\frac{i}{2} \Omega \\ i(\Omega + \beta^*) & -i(\Omega + \beta) & -\gamma(1 + 2\tilde{N}) \end{pmatrix}, \quad (19)$$

and the initial values for the correlation functions are

$$\begin{aligned} \langle \sigma_+ \sigma_- \rangle_{ss} &= \frac{1}{2}(1 + \langle \sigma_z \rangle_{ss}), \\ \langle \sigma_+ \sigma_+ \rangle_{ss} &= 0, \\ \langle \sigma_+ \sigma_z \rangle_{ss} &= -\langle \sigma_z \sigma_+ \rangle_{ss} = -\langle \sigma_+ \rangle_{ss}, \end{aligned} \quad (20)$$

where

$$\langle \sigma_z \rangle_{ss} = -\gamma \frac{\gamma^2 \left(\frac{1}{4} + \tilde{N}(\tilde{N} + 1) - |\tilde{M}|^2 + \delta^2 \right)}{d}, \quad (21)$$

$$\langle \sigma_+ \rangle_{ss} = i \frac{\Omega}{2d} \gamma^2 \left(\frac{1}{2} + \tilde{N} + \tilde{M}^* - i\delta \right), \quad (22)$$

$$d = \gamma^3(1 + 2\tilde{N})\left(\frac{1}{4} + \tilde{N}(\tilde{N} + 1) - |\tilde{M}|^2 + \delta^2\right) \quad (23)$$

$$+ \gamma\Omega\left[\left(\frac{1}{2} + \tilde{N} + \text{Re}\tilde{M}\right)(\Omega + \text{Re}\beta) + \text{Im}\beta(\text{Im}\tilde{M} + \delta)\right].$$

Taking the Laplace transform of (18), we obtain the system of algebraic equations for the transformed variables which can be easily solved [23]. For the Laplace transform of the correlation function $\langle\sigma_+(0)\sigma_-(\tau)\rangle_{ss}$, we get

$$\begin{aligned} F(z) = \frac{1}{zd(z)} & \left\{ -i\langle\sigma_+\rangle_{ss} \frac{\Omega}{2} \left[\gamma^2 \left(\frac{1}{2} + \tilde{N} + \tilde{M} + i\delta \right) \right. \right. \\ & \left. \left. + \gamma \left(\frac{3}{2} + \tilde{N} + \tilde{M} + i\delta \right) z + z^2 \right] \right. \\ & \left. + \frac{1}{2}(1 + \langle\sigma_z\rangle_{ss}) \left[\left[\gamma^2(1 + 2\tilde{N}) \left(\frac{1}{2} + \tilde{N} + i\delta \right) \right. \right. \right. \\ & \left. \left. + \frac{1}{2}\Omega(\Omega + \beta) \right] z + \gamma \left(\frac{3}{2} + 3\tilde{N} + i\delta \right) z^2 + z^3 \right] \right\}, \end{aligned} \quad (24)$$

and similarly for the Laplace transform of the difference $\langle\sigma_-(\tau)\sigma_+(0)\rangle_{ss} - \langle\sigma_+(0)\sigma_-(\tau)\rangle_{ss}$, we get

$$\begin{aligned} A(z) = \frac{1}{d(z)} & \left\{ i\langle\sigma_+\rangle_{ss} \Omega \left[\gamma \left(\frac{1}{2} + \tilde{N} + \tilde{M} + i\delta \right) + z \right] \right. \\ & \left. - \langle\sigma_z\rangle_{ss} \left[\gamma^2(1 + 2\tilde{N}) \left(\frac{1}{2} + \tilde{N} + i\delta \right) \right. \right. \\ & \left. \left. + \frac{1}{2}\Omega(\Omega + \beta) + \gamma \left(\frac{3}{2} + 3\tilde{N} + i\delta \right) z + z^2 \right] \right\}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} d(z) = d + & \left[\frac{5}{4} + 5\tilde{N}(\tilde{N} + 1) - |\tilde{M}|^2 + \delta^2 \right. \\ & \left. + \Omega(\Omega + \text{Re}\beta) \right] z + 2(1 + 2\tilde{N})z^2 + z^3 \end{aligned} \quad (26)$$

with d given by (23).

The spectra $\mathcal{F}(\omega)$ and $\mathcal{A}(\omega)$ are then given by

$$\begin{aligned} \mathcal{F}(\omega) &= \frac{1}{\pi} \text{Re} \{ F(z) |_{z=-i(\omega-\omega_L)} \}, \\ \mathcal{A}(\omega) &= \frac{1}{\pi} \text{Re} \{ A(z) |_{z=-i(\omega-\omega_L)} \}. \end{aligned} \quad (27)$$

For the simplest case of resonant driving field, $\Delta = 0$, and the squeezed vacuum phase $\phi = 0, \pi$ we have \tilde{N} , \tilde{M} , and β real, and $\delta = 0$. In this case, the characteristic polynomial (26) factorizes and its roots are given by

$$z_0 = -\gamma_x, \quad z_{\pm} = -\frac{1}{2}(\gamma_y + \gamma_z) \pm \Omega_R, \quad (28)$$

where

$$\gamma_x = \gamma \left(\frac{1}{2} + \tilde{N} + \tilde{M} \right), \quad \gamma_y = \gamma \left(\frac{1}{2} + \tilde{N} - \tilde{M} \right), \quad (29)$$

$$\gamma_z = \gamma_x + \gamma_y,$$

$$\Omega_R = \sqrt{\left| \Omega(\Omega + \beta) - \frac{1}{4}\gamma_x^2 \right|}, \quad (30)$$

$$\begin{aligned} \tilde{N} &= \frac{1}{2} \{ N(\omega_L) + N(\omega_L + \Omega) \\ &\mp (|M(\omega_L)| - |M(\omega_L + \Omega)|) \}, \end{aligned} \quad (31)$$

$$\tilde{M} = \pm \frac{1}{2} \{ (|M(\omega_L)| + |M(\omega_L + \Omega)|) \mp [N(\omega_L) - N(\omega_L + \Omega)] \}, \quad (32)$$

$$\beta = \delta_N \pm \delta_M. \quad (33)$$

In (31)–(33), the upper sign corresponds to $\phi = 0$ and the lower sign to $\phi = \pi$. The roots (28) are all real for $\Omega(\Omega + \beta) - \gamma_x^2/4 < 0$, and, if $\Omega(\Omega + \beta) - \gamma_x^2/4 > 0$, z_{\pm} becomes a complex conjugate pair with Ω_R replaced by $i\Omega_R$. They define the widths of the spectral lines and the effective Rabi frequency. It is clear that $\Omega(\Omega + \beta) - \gamma_x^2/4 = 0$ is a threshold at which the character of the solution changes.

Above the threshold $\Omega(\Omega + \beta) - \gamma_x^2/4 > 0$, the incoherent part of the fluorescence spectrum (the coherent delta function part of the spectrum is subtracted) is given by the following formula

$$\begin{aligned} \mathcal{F}(\omega) &= F_0 \frac{1}{\pi} \frac{\gamma_x}{(\omega - \omega_L)^2 + \gamma_x^2} \\ &+ \left\{ \frac{1}{\pi} \frac{F_a \left(\frac{\gamma_y + \gamma_z}{2} \right) + F_d(\omega - \omega_L + \Omega_R)}{(\omega - \omega_L + \Omega_R)^2 + \left(\frac{\gamma_y + \gamma_z}{2} \right)^2} \right. \\ &\left. + \frac{1}{\pi} \frac{F_a \left(\frac{\gamma_y + \gamma_z}{2} \right) - F_d(\omega - \omega_L - \Omega_R)}{(\omega - \omega_L - \Omega_R)^2 + \left(\frac{\gamma_y + \gamma_z}{2} \right)^2} \right\}, \end{aligned} \quad (34)$$

where

$$F_0 = \frac{D - \gamma\gamma_y}{4D}, \quad F_a = \frac{D(D - \gamma\gamma_y) - \gamma^2\Omega^2}{8D^2}, \quad (35)$$

$$F_d = \frac{D(D - \gamma\gamma_y)\gamma_x + \gamma\Omega^2[2D - \gamma(\gamma_y + \gamma_z)]}{16D^2\Omega_R},$$

$$D = \Omega(\Omega + \beta) + \gamma_y\gamma_z, \quad \Omega_R = \sqrt{\Omega(\Omega + \beta) - \frac{1}{4}\gamma_x^2}. \quad (36)$$

The absorption spectrum $\mathcal{A}(\omega)$ has the same structure as $\mathcal{F}(\omega)$, but with the replacement $F_i \rightarrow A_i$ according to

$$A_0 = \frac{\gamma\gamma_y}{2D}, \quad A_a = \frac{\gamma\gamma_y}{4D}, \quad A_d = \frac{\gamma(2\Omega^2 - \gamma_x - \gamma_z)}{8D\Omega_R}. \quad (37)$$

Formula (34) reveals the well-known three-peak structure. For large Rabi frequency Ω , the amplitudes F_0 and F_a become independent of Ω and $F_d \sim \Omega^{-1}$, and the fluorescence spectrum exhibits three Lorentzian lines: the central line at ω_L and two sidebands at $\omega_L \pm \Omega$. In the case of the absorption spectrum, A_0 and A_a go as Ω^{-2} , and $A_d \sim \Omega^{-1}$ is the dominant term in the spectrum. The absorption spectrum in the strong field limit shows dispersion features at the sidebands. The widths of the lines as well as their amplitudes depend on the bandwidth of the squeezed vacuum and can be calculated explicitly using our formulas.

Below the threshold, $\Omega(\Omega + \beta) - \gamma_x^2/4 < 0$, and for $\phi = 0, \pi$, and $\Delta = 0$, the spectrum takes the form

$$\mathcal{F}(\omega) = F_0 \frac{1}{\pi} \frac{\gamma_x}{(\omega - \omega_L)^2 + \gamma_x^2} + \left\{ \frac{1}{\pi} \frac{F_+ \left(\frac{\gamma_y + \gamma_z}{2} + \Omega_R \right)}{(\omega - \omega_L)^2 + \left(\frac{\gamma_y + \gamma_z}{2} + \Omega_R \right)^2} + \frac{1}{\pi} \frac{F_- \left(\frac{\gamma_y + \gamma_z}{2} - \Omega_R \right)}{(\omega - \omega_L)^2 + \left(\frac{\gamma_y + \gamma_z}{2} - \Omega_R \right)^2} \right\} \quad (38)$$

with

$$F_0 = \frac{D - \gamma\gamma_y}{4D},$$

$$F_{\pm} = \mp \frac{1}{16D^2\Omega_R} \{ D(D - \gamma\gamma_y)(\gamma_x \mp 2\Omega_R) + \gamma\Omega^2[2D - \gamma(\gamma_y + \gamma_z \mp 2\Omega_R)] \}, \quad (39)$$

$$D = \Omega(\Omega + \beta) + \gamma_y\gamma_z, \quad \Omega_R = \sqrt{\frac{1}{4}\gamma_x^2 - \Omega(\Omega + \beta)}, \quad (40)$$

and for the absorption spectrum $\mathcal{A}(\omega)$ we should replace F 's by

$$A_0 = \frac{\gamma\gamma_y}{2D}, \quad A_{\pm} = \pm \frac{\gamma(2\Omega^2 - \gamma_x\gamma_z \pm 2\Omega_R\gamma_y)}{8D\Omega_R}. \quad (41)$$

In this case, there are three Lorentzian contributions to the spectrum, all of them centered at the laser frequency, but with different linewidths.

Formulas (34) and (38) are analytical solutions for the fluorescence and probe absorption spectra for a resonantly driven atom in the finite bandwidth squeezed vacuum. It is clear that, on resonance and for $\phi = 0$, the spectrum is symmetric with respect to the laser frequency $\omega_L = \omega_A$. Below the threshold, it shows Lorentzian shape contributions with different widths at the laser frequency, and above the threshold it exhibits a Lorentzian line at the laser frequency, and Lorentzian as well as dispersion features at the Rabi sidebands. For a finite bandwidth squeezed vacuum, the widths and the amplitudes of the lines are defined by \tilde{N} , \tilde{M} , and β given by (31)–(33). They take the form

$$\gamma_x = \gamma \left\{ \frac{1}{2} + N(\omega_L + \Omega) \pm |M(\omega_L + \Omega)| \right\}, \quad (42)$$

$$\gamma_y = \gamma \left\{ \frac{1}{2} + N(\omega_L) \mp |M(\omega_L)| \right\}, \quad (43)$$

where the upper sign corresponds to $\phi = 0$ and the lower sign to $\phi = \pi$. For broadband squeezing, $N(\omega)$ and $M(\omega)$ do not depend on ω , which means that $\tilde{N} = N$ and $\tilde{M} = \pm|M|$ are constants describing the broadband squeezing. In this case, the shifts δ_N and δ_M are zero, and consequently $\beta = 0$. For ordinary vacuum $\gamma_x = \gamma_y = \gamma/2$ and the spectrum simplifies to the standard form [29, 30].

From (42), it is clear that the width of the central line, as well as the effective Rabi frequency, are defined by the squeezing properties on the sidebands only, while the widths of the sidebands depend on the squeezing properties at the laser frequency as well as at the sidebands. This feature has been found by Yeoman and Barnett [19] who discussed the resonance fluorescence spectrum. When squeezing at the sidebands is large, γ_x for $\phi = 0$ is large and the first contribution to

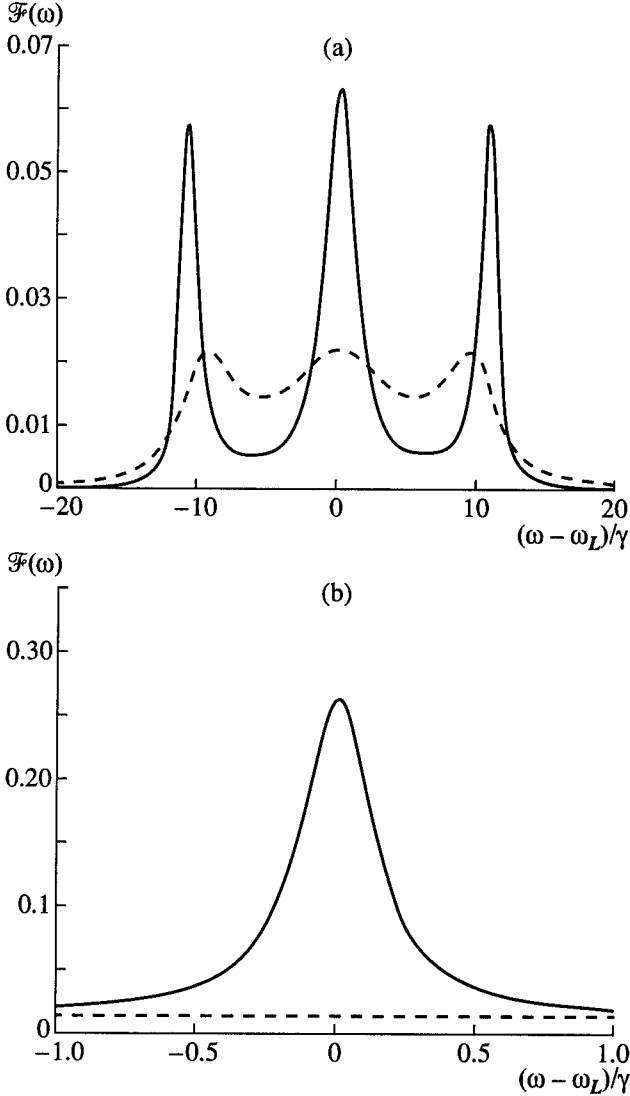


Fig. 1. Fluorescence spectrum for $\epsilon/\gamma_c = 0.5$ ($N(\omega_A) = 1.78$, $|M(\omega_A)| = 2.22$), $\phi = 0$, solid line—finite bandwidth with $\gamma_c/\gamma = 10$, dashed line—broadband squeezing: (a) above threshold ($\Omega = 10$) and (b) below threshold ($\Omega = 0.5$).

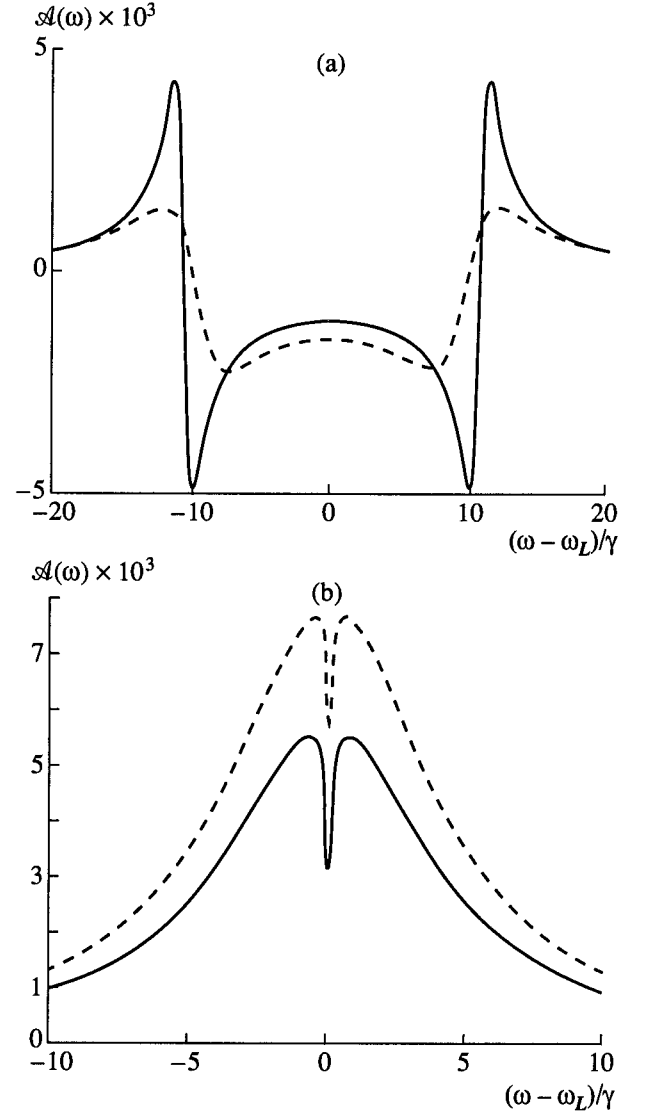


Fig. 2. Absorption spectrum for $\epsilon/\gamma_c = 0.5$ ($N(\omega_A) = 1.78$, $|M(\omega_A)| = 2.22$), $\phi = 0$, solid line—finite bandwidth with $\gamma_c/\gamma = 10$, dashed line—broadband squeezing: (a) above threshold ($\Omega = 10$) and (b) below threshold ($\Omega = 0.5$).

(34) and (38) is just a flat Lorentzian with the width comparable to the other contributions. For $\phi = \pi$, γ_x is small, which means that the first contribution to the spectrum is the peak at the laser frequency which is much narrower than the other contributions.

In Fig. 1, we have plotted examples of the resonance fluorescence spectrum for both the above threshold (Fig. 1a) and the bottom threshold (Fig. 1b) situations, for $\phi = 0$. The solid lines represent the spectrum for the finite bandwidth squeezed vacuum calculated according to our formulas, which is compared to that obtained for broadband squeezing. The parameters we used to calculate the spectrum are: $\epsilon/\gamma_c = 0.5$, which gives $N(\omega_A) = 1.78$ and $|M(\omega_A)| = 2.22$, $\Delta = 0$, $\gamma_c/\gamma = 10$ for narrow bandwidth (solid lines), $\gamma_c/\gamma = 100000$ for

broadband squeezing (dashed lines), $\Omega = 10$ for Fig. 1a, and $\Omega = 0.5$ for Fig. 1b. This example shows that the character of the spectrum is generally preserved when the bandwidth of squeezing becomes finite, but the parameters of the spectral lines such as their widths can be essentially modified.

In Fig. 2, we illustrate the probe absorption spectrum for the same set of parameters as Fig. 1. Figure 2a shows modification of the dispersive profile of the absorption spectrum for strong driving fields, and Fig. 2b shows a hole-burning feature discussed for broadband squeezing by Zhou *et al.* [31] that exist also for the narrow bandwidth, but is not as deep as for the broad bandwidth. Modification of the Rabi sidebands shown in Fig. 2a agrees qualitatively with that obtained

by Bosticky *et al.* [22] under the secular approximation, which requires sufficiently strong driving fields.

Generally, the roots of the characteristic polynomial are rather complicated, and the spectrum cannot be written in such a simple form as (34) or (38). Nevertheless, the analytical formulas for the Laplace transforms of the atomic correlation functions $F(z)$ and $A(z)$, given by (24) and (25), are still available, and the spectrum can be obtained according to (27). For nonzero detuning and/or squeezing phase $\phi \neq 0, \pi$, the spectrum is no longer symmetric and exhibits a number of interesting features which appear for the driving fields with the Rabi frequencies comparable to the atomic linewidth. For broadband squeezing, such features have recently been discussed by Ficek *et al.* [32]. For such fields, the secular approximation is not valid, but our approach is still applicable and can be used to find the modifications of the spectra when the bandwidth of the squeezed vacuum becomes finite.

4. CONCLUSIONS

We have derived simple analytical formulas for the resonance, fluorescence, and absorption spectra of a driven two-level atom damped to a squeezed vacuum with finite bandwidth. The derivation is based on the master equation which is valid for the bandwidth of the squeezed vacuum much larger than the natural linewidth of the atom but not necessarily larger than the Rabi frequency of the driving field. This allows us to study the spectra for both weak and strong fields. The formulas obtained in the paper give better insight into the physical origin of the spectral features that appear when the atom is damped to a squeezed vacuum with finite bandwidth. If the squeezing bandwidth becomes large, our results reproduce the results known for broadband squeezing. We have shown examples of the spectra above and below the threshold for the Rabi oscillations. This threshold depends on the parameters describing squeezed vacuums.

The master equation approach presented here leads to rather simple analytical expressions for the spectra, but the applicability of the approach is restricted by the Markov approximation used to derive the master equation, which requires the bandwidth of the squeezed vacuum to be much larger than the atomic linewidth. Violating this requirement can even lead to unphysical results. For the bandwidth of the squeezed vacuum, comparable with the atomic linewidth, the non-Markovian approach is necessary.

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