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Some effects of non-classical light on an atom

We consider a three-level atom in a V configuration damped to a broadband squeezed vacuum and driven on one of its allowed transitions (pump transition) by a coherent laser field. The resonance fluorescence spectrum and the probe absorption spectrum on the other allowed transition (probe transition) is studied for two cases: (i) carrier frequency of squeezed vacuum is tuned to the probe transition, and (ii) carrier frequency of the squeezed vacuum is tuned to the pump transition.

1. INTRODUCTION

Non-classical light exhibits a number of features that cannot be observed with classical light. The non-classical light can be generated in nonlinear optical processes. Properties of such non-classical light have been studied for many years by Professor Jan Perina and his collaborators in Olomouc. The subject and results of many research works in this field have been covered in details in an excellent book of Professor Perina on „*Quantum Statistics of Linear and Nonlinear Optical Phenomena*", the second edition of which appeared in 1991 [1]. The literature related to non-classical light contains already enormous number of papers and is still growing. References to a substantial part of the literature can be found in Professor's Perina book. Beside the problems associated with the properties of non-classical light itself that are a subject of intensive research, the old problems that involved ‚classical' fields (ordinary vacuum, for example, is considered in this respect as classical) are reconsidered with the non-classical light replacing the classical one (squeezed vacuum, for example, instead of ordinary vacuum), which leads to new and sometimes unexpected behavior of the well known systems. In this paper we consider a three-level atomic system in the V configuration which is coherently pumped on one of the allowed transitions (pump transition) and is damped to the squeezed vacuum instead of the ordinary vacuum. Moreover, we assume that the two allowed atomic transitions are coupled by a non-zero coherence transfer rate. We study resonance fluorescence and probe absorption spectra on undriven transition (probe transition).

The Mollow treatment [2] of a driven system of two level atoms with an off resonant field predicted that the absorption spectrum contains one absorption and one emission peak at the Rabi sidebands and a small dispersion profile at the central frequency. However, for a resonant driving field the central component of the spectrum disappears and the absorption spectrum exhibit dispersion-like profiles located at the Rabi sidebands. When atoms are damped into a squeezed vacuum, they interact with a modified electromagnetic vacuum and thus, their radiative properties change. It is well known [3-8] that there are essential differences between the atomic decay in an ordinary vacuum and the corresponding decay in a squeezed vacuum. For example, the atomic dipole decay rate depends on its phase difference with respect to the squeezed vacuum, the resonance fluorescence and probe absorption spectra also depend on the relative phase between the coherent driving field and the squeezed vacuum. The line-widths of the spectra can be drastically reduced for a particular choice of this phase difference. Recently, it has been shown [9], that the fluorescence spectrum can be asymmetric even if the atoms are driven by a resonant field. This feature arises from the fact that the relative phase between the squeezed vacuum and the driving field, if chosen between 0 and π, makes unequal populations of the dressed states. Moreover a small change in the Rabi frequency, phase, and detuning will result in the spectrum loosing its dispersive profiles. Ficek et al. [10] has studied the asymmetric probe-absorption spectrum and amplification without population inversion of a two level atom damped by a broadband squeezed vacuum. They have shown that a resonant driving field which is out of phase with the squeezed vacuum can be asymmetric. This feature is again due to the fact that unequal populations of the dressed states of the driven system have been produced. They have also shown that the amplification of the probe field at the central frequency is not due to any population inversion in both bare and dressed states, but it is rather related to the coherent population oscillations, and it cannot appear in the absence of the squeezed vacuum.

Since the squeezed vacuum is characterized by correlated pairs of photons, it is quite natural to expect that two-photon transitions in multilevel atoms should be significantly affected by the presence of the squeezed vacuum. Such a situation occurs, for instance, when the the carrier frequency of the squeezed vacuum is chosen so as to its double is tuned to resonance between the lower and upper states of the three-level cascade system. The two-photon correlations present in the squeezed vacuum have in this case dramatic influence on the steady-state populations of the atom [6,11-13]. The results for three-level systems driven by two independent laser beams show [7, 14,15] that the relative heights and widths of the peaks in the resonance fluorescence spectra can be sub-natural or supernatural depending on the relative phase between the driving fields and the squeezed vacuum, similarly as in two-level systems. Recently, Ferguson et al. [16] have examined the effect of a single broad-band squeezed vacuum on the stationary populations and coherences in a three-level atom in lambda configuration that is driven by two independent laser fields. They also included the coherence transfer rates that couple the two allowed atomic transitions. The squeezed vacuum together with the nonzero atomic coherence transfer rate lead to the nonzero steady-state atomic coherences [17] in a three-level atom under certain conditions.

In this paper, we will study the effect of the squeezed vacuum on the fluorescence and absorption spectra for a three level V-type system driven by a coherent laser field on one of the allowed transitions, which we call the pump transition/and damped to a squeezed vacuum which is tuned to either the probe transition or to the pump transition. The spectra are calculated for the atomic transition that is not pumped by the coherent field, which we call the probe transition. We also take into account the atomic coherence transfer rate. We discuss some new features of the spectra that are due to the squeezed vacuum.

2. THE MODEL AND EQUATIONS OF MOTION

We consider an atom with three levels in a V configuration shown schematically in *Fig.1*. The system has the lower state $|1\rangle$, and the two upper states $|2\rangle$ and $|3\rangle$. The atom is pumped by a coherent field with the Rabi frequency Ω (our Ω is in fact one half of the resonant Rabi frequency) on the transition $|1\rangle \leftrightarrow |3\rangle$ and probed by a weak field with the strength Ω_p on the transition $|1\rangle \leftrightarrow |2\rangle$. Moreover, the atom is damped to a broadband squeezed vacuum with the frequency ω_s. The master equation for such a system takes the form

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H,\rho\right] + \mathcal{L}\rho, \tag{1}$$

where H *is* the Hamiltonian of the system, which in the rotating wave approximation is given by

$$H = \hbar\omega_{21}|2\rangle\langle 2| + \hbar\omega_{31}|3\rangle\langle 3| + \hbar\Omega e^{i\omega_1 t}|1\rangle\langle 3| + \\ + \hbar\Omega_p e^{i\omega_2 t}|1\rangle\langle 2| + H.c.. \tag{2}$$

In equation (2) ω_{i1} is the energy of the level i (level 1 is assumed to have zero energy), ω_1

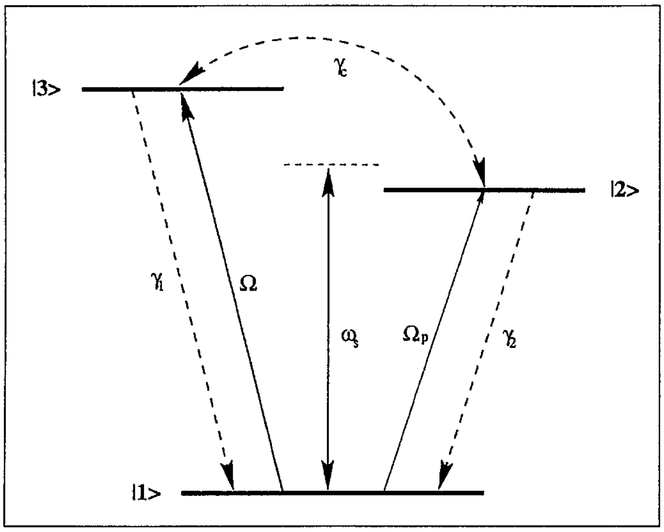

Figure 1: Schematic diagram of the system

is the frequency of the pump field acting on the atomic transition 1, i.e., the transition $|1\rangle \leftrightarrow |3\rangle$, ω_2 is the frequency of the probe field acting on the atomic transition 2, i.e., the transition $|1\rangle \leftrightarrow |2\rangle$. The second term $\mathcal{L}\rho$ which describes various damping terms, is the effect of the reservoir on the atom and is given by [16]

$$\begin{aligned}\mathcal{L}\rho = &-\tfrac{1}{2}(N+1)\sum_{i,j}\Gamma_{ij}\left(\rho\sigma_i^+\sigma_j^- + \sigma_i^+\sigma_j^-\rho - 2\sigma_j^-\rho\sigma_i^+\right)\\ &-\tfrac{1}{2}N\sum_{i,j}\Gamma_{ij}\left(\rho\sigma_i^-\sigma_j^+ + \sigma_i^-\sigma_j^+\rho - 2\sigma_j^+\rho\sigma_i^-\right)\\ &-\tfrac{1}{2}M\sum_{i,j}\Gamma_{ij}\left(\rho\sigma_i^+\sigma_j^+ + \sigma_i^+\sigma_j^-\rho - 2\sigma_i^+\rho\sigma_j^+\right)e^{-i2\omega_s t}\\ &-\tfrac{1}{2}M^*\sum_{i,j}\Gamma_{ij}\left(\rho\sigma_i^-\sigma_j^- + \sigma_i^-\sigma_j^-\rho - 2\sigma_i^-\rho\sigma_j^-\right)e^{i2\omega_s t}\end{aligned} \tag{3}$$

The parameters Γ_{ij} are the relaxation rates; for $i = j$ they describe spontaneous damping rates $\Gamma_{11} = \gamma_1$, $\Gamma_{22} = \gamma_2$ from the level $|3\rangle \leftrightarrow |1\rangle$ and $|2\rangle \leftrightarrow |1\rangle$, respectively, for $i \neq j$ we have the cross relaxation rate $\Gamma_{12} = \Gamma_{21} = \gamma_c$, that couples the two atomic transitions and is given by [16]

$$\gamma_c = \frac{\mu_{13}\cdot\mu_{12}}{12\pi\varepsilon_0\hbar c^3}\left(\omega_{31}^3 + \omega_{21}^3\right). \tag{4}$$

The operators s_i^-, s_i^+ ($i = 1, 2$) are the atomic lowering and raising operators for the transition i, i.e., $\sigma_1^- = |1\rangle\langle 3|$, $\sigma_2^- = |1\rangle\langle 2|$, etc. The parameter N *is* the mean photon number of the reservoir, which in our case is the broad-band squeezed vacuum, and $M = |M| \exp(i\phi)$ is a complex number describing the photon correlations present in the squeezed vacuum with the carrier frequency ω_s and the phase ϕ. The parameter M must satisfy the inequality $|M| \leq \sqrt{N(N+1)}$.

The master equation (1), under the assumption that $\Omega_p = 0$, leads to the following set of equations for the atomic density matrix elements (in a frame rotated with respect to ω_1 and ω_2)

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} = &-\left(N(\gamma_1+\gamma_2)+(N+1)\gamma_1\right)\rho_{11} + i\Omega\rho_{13} + (N+1)(\gamma_1+\gamma_2)\rho_{22},\\
&+(N+1)\gamma_c e^{(i\Delta_{12}t)}\rho_{23} - i\Omega\rho_{31} + (N+1)\gamma_c e^{(-i\Delta_{12}t)}\rho_{32} + (N+1)\gamma_1,\\
\frac{\partial}{\partial t}\rho_{12} = &-(\gamma_{12}+i\delta_2)\rho_{12} - \tfrac{1}{2}(N+1)\gamma_c e^{(\Delta_{12}t)}\rho_{13} + M^*\gamma_2 e^{(2i\Delta_{s1}t)}\rho_{21},\\
&+M^*\gamma_c e^{(2i\Delta t)}\rho_{31} - i\Omega\rho_{32},\\
\frac{\partial}{\partial t}\rho_{13} = &\,2i\Omega\rho_{11} - \tfrac{1}{2}(N+1)\gamma_c e^{(-i\Delta_{12}t)}\rho_{12} - (\gamma_{13}+i\delta_1)\rho_{13},\\
&+i\Omega\rho_{22} + M^*\gamma_c e^{(2i\Delta t)}\rho_{21} + M^*\gamma_1 e^{(2i\Delta_{s2}t)}\rho_{31} - i\Omega,\\
\frac{\partial}{\partial t}\rho_{22} = &\,N\gamma_2\rho_{11} - (N+1)\gamma_2\rho_{22} - \tfrac{1}{2}(N+1)\gamma_c e^{(i\Delta_{12}t)}\rho_{23},\\
&-\tfrac{1}{2}(N+1)\gamma_c e^{(-i\Delta_{12}t)}\rho_{23},\\
\frac{\partial}{\partial t}\rho_{23} = &\,\tfrac{1}{2}(3N+1)\gamma_c e^{-i\Delta_{12}t}\rho_{11} + i\Omega\rho_{21} - (N+1)\gamma_c e^{-i\Delta_{12}t}\rho_{22},\\
&-\left(\gamma_{23} - i(\delta_2-\delta_1)\right)\rho_{23} - \tfrac{1}{2}(N+1)\gamma_c e^{-i\Delta_{12}t}.
\end{aligned} \tag{5}$$

where the parameters γ_{ij} are given by:

$$\begin{aligned}\gamma_{12} &= (N+1)\gamma_2 + \tfrac{1}{2}N\gamma_1,\\ \gamma_{13} &= (N+1)\gamma_1 + \tfrac{1}{2}N\gamma_2,\\ \gamma_{23} &= \tfrac{1}{2}(N+1)(\gamma_2+\gamma_1),\end{aligned} \tag{6}$$

$\delta_1 = \omega_1 - \omega_{31}$, $\delta_2 = \omega_2 - \omega_{21}$, $\Delta_{12} = \omega_1 - \omega_2$, $\Delta_{si} = \omega_s - \omega_i$, $\Delta = 2\omega_s - \omega_1 - \omega_2$, ρ_{ji} is the complex conjugate of ρ_{ij}, and the trace condition $\rho_{11} + \rho_{22} + \rho_{33} = 1$ have been used in (5). Since we have assumed $\Omega = 0$, we have actually assumed that the atomic coherences ρ_{12} and ρ_{21} have been rotated with respect to the atomic frequency ω_{21} instead of the frequency ω_2, so $\delta_2 = 0$, and we will simplify the notation using $\delta_2 = \delta$ later on.

3. Resonance fluorescence and probe absorption spectrum

We study here the radiative properties of the atom associated with the atomic transition $|1\rangle \leftrightarrow |2\rangle$, which is not pumped by the coherent light. The only mechanism that can populate the state $|2\rangle$ is due to the nonzero value of the mean number of photons N in the squeezed vacuum. However, the squeezed vacuum introduces some extra couplings to the equations (5) through the parameter M, which modify the radiative properties of the atom. We calculate the resonance fluorescence as well as absorption spectrum of light on the probe transition showing the appearance of an additional structure that is introduced by the squeezed vacuum.

The fluorescence spectrum is defined by the Fourier transform of the correlation function

$$\mathcal{F}(t,t+\tau) = \left\langle E^{(-)}(r,t)E^{(+)}(r,t+\tau)\right\rangle \tag{7}$$

of the electric field emitted by the atom, where $E^{(\pm)}$ are the positive and negative frequency parts of the total field operator. The field correlation function (7) is proportional to the atomic correlation function

$$\mathcal{F}(t,t+\tau) \sim \left\langle \sigma^+(t)\sigma^-(t+\tau)\right\rangle. \tag{8}$$

According to the quantum regression theorem, this two-time correlation function can be calculated from the equations of motion for the expectation values of the atomic lowering operator $\langle\sigma^-(t+\tau)\rangle$. Once the atomic correlation function (8) is known, the steady-state resonance fluorescence spectrum can be calculated as the Fourier transform of this function from the formula

$$F(\omega)=\mathrm{Re}\int_0^\infty\langle\sigma^+(t)\sigma^-(t,t+\tau)\rangle e^{i\omega\tau}d\tau. \qquad (9)$$

Similarly, the absorption spectrum can be calculated according to the formula [2]

$$S(\omega)=\mathrm{Re}\int_0^\infty\langle[\sigma^+(t+\tau),\sigma^-(t)]\rangle e^{i\omega\tau}d\tau. \qquad (10)$$

The correlation functions (9) and (10) are calculated using the quantum regression theorem in the absence of the probe field using equations (5). Generally, equations (5) cannot be solved directly because the matrix of the coefficients is time dependent. To get rid of this time dependence we choose the carrier frequency ω_s of the squeezed vacuum to be resonant, or nearly resonant, with one of the allowed transitions, and we discard in equations (5) all rapidly oscillating terms (secular approximation).

3.1 Squeezed vacuum frequency tuned to the probe transition

We assume that the atom is non-degenerate, so the fast oscillating terms that oscillate at the frequency Δ_{12} can be neglected, and if the squeezed vacuum is tuned to the probe transition ($\Delta_{s1}=0$), the terms that oscillate at the frequencies Δ_{s2} and Δ are also fast oscillating, and they can thus be neglected. As a result the equations (5) can be written in the form

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11}&=-\left(N(\gamma_1+\gamma_2)+(N+1)\gamma_1\right)\rho_{11}+i\Omega\rho_{13}+(N+1)(\gamma_2+\gamma_1)\rho_{22},\\
&\quad-i\Omega\rho_{31}+(N+1)\gamma_1,\\
\frac{\partial}{\partial t}\rho_{12}&=-\gamma_{12}\rho_{12}+M^*\gamma_2\rho_{21}-i\Omega\rho_{32},\\
\frac{\partial}{\partial t}\rho_{13}&=2i\Omega\rho_{11}-(\gamma_{13}+i\delta)\rho_{13}+i\Omega\rho_{22}-i\Omega,\\
\frac{\partial}{\partial t}\rho_{22}&=N\gamma_2\rho_{11}-(N+1)\gamma_2\rho_{22},\\
\frac{\partial}{\partial t}\rho_{23}&=i\Omega\rho_{21}-(\gamma_{23}+i\delta)\rho_{23}.
\end{aligned} \qquad (11)$$

The steady-state solution to equations (11) can be found by putting their left hand side equal to zero and solving the resulting algebraic set of equations. In this way we get formulas for the steady-state values of the populations ρ_{22}, ρ_{11} and the coherence ρ_{31}. The steady-state value for the coherence ρ_{12} is equal to zero, as one could expect because this transition is not pumped. The results are found to be

$$\begin{aligned}
\rho_{11}&=\frac{(N+1)\left((N+1)\gamma_1\gamma_{13}+2\Omega^2\right)}{(N+1)(3N+1)\gamma_1\gamma_{13}+2(3N+2)\Omega^2},\\
\rho_{22}&=\frac{N\left((N+1)\gamma_1\gamma_{13}+2\Omega^2\right)}{(N+1)(3N+1)\gamma_1\gamma_{13}+2(3N+2)\Omega^2},\\
\rho_{31}&=\frac{-i(N+1)\gamma_1\Omega}{(N+1)(3N+1)\gamma_1\gamma_{13}+2(3N+2)\Omega^2}.
\end{aligned} \qquad (12)$$

It is clear from the solutions (12) that they depend on the mean number of photons in the squeezed vacuum N, but they do not depend on the parameter M describing correlations in the squeezed field. For $\Omega=0$, $\rho_{31}=0$, and the atomic populations are the same as in a thermal field with the same mean number of photons. This means that the steady-state expectation values of the atomic operators are not affected by the presence of photon correlations in the squeezed vacuum. The results do not depend also on the coherence transfer rate γ_c given by (4). The situation can, however, be different when the fluorescence and/or absorption spectrum is calculated. The spectra are defined by the two-time correlation functions (9) and (10) that can be sensitive to the presence of correlations in the squeezed light. We will show that this is really the case.

We calculate the fluorescence spectrum and the absorption spectrum for the probe transition, using formulas (9) and (10), for the case when the squeezed vacuum frequency is resonant with the atomic transition $|1\rangle\leftrightarrow|2\rangle$. This means that we calculate the spectra for the atomic transition that is not pumped by the laser field. The mechanism that populates state $|2\rangle$ is due to the squeezed vacuum field with the mean number of photons N, as it is clear from (12). The spectra are given by

$$F(\omega)=\mathrm{Re}\left\{\left.\frac{(z+\gamma_{23}-i\delta)\rho_{22}}{A_-(z,\Omega)-\dfrac{\gamma_2^2\left((z+\gamma_{23})^2+\delta^2\right)}{A_+(z,\Omega)}|M|^2}\right|_{z=i(\omega-\omega_{21})}\right\}, \qquad (13)$$

$$S(\omega)=\mathrm{Re}\left\{\left.\frac{(z+\gamma_{23}-i\delta)(\rho_{11}-\rho_{22})-i\Omega\rho_{31}}{A_-(z,\Omega)-\dfrac{\gamma_2^2\left((z+\gamma_{23})^2+\delta^2\right)}{A_+(z,\Omega)}|M|^2}\right|_{z=i(\omega-\omega_{21})}\right\}, \qquad (14)$$

where

$$A_\pm(z,\Omega)=z^2+(\gamma_{12}+\gamma_{23})z+\gamma_{12}\gamma_{23}+\Omega^2\pm i\delta(z+\gamma_{23}). \qquad (15)$$

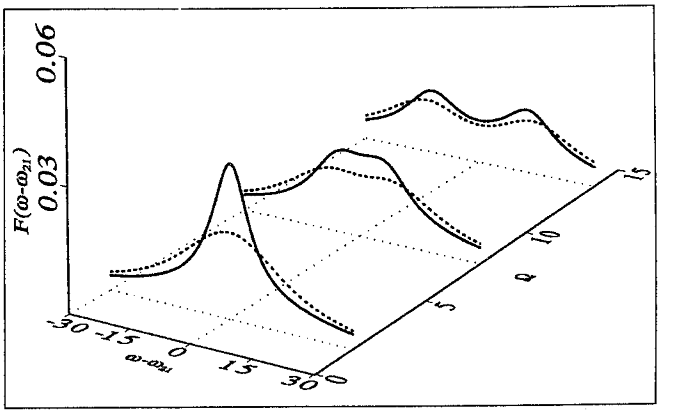

Fig. 2 The fluorescence spectrum on the probe transition when the squeezed vacuum frequency is tuned to the probe transition for different values of Ω = 1.7, 7.7, 13.7. The other parameters are: $N=1$, $M=\sqrt{2}$, $\gamma_1=\gamma_2=1$, $\Omega=10$ and $\delta=0$. Solid line represents the spectrum for the squeezed vacuum and dotted line for thermal field

It is clear from (13) and (14) that for $M = 0$, the denominator is a polynomial of the second order in z. This means that for the ordinary vacuum or thermal light the spectrum can show only two Lorentzian peaks. The situation is different for $M \neq 0$ in which case there can generally be four different Lorentzian contributions to the fluorescence spectrum.

In the case when the pump field is resonant with the atomic transition $|1\rangle \leftrightarrow |2\rangle$ ($\delta = 0$) the fluorescence spectrum can be rewritten in the simple form

$$F(\omega)=\frac{\rho_{22}}{2}\mathrm{Re}\left\{\left[\frac{z+\gamma_{23}}{(z+\gamma_{12}+M\gamma_2)(z+\gamma_{23})+\Omega^2}+\frac{z+\gamma_{23}}{(z+\gamma_{12}-M\gamma_2)(z+\gamma_{23})+\Omega^2}\right]_{z=i(\omega-\omega_{21})}\right\}. \qquad (16)$$

It is clear from the expression (16), which is a sum of two terms each of which has the second order polynomial in the denominator and thus represents two lines, that generally the spectrum is composed of four lines which are symmetrically disposed with respect to the frequency ω_{21}. To confirm this statement we decompose the spectrum (16) into the four Lorentzians explicitly. Beside the four Lorentzians the spectrum contains also some dispersion-like terms, which, however, are small in the strong field limit. The strong field limit means in this case that the following condition is satisfied

$$4\left(\gamma_{12}\gamma_{23}+\Omega^2\right)>\left(\gamma_{12}+\gamma_{23}+|M|\gamma_2\right)^2. \qquad (17)$$

In the strong field limit the spectrum (16) can be rewritten in the form

$$\begin{aligned}F(\omega)=\frac{\rho_{22}}{4}\Bigg\{&\left[\frac{\gamma_+}{(\omega-\omega_{21}-\Omega_+)^2+\gamma_+^2}+\frac{\gamma_+}{(\omega-\omega_{21}-\Omega_+)^2+\gamma_+^2}\right]+\\&+\left[\frac{\gamma_-}{(\omega-\omega_{21}-\Omega_-)^2+\gamma_-^2}+\frac{\gamma_-}{(\omega-\omega_{21}-\Omega_-)^2+\gamma_-^2}\right]+\\&+\frac{1}{\Omega_+}\left[\frac{(\gamma_+-\gamma_{23})(\omega-\omega_{21}-\Omega_+)}{(\omega-\omega_{21}-\Omega_+)^2+\gamma_+^2}-\frac{(\gamma_+-\gamma_{23})(\omega-\omega_{21}+\Omega_+)}{(\omega-\omega_{21}+\Omega_+)^2+\gamma_+^2}\right]+\\&+\frac{1}{\Omega_-}\left[\frac{(\gamma_--\gamma_{23})(\omega-\omega_{21}-\Omega_-)}{(\omega-\omega_{21}-\Omega_-)^2+\gamma_-^2}-\frac{(\gamma_--\gamma_{23})(\omega-\omega_{21}+\Omega_-)}{(\omega-\omega_{21}+\Omega_-)^2+\gamma_-^2}\right]\Bigg\}.\end{aligned} \qquad (18)$$

where

$$\gamma_\pm=\tfrac{1}{2}\left(\gamma_{12}+\gamma_{23}\pm|M|\gamma_2\right), \qquad (19)$$

$$\Omega_\pm=\tfrac{1}{2}\sqrt{4\left(\gamma_{12}\gamma_{23}+\Omega^2\right)-\gamma_\pm^2}. \qquad (20)$$

It is clear from equation (18) that for the squeezed vacuum ($|M| \neq 0$) the resonance fluorescence spectrum on the probe transition consist of four Lorentzian lines located symmetrically with respect to the transition frequency ω_{21} at the frequencies $\Omega_\pm$ and having the widths $\gamma_\pm$. Beside the four Lorentzians the spectrum contains some dispersion-like terms, which are of the order of $\Omega_\pm^{-1}$ and are negligible if $\Omega_\pm >> \gamma_\pm$. The presence of the four lines in the fluorescence spectrum is the result of the squeezed vacuum. This effect is associated with the different damping rates for the two quadrature components of the atomic dipole moment in the squeezed vacuum. In the case of thermal field, $|M| = 0$, with the mean number of photons N the four lines merge into two lines. Since the difference $\Omega_+ - \Omega_-$ in the strong field limit is small and the width γ_+ is large, the peaks associated with the frequencies Ω_+ and Ω_- cannot be separated in practice. The two peaks merge into one peak, but the resulting peak is narrower than the peak obtained for the thermal field with the same mean number of photons. Similar decomposition can be performed for the absorption spectrum.

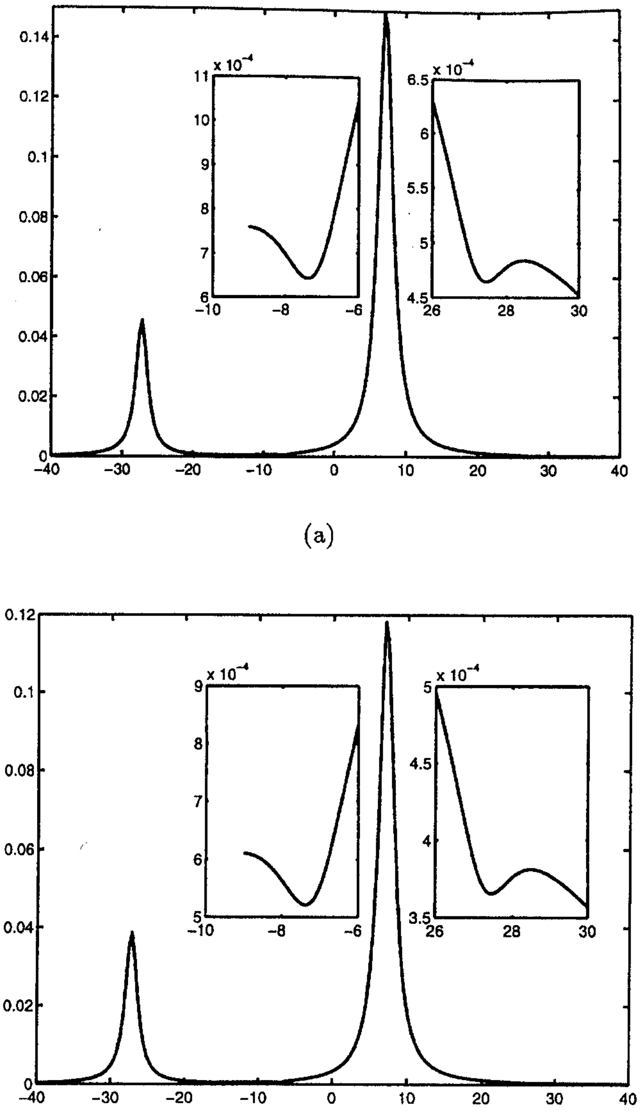

Fig. 3 (a) - The absorption spectrum $S(\omega - \omega_{21})$ on the transition $1 \leftrightarrow 2$ for the squeezed vacuum frequency tuned to the probe transition (b) - Fluorescence spectrum $F(\omega - \omega_{21})$ for the transition $1 \leftrightarrow 2$. The parameters are: $\gamma_1 = 0.01$, $\gamma_2 = 1$, $\Omega_2 = 14$, $\delta_2 = -20$, $N = 0.8$, $M = \sqrt{N(N+1)}$

Examples of the fluorescence spectrum are shown in Fig. 2. It is seen that in the presence of the squeezed vacuum the fluorescence spectrum is narrower than the corresponding spectrum for thermal light with the same number of photons. For strong driving fields the spectrum splits into two lines. Each of the two lines is composed of two components with different widths, but effectively they appear as a single line because the splitting due to the different widths resulting in the squeezed vacuum are too small to be resolved within the line-width. To see the presence of the extra structure in the spectrum the driving field must be considerably detuned from the resonance. In the case of large detuning the lines should be separated and one can expect the appearance of additional structure in the spectrum. That it is really the case we have shown in Fig.3, where we have plotted the fluorescence as well as absorption spectra for large detuning to make the extra structure visible. As it is seen there are really small dips that appeared in the spectra. The additional structure is, however, hardly visible, and the most essential result of the squeezed vacuum is the narrowing of the spectrum that is clearly seen in Fig. 2.

3.2 Squeezed vacuum frequency tuned to the pump transition

In the case when the squeezed vacuum frequency is close to resonance with the pump transition $|1\rangle \leftrightarrow |3\rangle$, we substitute $\Omega_p = 0$ and $\Delta_{s2} = 0$ in equation (5), and we neglect the fast oscillating terms that oscillate at the frequencies Δ_{12} and Δ. The master equation (5) gives us then

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= -\left(N(\gamma_1+\gamma_2)+(N+1)\gamma_1\right)\rho_{11} + i\Omega\rho_{13} + (N+1)(\gamma_2+\gamma_1)\rho_{22}, \\
&\quad - i\Omega\rho_{31} + (N+1)\gamma_1, \\
\frac{\partial}{\partial t}\rho_{12} &= -\gamma_{12}\rho_{12} - i\Omega\rho_{32}, \\
\frac{\partial}{\partial t}\rho_{13} &= 2i\Omega\rho_{11} - (\gamma_{13}+i\delta)\rho_{13} + i\Omega\rho_{22} + M^*\gamma_1\rho_{31} - i\Omega, \qquad (21) \\
\frac{\partial}{\partial t}\rho_{22} &= N\gamma_2\rho_{11} - (N+1)\gamma_2\rho_{22}, \\
\frac{\partial}{\partial t}\rho_{23} &= i\Omega\rho_{21} - (\gamma_{23}+i\delta)\rho_{23}.
\end{aligned}$$

The steady-state solutions to (21) can be found analytically in general form, but the formulas are rather involved, so we only adduce them here for the special case when $\delta = 0$.

$$\rho_{11} = \frac{(N+1)\left[2\gamma_{13}^{(-)}(\phi)\Omega^2 + (N+1)\gamma_{23}^{(-)}(0)\gamma_{23}^{(+)}(0)\gamma_1\right]}{2(3N+2)\gamma_{13}^{(-)}(\phi)\Omega^2 + (3N+1)(N+1)\gamma_{13}^{(-)}(0)\gamma_{13}^{(+)}(0)\gamma_1},$$

$$\rho_{22} = \frac{N\left[2\gamma_{13}^{(-)}(\phi)\Omega^2 + (N+1)\gamma_{23}^{(-)}(0)\gamma_{23}^{(+)}(0)\gamma_1\right]}{2(3N+2)\gamma_{13}^{(-)}(\phi)\Omega^2 + (3N+1)(N+1)\gamma_{13}^{(-)}(0)\gamma_{13}^{(+)}(0)\gamma_1}, \qquad (22)$$

$$\rho_{31} = \frac{-i(N+1)\left(|M|\gamma_1 e^{i\phi} - \gamma_{13}\right)\gamma_1\Omega}{2(3N+2)\gamma_{13}^{(-)}(\phi)\Omega^2 + (3N+1)(N+1)\gamma_{13}^{(-)}(0)\gamma_{13}^{(+)}(0)\gamma_1},$$

where

$$\gamma_{mn}^{(\pm)} = \gamma_{mn} \pm |M|\gamma_1\cos\phi. \qquad (23)$$

It is seen from (22) that when the squeezed vacuum acts on the same transition as the coherent field the atomic density matrix elements depend on the phase ϕ of the squeezed light. This is the effect known from the two-level system damped to the squeezed vacuum [4,18], which leads to different damping rates for the two quadrature components of the atomic dipole associated with the transition $|1\rangle \leftrightarrow |3\rangle$.

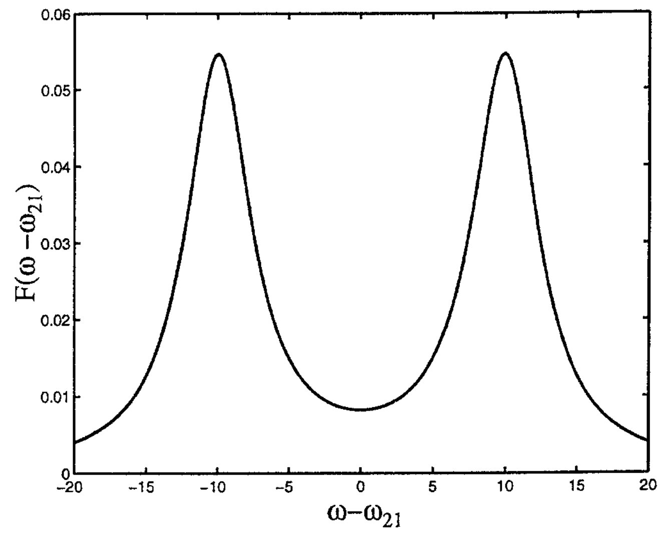

Fig. 4 The fluorescence spectrum on the probe transition when the squeezed vacuum frequency is tuned to the pump transition. There is no difference in the scale of the figure between the thermal field and the squeezed vacuum. The parameters are: $\gamma_1 = 1$, $\gamma_2 = 0.001$, $\delta = 0$, $\Omega = 10$, $\phi = \pi/10$, $N = 5$, and $M = \sqrt{30}$

The fluorescence and absorption spectra for this case are given by

$$F(\omega) = \mathrm{Re}\left\{\left.\frac{(z+\gamma_{23}-i\delta)\rho_{22}}{(z+\gamma_{12})(z+\gamma_{23}-i\delta)+\Omega^2}\right|_{z=i(\omega-\omega_{21})}\right\}, \qquad (24)$$

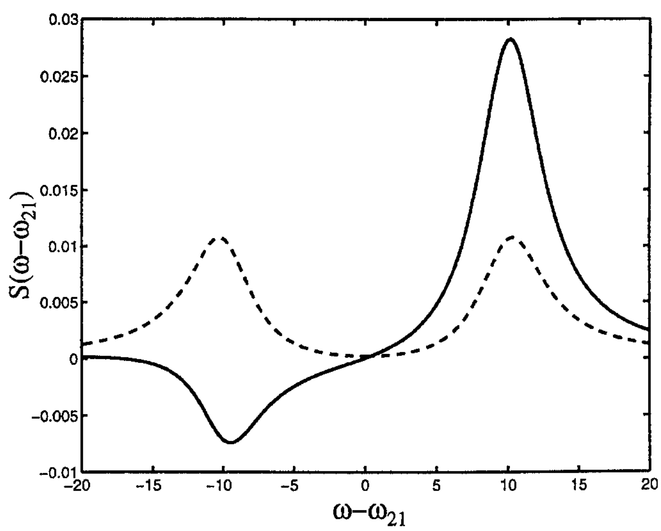

Fig. 5 The absorption spectrum on the probe transition when the squeezed vacuum frequency is tuned to the pump transition for $M = \sqrt{30}$ (solid line), and for $M = 0$ (dashed line). The other parameters are the same as in Fig. 4

$$S(\omega) = \mathrm{Re}\left\{\left.\frac{(z+\gamma_{23}-i\delta)(\rho_{11}-\rho_{22}) - i\Omega\rho_{31}}{(z+\gamma_{12})(z+\gamma_{23}-i\delta)+\Omega^2}\right|_{z=i(\omega-\omega_{21})}\right\}. \qquad (25)$$

From equations (24) and (25) it is clear that when the squeezed vacuum acts on the pump transition, both the fluorescence and absorption spectra on the probe transition do not exhibit, contrary to the previous case, any additional structure. Their dependence on the squeezed vacuum parameter M is only through the atomic populations given by (22). For high Rabi frequencies the population ρ_{22} of the upper lasing level is independent of M as well as damping rates, and it is given by

$$\rho_{22} = \frac{N}{3N+2}. \tag{26}$$

The same is also true for low Rabi frequencies, for which the population ρ_{22} is equal to

$$\rho_{22} = \frac{N}{2(3N+1)(3N+2)}. \tag{27}$$

The examples of the fluorescence and absorption spectra for the pump transition are shown in Fig. 4 and Fig. 5. The fluorescence spectrum shown in Fig. 4 exhibits two peaks the presence of which is caused by the splitting of the state $|1\rangle$ in the strong field. For the choice of atomic parameters in Fig. 4 there is no visible difference between the fluorescence in the squeezed vacuum and a thermal field with the same number of photons. The absorption spectrum illustrated in Fig. 5 also reveals the two-peak structure but with absorption on one side of the resonance and gain (negative absorption) on the other side. This is the gain without population inversion in the bare atomic states. Since the atomic populations (22) depend on the phase of the squeezed vacuum, the amplification of the probe light depends crucially on this phase. The structure of the spectra is better understood in the dressed-state picture discussed in the next Section.

4. DRESSED STATE PICTURE

In the strong field limit the dressed-atom picture [19, 20] provides useful description of the interaction of atom with strong fields. In this approach, eigenstates of the coupled atom+driving field system, referred to as the dressed states, serve as the basis for the whole system. For our model the semi-classical dressed states are the eigenvectors $|2\rangle$ and $|\pm\rangle$ of the Hamiltonian

$$H = \hbar \begin{bmatrix} 0 & 0 & \Omega \\ 0 & \omega_{21} & 0 \\ \Omega & 0 & -\delta \end{bmatrix}. \tag{28}$$

The dressed states are given by

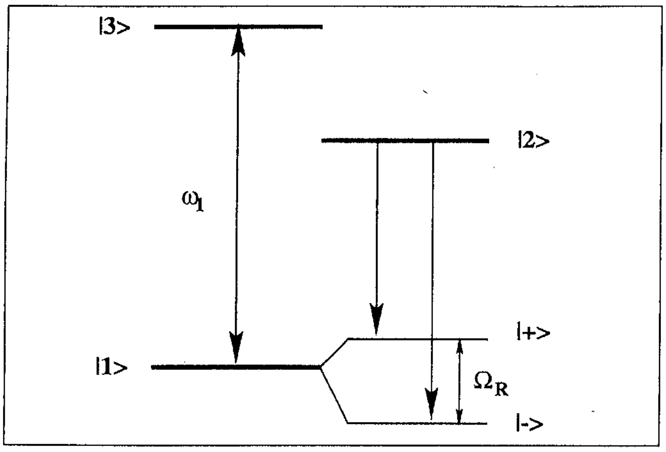

Fig. 6 The semi-classical dressed states of the atom pumped by the strong field with the Rabi frequency Ω_R on the $1 \leftrightarrow 3$ transition and the possible transitions leading to fluorescence on the $1 \leftrightarrow 2$ transition

$$|\pm\rangle = \frac{1}{\sqrt{2}}\sqrt{1 \pm \frac{\delta}{\Omega_R}}|1\rangle \pm \frac{1}{\sqrt{2}}\sqrt{1 \mp \frac{\delta}{\Omega_R}}|3\rangle \tag{29}$$

with eigenvalues equal, respectively, to

$$E_2 = \hbar\omega_{21}, \qquad E_\pm = -\frac{1}{2}\hbar\delta \pm \frac{1}{2}\hbar\Omega_R, \tag{30}$$

and the Rabi frequency Ω_R equal to

$$\Omega_R = \sqrt{\delta^2 + 4\Omega^2}. \tag{31}$$

In the case when the squeezed vacuum is tuned to the probe transition there are two peaks in the fluorescence spectrum that can be easily explain as the transitions from the atomic level $|2\rangle$ to the two dressed states $|\pm\rangle$. For thermal field, when $|M| = 0$, and for $\delta = 0$, these are exactly two peaks as it is seen from the analytical formula (18). In the squeezed vacuum each of the peaks is composed of two separate peaks with different widths. It is, however, difficult to separate the two components of each peak because of the small distance between the components and the large width of one of them. A very weak additional structure in the fluorescence as well as absorption spectrum can be seen from Fig. 3 for large values of the detuning δ. In the case when the squeezed vacuum frequency is tuned to the pump transition the fluorescence spectrum on the probe transition shows only two peaks that reflect the structure of dressed states and the form of the spectrum is not affected by the squeezed vacuum parameter M. The squeezed vacuum modifies in this case only the steady-state solutions for the atomic populations and coherences, as it is seen from formulas (22).

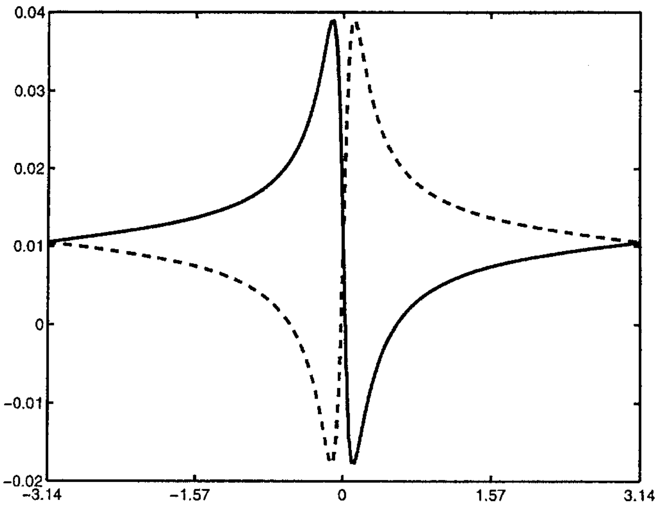

Fig. 7 The absorption spectrum at the frequency $\omega = \omega_{21} - \Omega$ (solid line) and at $\omega = \omega_{21} + \Omega$ (dashed line) versus the squeezed vacuum phase, when the squeezed vacuum is tuned to the pump transition. The parameters are: $\gamma_1 = 1$, $\gamma_2 = 0{,}001$, $\Omega = 10$, $\delta = 0$, $N = 5$, $M = \sqrt{30}$

The absorption spectrum shown in Fig. 5 reflects the same structure of the dressed states exhibiting two absorption peaks in the thermal field. In the squeezed vacuum, however, we can observe narrowing of the absorption peak on one side, and the negative absorption, i.e., the gain on the other side of the resonance. Amplification without population inversion can occur on this side. The amplification of the probe field depends strongly on the phase of the squeezed vacuum and the mechanism behind this effect is shown

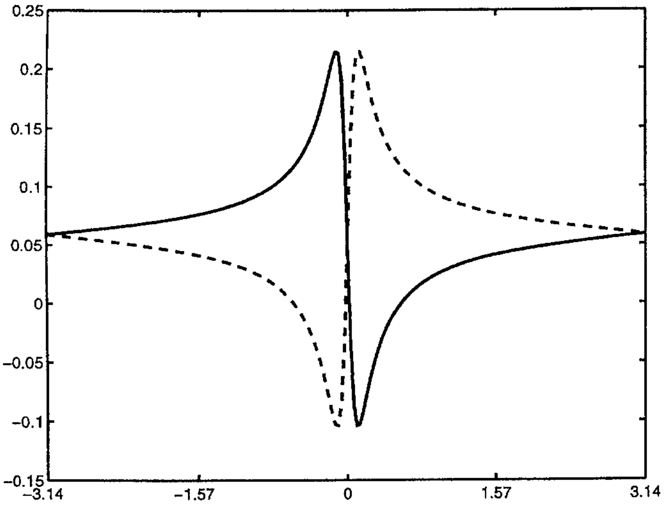

Fig. 8 The dressed-state population inversion $\rho_{--} - \rho_{22}$ (solid line) and $\rho_{++} - \rho_{22}$ (dashed line) versus the squeezed vacuum phase, when the squeezed vacuum is tuned to the pump transition. The parameters are are the same as Fig. 7.

in Fig. 8. There is population inversion between in the dressed states leading to the gain, and there is no population inversion on the absorption side. At low N the absorption spectrum shows absorption of the probe field. By increasing N we can get amplification or absorption depending on the phase of the squeezed vacuum.

We can see from our formulas for the fluorescence and absorption spectra, for the two cases considered in this paper, that for the case when the squeezed vacuum is tuned to the probe transition, the spectrum itself depends on $|M|$, but the steady-state populations and coherences do not depend on the squeezed vacuum parameter M. Quite differently, in the case when the squeezed vacuum frequency is tuned to the pump transition, the spectrum on the probe transition depends on the squeezed vacuum parameter M through the steady-state populations and coherence only.

5. CONCLUSIONS

In this paper we have studied the radiative properties of a three-level atom in V configuration that is pumped by a coherent laser field on one of the allowed transitions and probed on the other transition. The main idea of the paper was to study resonance fluorescence and absorption spectra on the probe transition taking into account the fact that the other transition is coherently pumped and assuming that the atom is damped to the squeezed vacuum instead of the ordinary vacuum or thermal field. Moreover, we have included the non-zero coherence transfer rate that couples the two atomic transitions. We have considered two different situations in which the carrier frequency of the squeezed vacuum was tuned either to the probe transition or to the pump transition. When the squeezed vacuum frequency is tuned to a particular atomic transition we have in fact the situation similar to the two-level atom damped to the squeezed vacuum, but the presence of the other atomic transition changes, sometimes in an essential way, the radiative properties of the atom with respect to those observed in the two-level atom. We have found analytical formulas for the fluorescence and absorption spectra and illustrated them graphically for certain choices of the atomic and field parameters. Our results show some similarities and differences with respect to the two-level atom spectra. We have found narrowing of the spectra in the squeezed vacuum, which is known from the two-level case, but we have also found a weak additional structure in the spectra. When the frequency of the squeezed vacuum is tuned to the pump transition and the pump field is strong, the absorption spectrum on the probe transition has two peaks at Rabi sidebands: one of them shows absorption and the other amplification of the probe field. In this case we get absorption without inversion in the bare atomic states, but as our analysis has shown, the mechanism for this gain is related to the population inversion in the dressed states. We have also shown that for the two cases of the squeezed vacuum considered in this paper the atomic coherence transfer rate has no influence on the atomic spectra. In the strong-field limit the dressed states appeared to be a good tool for explaining the features of the spectra.

Acknowledgments

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