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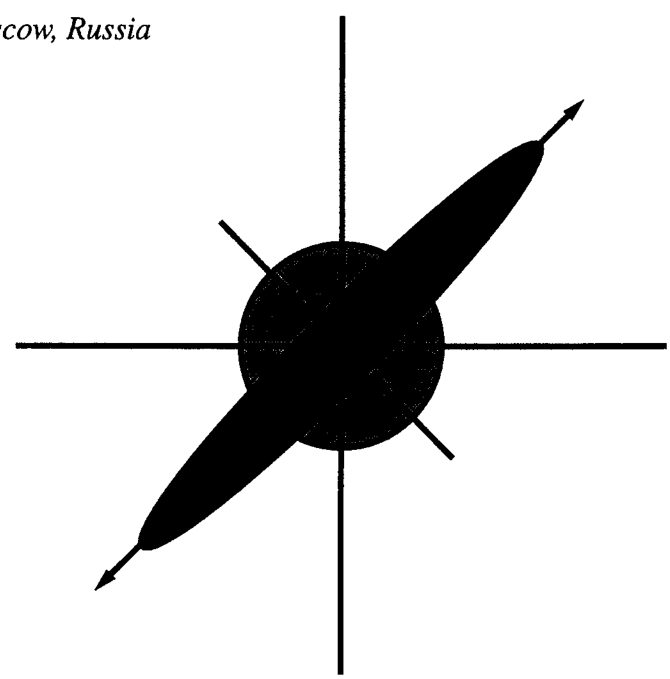

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RADIATIVE PROPERTIES OF ATOMS IN THE PRESENCE OF SQUEEZED VACUUM

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Abstract

We study radiative properties of a three-level atom in a broad-band squeezed vacuum. The atom in lambda configuration and with a non-zero coherence transfer rate between the two allowed transitions is damped to a broad-band squeezed vacuum and driven by a coherent external field. The carrier frequency of the squeezed vacuum is tuned to half of the sum of the two transition frequencies. The atomic fluorescence and absorption spectra are modified by the presence of the squeezed vacuum. We show an interesting interplay of the coherence transfer rate and squeezing in modifying the spectra.

1 Introduction

It is well known that three-level atoms subjected to a squeezed vacuum exhibit a number of interesting differences with respect to the atoms placed in ordinary vacuum [1-6]. There are, for example, spectacular qualitative changes in the steady-state populations such as two-photon population inversions [1-3]. The results for three-level systems driven by two independent laser beams [4-6] show that the relative heights and widths of the peaks in the fluorescence spectra can be subnatural or supernatural depending on the relative phase between the driving fields and squeezed vacuum, similarly as in two-level systems.

Recently, Ferguson et al. [7] have examined the effect of a single broad-band squeezed vacuum on the stationary populations and coherences in a three-level atom of lambda configuration that is driven by two independent laser fields. They also included the coherence transfer rates that couple the two allowed atomic transitions. In this paper we follow the approach used in [7], and we consider a three-level atom in lambda configuration that is damped to a single broad-band squeezed vacuum, with the bandwidth that is much larger than the distance between the two lower levels, but instead of two fields driving two allowed transitions, we assume that only one atomic transition is driven. We have found [8] that there is a non-zero steady-state atomic coherence associated with the other transition. Moreover, we have found interesting modifications of the atomic spectra resulting from the interplay of the atomic coherence transfer rate and squeezing.

2 The model

We consider the system which is schematically shown in Fig. 1. A non-degenerate three-level atom with the allowed transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ is coherently driven by a resonant field on the transition $1 \leftrightarrow 3$ and damped into a broad-band squeezed vacuum the carrying frequency of which is equal to ω_s. We assume that the distance (frequency) between the two lower states is bigger than the Rabi frequency Ω of the driving field and the atomic damping rates, but the squeezed vacuum is considered as a single broad-band squeezed vacuum, i.e., we assume that its bandwidth is larger than this distance, so both of the lower states are subjected to the same vacuum. We also assume that the atomic system has non-zero coherence transfer rate γ_c that couples the two allowed transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$.

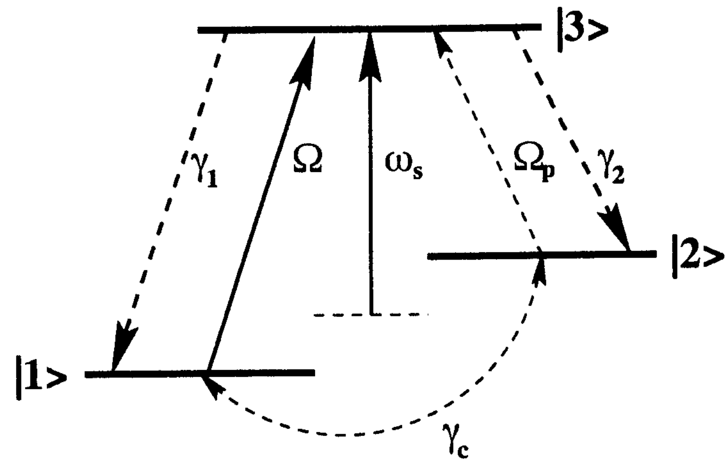

FIG. 1. Schematic diagram of the system

Evolution of the system is described by the master equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho, \tag{1}$$

where the Hamiltonian of the system is given by

$$H = \hbar\,\omega_{31}\,|3\rangle\langle 3| + \hbar\,\omega_{21}\,|2\rangle\langle 2| + \{\hbar\,\Omega e^{-i\omega_1 t}\,|3\rangle\langle 1| + \hbar\,\Omega_p e^{-i\omega_2 t}|3\rangle\langle 2| + H.c.\}, \tag{2}$$

and the irreversible part of the master equation is given by

$$\begin{aligned}\mathcal{L}\rho \;=\; & -\frac{1}{2}(N+1)\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho\, S_i^+) \\ & -\frac{1}{2}N\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^- S_j^+ + S_i^- S_j^+ \rho - 2S_j^+ \rho\, S_i^-) \\ & -\frac{1}{2}M\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^+ S_j^+ + S_i^+ S_j^+ \rho - 2S_j^+ \rho\, S_i^+)e^{-i2\omega_s t} \\ & -\frac{1}{2}M^*\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^- S_j^- + S_i^- S_j^- \rho - 2S_j^- \rho\, S_i^-)e^{i2\omega_s t},\end{aligned} \tag{3}$$

with $S_1^- = |3\rangle\langle 1|$, $S_2^- = |3\rangle\langle 2|$. The sum over i, j is with $i, j = 1, 2$ only. The parameters N and $M = |M|\exp(i\phi)$ characterize squeezed vacuum, where $|M|^2 \le N(N+1)$, and ω_s is the carrier frequency of the squeezed vacuum, and ϕ is its phase. We have assumed that N and M do not depend on the frequency. The damping rates Γ_{ij} are the spontaneous emission rates $\Gamma_{11} = \gamma_1$, $\Gamma_{22} = \gamma_2$ for the two allowed transitions, and $\Gamma_{12} = \Gamma_{21} = \gamma_c$ is the coherence transfer rate given by [7]

$$\gamma_c = \frac{\boldsymbol{\mu}_{13} \cdot \boldsymbol{\mu}_{23}}{12\pi\epsilon_0\hbar c^3}(\omega_{31}^3 + \omega_{32}^3)\,. \tag{4}$$

The laser driving field is described by the Rabi frequency Ω, and the probe field by the Rabi frequency Ω_p (actually our Ω's are halves of the Rabi frequencies). We assume here that the relation $2\omega_s - \omega_1 - \omega_2 = 0$ is satisfied, where ω_1 is the frequency of the driving field, and ω_2 is the frequency of the probe field, or in our case this is the frequency of the atomic dipole moment on the $2 \leftrightarrow 3$ transition. On neglecting all rapidly oscillating terms, and assuming that there is no probe field ($\Omega_p = 0$) and the driving field is resonant, the master equation (1) leads to the following equations

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= -N\gamma_1\rho_{11} + (N+1)\gamma_1\rho_{33} + i\,\Omega\,(\rho_{13} - \rho_{31})\,, \\
\frac{\partial}{\partial t}\rho_{12} &= -\gamma_{12}\,\rho_{12} - i\,\Omega\,\rho_{32}\,, \\
\frac{\partial}{\partial t}\rho_{13} &= -\gamma_{13}\,\rho_{13} + M^*\gamma_c\,\rho_{32} + i\,\Omega\,(\rho_{11} - \rho_{33})\,, \\
\frac{\partial}{\partial t}\rho_{22} &= -N\gamma_2\,\rho_{22} + (N+1)\,\gamma_2\,\rho_{33}\,, \\
\frac{\partial}{\partial t}\rho_{23} &= -\gamma_{23}\,\rho_{23} + M^*\gamma_c\,\rho_{31} + i\,\Omega\,\rho_{21}\,, \\
\frac{\partial}{\partial t}\rho_{33} &= N\gamma_1\,\rho_{11} + N\gamma_2\,\rho_{22} - (N+1)\,(\gamma_1 + \gamma_2)\,\rho_{33} - i\,\Omega\,(\rho_{13} - \rho_{31})\,,
\end{aligned} \tag{5}$$

where the damping parameters γ_{ij} are given by

$$\begin{aligned}
\gamma_{12} &= \frac{1}{2}(\gamma_1 + \gamma_2)\,N\,, \\
\gamma_{13} &= \left(\gamma_1 + \frac{1}{2}\gamma_2\right)N + \frac{1}{2}(\gamma_1 + \gamma_2)\,, \\
\gamma_{23} &= \left(\gamma_2 + \frac{1}{2}\gamma_1\right)N + \frac{1}{2}(\gamma_1 + \gamma_2)\,.
\end{aligned} \tag{6}$$

The relations $\rho_{ij} = \rho_{ji}^*$, and $\rho_{11} + \rho_{22} + \rho_{33} = 1$ hold for the atomic density matrix elements. It is interesting to note that due to the non-zero coherence transfer rate γ_c and non-zero squeezing parameter M, there are in equations (5) terms that couple the two atomic coherences ρ_{13} and ρ_{32}, which in consequence leads to non-zero steady-state values of them. This feature of the system is our main interest in this paper.

3 Steady-state solutions

The steady-state solutions of equations (5) give for the atomic coherences the following formulas

$$
\begin{aligned}
\rho_{12} &= \frac{\Omega^2 M \gamma_c}{\tilde{\gamma}_{13}(\gamma_{12}\gamma_{23} + \Omega^2)}(\rho_{11} - \rho_{33}), \\
\rho_{13} &= \frac{i\Omega}{\tilde{\gamma}_{13}}(\rho_{11} - \rho_{33}), \\
\rho_{23} &= \frac{-i\Omega M^* \gamma_c \gamma_{12}}{\tilde{\gamma}_{13}(\gamma_{12}\gamma_{23} + \Omega^2)}(\rho_{11} - \rho_{33}),
\end{aligned} \tag{7}
$$

where the parameter $\tilde{\gamma}_{13}$ is given by

$$
\tilde{\gamma}_{13} = \gamma_{13} - \frac{\gamma_{12} \mid M \mid^2 \gamma_c^2}{\gamma_{12}\gamma_{23} + \Omega^2}, \tag{8}
$$

and the steady-state population inversion is equal to

$$
\rho_{11} - \rho_{33} = \frac{N\gamma_1\tilde{\gamma}_{13}}{N(3N+2)\gamma_1\tilde{\gamma}_{13} + 2\Omega^2(3N+1)}. \tag{9}
$$

It is seen from formulas (7) that the squeezed vacuum, $M \neq 0$, in the presence of the non-zero coherence transfer rate, $\gamma_c \neq 0$, have a dramatic effect on the atomic coherences. The atomic coherences ρ_{12} and ρ_{23} that are not directly driven by the pump field acquire non-zero steady-state values that are proportional to the product of the value of squeezing M and the coherence transfer rate γ_c as well as to the atomic inversion $\rho_{11} - \rho_{33}$. This is a very interesting result showing that at certain circumstances squeezed vacuum, together with the coherence transfer rate, can play a role similar to a coherent field producing nonzero dipole moment on the transition that is not pumped by the coherent field.

4 Absorption and fluorescence spectra

Generally, the absorption and fluorescence spectra can be calculated for both atomic transitions. Their analytical form, however, is quite complicated and we do not write them here. For small Rabi frequencies there is no difference between the absorption spectra calculated for the pumped transition ($1 \leftrightarrow 3$) and the probe transition ($1 \leftrightarrow 2$). The absorption spectra for the two transitions differ for strong Rabi frequencies. The absorption spectrum for the pumped transition is similar to that for the two-level atom, whereas the spectrum for the probe transitions shows large absorption at both Rabi sidebands.

It can be shown that for small Rabi frequencies, the squeezed vacuum described by the parameter M in connection with the coherence transfer rate γ_c always leads to narrowing of the central peak of both absorption and fluorescence spectra. The spectrum has a form of two Lorentzians the widths of which are equal to the modified due to the squeezed vacuum damping rates of the two quadratures of the atomic coherence ρ_{13}. Example of such narrowing is shown in Fig. 2.

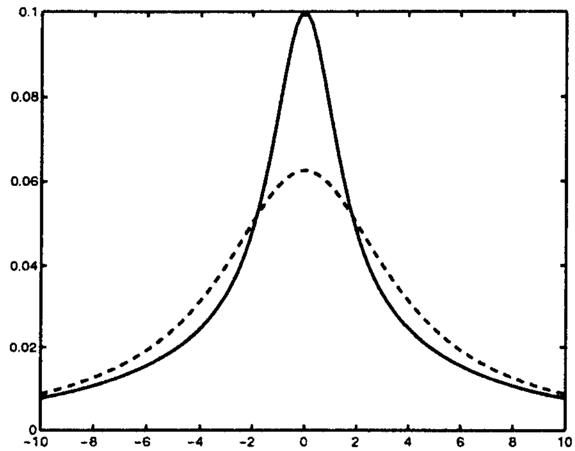

FIG. 2. Fluorescence spectrum for the transition $2 \leftrightarrow 3$ in: (i) the squeezed vacuum with $M = \sqrt{N(N+1)}$ (solid line), (ii) thermal field (dashed line). The parameters are: $\Omega = 0.1$, $\gamma_1 = \gamma_2 = \gamma_c = 1$, $N = 2$.

The absorption spectrum of a two-level atom in a squeezed vacuum with the carrier frequency resonant with the atomic transition (without coherent pumping) takes exceptionally simple form [9] of two Lorentzians. It is interesting to compare the spectrum for the three-level atom in the squeezed vacuum without coherent pumping with the corresponding spectrum for the two-level atom. For $\Omega = 0$, $\gamma_1 = \gamma_2 = \Gamma$, the fluorescence and absorption spectra for the three-level system take the following forms: for the fluorescence spectrum

$$\begin{aligned} F(\omega - \omega_{31}) &= \frac{\rho_{33}}{2}\left[\frac{\Gamma_+}{(\omega-\omega_{31})^2+\Gamma_+^2}\right] + \\ &\quad \frac{\rho_{33}}{2}\left[\frac{\Gamma_-}{(\omega-\omega_{31})^2+\Gamma_-^2}\right], \end{aligned} \tag{10}$$

and for the absorption spectrum

$$\begin{aligned} A(\omega - \omega_{31}) &= \frac{\rho_{11}-\rho_{33}}{2}\left[\frac{\Gamma_+}{(\omega-\omega_{31})^2+\Gamma_+^2}\right] + \\ &\quad \frac{\rho_{11}-\rho_{33}}{2}\left[\frac{\Gamma_-}{(\omega-\omega_{31})^2+\Gamma_-^2}\right], \end{aligned} \tag{11}$$

where the parameter $\Gamma_\pm$ is given by

$$\Gamma_\pm = \frac{1}{2}\left[(3N+2)\Gamma \pm 2|M|\gamma_c\right], \tag{12}$$

and where the atomic populations are given by

$$\rho_{11} = \frac{N+1}{(3N+2)}, \tag{13}$$

$$\rho_{33} = \frac{N}{(3N+2)}. \tag{14}$$

To get the spectra for the two-level system one has to make the transformations

$$\begin{aligned}\gamma_c &\rightarrow \Gamma\,, \\ (3N+2) &\rightarrow (2N+1)\,, \\ \rho_{33} &\rightarrow \rho_{22}\,, \\ \omega_{31} &\rightarrow \omega_{21}\,.\end{aligned}$$

We can note that the coherence transfer rate in the three-level system is crucial to get the same effect of narrowing the spectral lines as in the two-level system. If the atom has $\gamma_c \neq 0$ it is similar to the two-level system, however, if the atom has $\gamma_c = 0$, so, the three level system in squeezed vacuum is not similar to two-level system in squeezed vacuum, but it is similar to the two-level system in thermal field. This correspondence $\gamma_c \rightarrow \Gamma$ is essential to get similar spectra for two- and three-level systems. This is another example of joint effect of squeezing and atomic coherence transfer rate.

5 Conclusion

In this paper, we have shown few examples of coordinated action of the broad-band squeezed vacuum and non-zero atomic coherence transfer rate that can lead to essential modification of the radiative properties of the atom.

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