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ANALYTICAL RESULTS FOR THE PROBE ABSORPTION SPECTRUM OF A DRIVEN TWO-LEVEL ATOM IN A SQUEEZED VACUUM WITH FINITE BANDWIDTH[1]

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Analytical formulas for the probe absorption spectrum of a driven two-level atom damped to a squeezed vacuum with finite bandwidth are derived. We use the master equation approach to describe the evolution of the strongly driven two-level atom coupled to the reservoir being a squeezed vacuum with finite bandwidth produced by a degenerate parametric oscillator (DPO). The master equation is derived under the Born and Markov approximation which require the squeezed vacuum bandwidth to be much larger than the atomic linewidth, but not necessarily larger than the Rabi frequency of the driving field. Our master equation takes into account the detuning of the laser field from the atomic resonance. Examples of the absorption spectra are plotted and compared to their equivalents for the broadband squeezing.

1. Introduction

Squeezed vacuum is a reservoir with strong correlations between field amplitudes at frequencies placed symmetrically with respect to a certain carrier frequency ω_s, and the evolution of a quantum system in such an unusual reservoir exhibits a number of new features. Since the first paper published by Gardiner on spectroscopy with a broadband squeezed vacuum field [1] much work has been done to find such new features in the resonance fluorescence and probe absorption spectra of two- and three-level atoms in a squeezed vacuum [2–8].

Most of the studies dealing with the problem of a two-level atom in a squeezed vacuum assume that the squeezed vacuum is broadband, i.e., the bandwidth of the squeezed vacuum is much larger than the atomic linewidth and the Rabi frequency of

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the driving field. Experimental realizations of squeezed states [9, 10, 11, 12], however, indicate that the bandwidth of the squeezed light is typically of the order of the atomic linewidth. The most popular schemes for generating squeezed light are those using a parametric oscillator operating below threshold, the output of which is a squeezed beam with a bandwidth of the order of the cavity bandwidth [13, 14]. There are two types of squeezed field that can be generated by such a parametric oscillator. If the oscillator works in a degenerate regime, the squeezed field has the profile with the maximum of squeezing at the central frequency and a small squeezing far from the center. In the non-degenerate regime, the profile has two peaks at frequencies symmetrically displaced from the central frequency. For strong driving fields and finite bandwidth of squeezing this means that the Rabi sidebands can feel quite different squeezing then the central line. A realistic description of radiative properties of the two-level atom in such a squeezed field must thus take into account the finite bandwidth of the squeezed field.

First studies of the finite-bandwidth effects have been performed by Gardiner et al. [13], Parkins and Gardiner [15] and Ritsch and Zoller [16]. The approaches were based on stochastic methods and numerical calculations, and were applied to analyze the narrowing of the spontaneous emission and absorption lines. The fundamental effect of narrowing has been confirmed, but the effect of finite bandwidth was to degrade the narrowing of the spectral lines rather than enhance it. Later, however, numerical simulations done by Parkins [17, 18] demonstrated that for strong driving fields a finite bandwidth of squeezing can have positive effect on the narrowing of the Rabi sidebands. He has found that there is a difference between the two types of squeezed light generated in either degenerate or non-degenerate regime of the parametric oscillator. In the former case it is possible to narrow either both of the Rabi sidebands or the central peak of the fluorescent spectrum, while in the latter case simultaneous narrowing of all three spectral peaks is possible.

Yeoman and Barnett [19] have recently proposed an analytical technique for investigating the behaviour of a coherently driven atom damped by a squeezed vacuum with finite bandwidth. In the approach, they have derived a master equation and analytic expressions for the fluorescent spectrum for the simple case of a two-level atom exactly resonant with the frequencies of both the squeezed field and the driving field. Their analytical results agree with that of Parkins [17, 18] and show explicitly that the width of the central peak of the fluorescent spectrum depends solely on the squeezing present at the Rabi sideband frequencies. They have assumed that the atom is classically driven by a resonant laser field for which the Rabi frequency is much larger than the bandwidth of the squeezed vacuum though this is still large compared to the natural linewidth. Unlike the conventional theory, based on uncoupled states, it is possible to obtain a master equation consistent with the Born-Markov approximation by first including the interaction of the atom with the driving field exactly, and then considering the coupling of this combined dressed atom system with the finite-bandwidth squeezed vacuum. The advantage of this dressed atom method over the more complex treatments based on adjoint equation or stochastic methods [17, 18, 20] is that simple analytical expressions for the spectra can be obtained, thus displaying explicitly the factors that determine the intensities of the spectral features and their widths. The idea of Yeoman and Barnett has recently been extended by Ficek et al. [21] to the case of a fully quantized dressed-atom model coupled to a finite bandwidth squeezed field inside an optical cavity. They have studied the fluorescence spectrum under the secular approximation [22] and have found that in the presence of a single-mode cavity the effect of squeezing on the fluorescence spectrum is more evident in the linewidths of the Rabi sidebands rather than in the linewidth of the central component. In the presence of a two-mode cavity and a two-mode squeezed vacuum the signature of squeezing is evident in the linewidths of all spectral lines. They have also established that the narrowing of the spectral lines is very sensitive to the detuning of the driving field from the atomic resonance. The dressed atom method,under the secular approximation, including a detuning of the driving field from the atomic resonance has also been applied to calculate the probe absorption spectra of a driven three-level atom in a narrow bandwidth squeezed vacuum [23].

Recently, Tanaś et al. [24] have extended the Yeoman and Barnett [19] technique to include a non-zero detuning of the driving field from the atomic resonance and derived the master equation for a two-level atom driven by a classical laser field and damped by a finite-bandwidth squeezed vacuum. In this paper, we use this master equation to study the probe absorption spectrum for the two-level atom driven by a classical external field and damped by a squeezed vacuum with finite bandwidth produced by a degenerate parametric oscillator (DPO). Using the quantum regression theorem, we derive analytical formulas for the probe absorption spectrum of the atom. We show that for the finite bandwidth squeezed vacuum the absorption spectrum is modified in an essential way with respect to the spectrum for broadband squeezing.

2. Master equation

We consider a two-level atom driven by a detuned monochromatic laser field and damped by a squeezed vacuum with finite bandwidth. Applying the approach of paper [24], which is based on the idea of Yeoman and Barnett [19], being in turn an extension of the model proposed by Carmichael and Walls [25] and Cresser [26], we derive a master equation of the system which includes squeezing bandwidth effects. In this approach, we first perform the dressing transformation to include the interaction of the atom with the driving field and next we couple the resulting dressed atom to the narrow bandwidth squeezed vacuum field. We derive the master equation under the Markov approximation which requires the squeezing bandwidth to be much greater than the atomic linewidth, but not necessarily greater than the Rabi frequency of the driving field and the detuning. For simplicity, we assume that the squeezing properties are symmetric about the central frequency of the squeezed field which, in turn, is exactly equal to the laser frequency. Our approach differs from that of Yeoman and Barnett in performing the Markov approximation in the time domain rather than the Laplace transform variable domain with pole approximation, and in adding a non-zero detuning.

We start from the Hamiltonian of the system which in the rotating-wave and electric-dipole approximations is given by

$$H = H_A + H_R + H_L + H_I\,, \tag{1}$$

where

$$H_A = \frac{1}{2}\hbar\omega_A\,\sigma_z = -\frac{1}{2}\hbar\Delta\,\sigma_z + \frac{1}{2}\hbar\,\omega_L\,\sigma_z \tag{2}$$

is the Hamiltonian of the atom,

$$H_R = \int_0^\infty \omega\, b^+(\omega)\, b(\omega)\,\mathrm{d}\omega \tag{3}$$

is the Hamiltonian of the vacuum field,

$$H_L = \frac{1}{2}\hbar\Omega\,[\sigma_+\exp(-i\omega_L t) + \sigma_-\exp(i\omega_L t)\,] \tag{4}$$

is the interaction between the atom and the classical laser field, and

$$H_I = i\hbar\int_0^\infty K(\omega)\,\left[\sigma_+\, b(\omega) - b^+(\omega)\,\sigma_-\,\right]\,\mathrm{d}\omega \tag{5}$$

is the interaction of the atom with the vacuum field. In (2)-(5), $K(\omega)$ is the coupling of the atom to the vacuum modes, $\Delta = \omega_L - \omega_A$ is the detuning of the driving laser field frequency ω_L from the atomic resonance ω_A, and σ_+, σ_-, and σ_z are the Pauli pseudo-spin operators describing the two-level atom. The laser driving field strength is given by the Rabi frequency Ω, while the operators $b(\omega)$ and $b^+(\omega)$ are the annihilation and creation operators for the vacuum modes satisfying the commutation relation

$$[b(\omega), b^+(\omega')] = \delta(\omega - \omega')\,. \tag{6}$$

For simplicity, we assume that the laser field phase is equal to zero ($\phi_L = 0$).

In order to derive the master equation we perform the two-step unitary transformation. In the first step we use the second part of the atomic Hamiltonian (2) and the free field Hamiltonian (3) to transform to the frame rotating with the laser frequency ω_L and to the interaction picture with respect to the vacuum modes. After this transformation our system is described by the Hamiltonian

$$H_0 + H_I^r(t)\,, \tag{7}$$

where

$$H_0 = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(\sigma_+ + \sigma_-)\,, \tag{8}$$

and

$$H_I^r(t) = i\hbar\int_0^\infty K(\omega)\,\left[\sigma_+\, b(\omega)\,\exp[i(\omega_L - \omega)\,t] - b^+(\omega)\,\sigma_-\,\exp[-i(\omega_L - \omega)\,t]\right]\mathrm{d}\omega\,. \tag{9}$$

The second step is the unitary dressing transformation performed with the Hamiltonian H_0, given by (8). The transformation

$$\sigma_\pm(t) = \exp[-\frac{i}{\hbar}H_0 t]\sigma_\pm\exp[\frac{i}{\hbar}H_0 t] \tag{10}$$

leads to the following time-dependent atomic raising and lowering operators

$$\sigma_{\pm}(t) = \frac{1}{2}\left[\sigma_a \pm (1 \mp \tilde{\Delta})\sigma_b \exp(i\Omega' t) \pm (1 \pm \tilde{\Delta})\sigma_c \exp(-i\Omega' t)\right], \tag{11}$$

where

$$\begin{aligned} \sigma_a &= \tilde{\Omega}\left[\tilde{\Omega}(\sigma_+ + \sigma_-) - \tilde{\Delta}\sigma_z\right], \\ \sigma_b &= \frac{1}{2}\left[(1-\tilde{\Delta})\sigma_+ - (1+\tilde{\Delta})\sigma_- - \tilde{\Omega}\sigma_z\right], \\ \sigma_c &= \frac{1}{2}\left[(1+\tilde{\Delta})\sigma_+ - (1-\tilde{\Delta})\sigma_- + \tilde{\Omega}\sigma_z\right], \end{aligned} \tag{12}$$

are the 'dressed' operators oscillating at frequencies 0, Ω' and $-\Omega'$, respectively, and

$$\tilde{\Omega} = \frac{\Omega}{\Omega'}, \quad \tilde{\Delta} = \frac{\Delta}{\Omega'}, \quad \Omega' = \sqrt{\Omega^2 + \Delta^2}. \tag{13}$$

For $\Delta = 0$, the transformation (11) reduces to that of Yeoman and Barnett [19]. Under the transformation (11) the interaction Hamiltonian takes the form

$$H_I(t) = i\hbar \int_0^{\infty} K(\omega)\Big[\sigma_+(t)b(\omega)\exp[i(\omega_L - \omega)\,t] - \tag{14}$$

$$b^+(\omega)\sigma_-(t)\exp[-i(\omega_L - \omega)\,t]\Big]\mathrm{d}\omega\,. \tag{15}$$

The master equation for the reduced density operator ρ of the system can be derived using standard methods [27]. In the Born approximation the equation of motion for the reduced density operator is given by [27]

$$\frac{\partial \rho^D}{\partial t} = -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{[H_I(t), [H_I(t-\tau), \rho_R(0)\rho^D(t-\tau)]]\right\}\,\mathrm{d}\tau\,, \tag{16}$$

where the superscript D stands for the dressed picture, $\rho_R(0)$ is the density operator for the field reservoir, Tr_R is the trace over the reservoir states and the Hamiltonian $H_I(t)$ is given by (15). We next make the Markov approximation [27] by replacing $\rho^D(t-\tau)$ in (16) by $\rho^D(t)$, substitute the Hamiltonian (15) and take the trace over the reservoir variables. We assume that the reservoir is in a squeezed vacuum state in which the operators $b(\omega)$ and $b^+(\omega)$ satisfy the relations [1]

$$\begin{aligned} \mathrm{Tr}_R[\rho_R(0)\,b(\omega)b^+(\omega')] &= [N(\omega)+1]\,\delta(\omega-\omega')\,, \\ \mathrm{Tr}_R[\rho_R(0)\,b^+(\omega)b(\omega')] &= N(\omega)\,\delta(\omega-\omega')\,, \\ \mathrm{Tr}_R[\rho_R(0)\,b(\omega)\,b(\omega')] &= M(\omega)\,\delta(2\,\omega_L-\omega-\omega')\,, \end{aligned} \tag{17}$$

where $N(\omega)$ and $M(\omega)$ are the parameters describing the squeezing and that the carrier frequency of the squeezed field is equal to the laser frequency ω_L. In the Markov approximation we can extend the upper limit of the integration over τ to infinity and next perform necessary integrations using the formula

$$\int_0^\infty \exp(\pm i\,\epsilon\,\tau)\,\mathrm{d}\tau = \pi\,\delta(\epsilon) \pm i\,\mathcal{P}\frac{1}{\epsilon}\,, \tag{18}$$

where $\mathcal{P}$ means the Cauchy principal value. After lengthy calculations we obtain the master equation which in the frame rotating with the laser frequency ω_L can be written as

$$\begin{aligned}\dot{\rho} &= \frac{1}{2}i\,\gamma\,\delta\,[\sigma_z,\rho] \\ &+ \frac{1}{2}\gamma\,\tilde{N}\,\left(2\,\sigma_+\rho\,\sigma_- - \sigma_-\sigma_+\rho - \rho\,\sigma_-\sigma_+\right) \\ &+ \frac{1}{2}\gamma\,(\tilde{N}+1)\,\left(2\,\sigma_-\rho\,\sigma_+ - \sigma_+\sigma_-\rho - \rho\,\sigma_+\sigma_-\right) \\ &- \gamma\,\tilde{M}\,\sigma_+\rho\,\sigma_+ - \gamma\,\tilde{M}^*\,\sigma_-\rho\,\sigma_- \\ &- \frac{1}{2}i\,\Omega\,[\sigma_+ + \sigma_-,\rho] + \frac{1}{4}i\,\left(\beta\,[\sigma_+,[\sigma_z,\rho]] - \beta^*[\sigma_-,[\sigma_z,\rho]]\right)\,,\end{aligned} \tag{19}$$

where γ is the natural atomic linewidth,

$$\tilde{N} = N(\omega_L+\Omega') + \frac{1}{2}\,(1-\tilde{\Delta}^2)\,\mathrm{Re}\,\Gamma_-\,, \tag{20}$$

$$\tilde{M} = M(\omega_L+\Omega') - \frac{1}{2}\,(1-\tilde{\Delta}^2)\Gamma_- + i\,\tilde{\Delta}\,\delta_M\,\mathrm{e}^{i\phi}\,, \tag{21}$$

$$\delta = \frac{\Delta}{\gamma} - \frac{1}{2}(1-\tilde{\Delta}^2)\,\mathrm{Im}\,\Gamma_- + \tilde{\Delta}\,\delta_N\,, \tag{22}$$

$$\beta = \gamma\tilde{\Omega}\left[\delta_N + \delta_M\,\mathrm{e}^{i\phi} - i\,\tilde{\Delta}\,\Gamma_-\right]\,, \tag{23}$$

$$\Gamma_- = N(\omega_L) - N(\omega_L+\Omega') - \left[|M(\omega_L)| - |M(\omega_L+\Omega')|\right]\mathrm{e}^{i\phi}\,, \tag{24}$$

$$\delta_N = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{N(x)}{x+\Omega'}\,\mathrm{d}x\,, \qquad \delta_M = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{|M(x)|}{x+\Omega'}\,\mathrm{d}x\,, \tag{25}$$

and ϕ is the phase of squeezing ($M(\omega) = |M(\omega)|\exp(i\phi)$). In the derivation of equation (19) we have assumed that the phase ϕ does not depend on frequency [28], and we have included the divergent frequency shifts (the Lamb shift) to the redefinition of the atomic transition frequency [27]. Moreover, we have assumed that the squeezed vacuum is symmetric about the central frequency ω_L, so that $N(\omega_L-\Omega') = N(\omega_L+\Omega')$, and a similar relation holds for $M(\omega)$.

The master equation (19) has the standard form known from the broadband squeezing approaches with the new effective squeezing parameters $\tilde{N}$ and $\tilde{M}$ given by (20) and (21). There are also new terms, proportional to β which are essentially narrow bandwidth modifications to the master equation. All the narrow bandwidth modifications are determined by the parameter Γ_- given by (24), which represents the difference between the squeezing values at the central line and the sidebands, and the shifts δ_N and δ_M defined in (25). They all become zero when the squeezing bandwidth goes to infinity.

The squeezing induced shifts δ_N and δ_M depend on the explicit form of $N(\omega)$ and $|M(\omega)|$. For a degenerate parametric oscillator (DPO) the squeezing properties are described by [13]

$$\begin{aligned} N(x) &= \frac{\lambda^2-\mu^2}{4}\left[\frac{1}{x^2+\mu^2}-\frac{1}{x^2+\lambda^2}\right], \quad (26)\\ |M(x)| &= \frac{\lambda^2-\mu^2}{4}\left[\frac{1}{x^2+\mu^2}+\frac{1}{x^2+\lambda^2}\right], \quad (27)\end{aligned}$$

where $x=\omega-\omega_L$, and λ and μ are related to the cavity damping rate, γ_c, and the real amplification constant, ϵ, of the parametric oscillator according to

$$\lambda = \gamma_c+\epsilon\,, \qquad \mu=\gamma_c-\epsilon\,.$$

The Cauchy principal values of the integrals (25) can be evaluated using the contour integration which gives

$$\delta_N = \delta_\mu-\delta_\lambda\,, \qquad \delta_M=\delta_\mu+\delta_\lambda\,, \quad (28)$$

where the form of δ_μ and δ_λ for the degenerate parametric oscillator is given by

$$\delta_\mu = \gamma\,\Omega'\,\frac{\lambda^2-\mu^2}{4}\,\frac{1}{\mu\,(\Omega'^2+\mu^2)}\,, \qquad \delta_\lambda=\gamma\,\Omega'\,\frac{\lambda^2-\mu^2}{4}\,\frac{1}{\lambda\,(\Omega'^2+\lambda^2)}\,. \quad (29)$$

From the master equation (19) we easily derive the optical Bloch equations for the mean values of the atomic operators

$$\begin{aligned}\langle\dot\sigma_-\rangle &= -\gamma\,(\frac{1}{2}+\tilde N-i\,\delta)\langle\sigma_-\rangle-\gamma\,\tilde M\,\langle\sigma_+\rangle+\frac{i}{2}\Omega\,\langle\sigma_z\rangle\,,\\ \langle\dot\sigma_z\rangle &= i\,(\Omega+\beta^*)\langle\sigma_-\rangle-i\,(\Omega+\beta)\,\langle\sigma_+\rangle-\gamma\,(1+2\,\tilde N)\,\langle\sigma_z\rangle-\gamma\,.\end{aligned} \quad (30)$$

The equation for $\langle\sigma_+\rangle$ is obtained as Hermitian conjugate of equation for $\langle\sigma_-\rangle$. Defining the Hermitian operators σ_x and σ_y as

$$\sigma_x = \frac{1}{2}(\sigma_-+\sigma_+)\,, \qquad \sigma_y=\frac{1}{2i}(\sigma_--\sigma_+)\,, \quad (31)$$

we get from (30) the following equations of motion for the atomic polarization quadratures

$$\begin{aligned}\langle\dot\sigma_x\rangle &= -\gamma\left(\frac{1}{2}+\tilde N+\mathrm{Re}\,\tilde M\right)\langle\sigma_x\rangle-\gamma\left(\mathrm{Im}\,\tilde M+\delta\right)\langle\sigma_y\rangle\,,\\ \langle\dot\sigma_y\rangle &= -\gamma\left(\mathrm{Im}\,\tilde M-\delta\right)\langle\sigma_x\rangle-\gamma\left(\frac{1}{2}+\tilde N-\mathrm{Re}\,\tilde M\right)\langle\sigma_y\rangle+\frac{1}{2}\,\Omega\,\langle\sigma_z\rangle\,,\\ \langle\dot\sigma_z\rangle &= 2\,\mathrm{Im}\,\beta\,\langle\sigma_x\rangle-2\,(\Omega+\mathrm{Re}\,\beta)\,\langle\sigma_y\rangle-\gamma\,(1+2\,\tilde N)\,\langle\sigma_z\rangle-\gamma\,.\end{aligned} \quad (32)$$

The Bloch Eqs.(32) show clearly the two different decay rates $\gamma_x = \gamma\left(\frac{1}{2} + \tilde{N} + \mathrm{Re}\,\tilde{M}\right)$ and $\gamma_y = \gamma\left(\frac{1}{2} + \tilde{N} - \mathrm{Re}\,\tilde{M}\right)$ for the two quadrature components of the atomic dipole $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ which are already known from the Gardiner paper [1], but now the squeezing parameters $\tilde{N}$ and $\tilde{M}$ are more complicated. We can also see that the purely narrow-bandwidth features represented by $\mathrm{Im}\tilde{M}$ and β introduce additional couplings between the components of the Bloch vector.

3. Steady-state solutions

The Bloch equations (32) can be easily solved for the steady-state values of the atomic variables, and the result is given by

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{1}{2}\gamma\frac{\Omega\,\gamma(\mathrm{Im}\,\tilde{M}+\delta)}{d}\,,\\
\langle\sigma_y\rangle_{ss} &= -\frac{1}{2}\gamma\frac{\Omega\,\gamma\left(\frac{1}{2}+\tilde{N}+\mathrm{Re}\,\tilde{M}\right)}{d}\,,\\
\langle\sigma_z\rangle_{ss} &= -\gamma\frac{\gamma^2\left(\frac{1}{4}+\tilde{N}(\tilde{N}+1)-|\tilde{M}|^2+\delta^2\right)}{d}\,,
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
d &= \gamma^3(1+2\tilde{N})\left(\frac{1}{4}+\tilde{N}(\tilde{N}+1)-|\tilde{M}|^2+\delta^2\right)\\
&+ \gamma\Omega\left[\left(\frac{1}{2}+\tilde{N}+\mathrm{Re}\,\tilde{M}\right)(\Omega+\mathrm{Re}\,\beta)+\mathrm{Im}\,\beta\,(\mathrm{Im}\,\tilde{M}+\delta)\right].
\end{aligned} \tag{34}$$

The steady-state solutions (33) exhibit a number of interesting features. It is seen that generally all the components of the Bloch vector have nonzero steady-state values. Even for the resonant driving field ($\Delta = 0$), we find from (21), (22) and (24) that

$$\mathrm{Im}\,\tilde{M} + \delta = |M(\omega_A)|\sin\phi\,, \tag{35}$$

indicating that even for $\Delta = 0$ the $\langle\sigma_x\rangle_{ss}$ component of the Bloch vector can have a non-zero steady-state solution provided the phase ϕ is different from 0 or π and there is a non-zero squeezing at the atomic resonance. This effect can lead to unequal populations of the dressed states of the system [29, 24]. The dressed states can be found by diagonalizing the Hamiltonian (8), which gives

$$\begin{aligned}
|1\rangle &= \sqrt{\frac{1+\tilde{\Delta}}{2}}\,|g\rangle + \sqrt{\frac{1-\tilde{\Delta}}{2}}\,|e\rangle\,,\\
|2\rangle &= -\sqrt{\frac{1-\tilde{\Delta}}{2}}\,|g\rangle + \sqrt{\frac{1+\tilde{\Delta}}{2}}\,|e\rangle\,,
\end{aligned} \tag{36}$$

where the dressed energies are: $E_1 = \hbar\Omega'/2$ and $E_2 = -\hbar\Omega'/2$, and $|g\rangle$ and $|e\rangle$ are the ground and the excited state of the atom, respectively. The populations of the dressed states can be expressed in terms of the expectation values $\langle\sigma_x\rangle_{ss}$ and $\langle\sigma_z\rangle_{ss}$ as follows

$$\begin{aligned}\rho_{11} &= \frac{1}{2}\left(1-\tilde{\Delta}\,\langle\sigma_z\rangle_{ss}\right)+\sqrt{1-\tilde{\Delta}^2}\,\langle\sigma_x\rangle_{ss}\,,\\ \rho_{22} &= \frac{1}{2}\left(1+\tilde{\Delta}\,\langle\sigma_z\rangle_{ss}\right)-\sqrt{1-\tilde{\Delta}^2}\,\langle\sigma_x\rangle_{ss}\,.\end{aligned} \tag{37}$$

For a resonant driving field ($\tilde{\Delta}=0$) the stationary populations of the dressed states depend solely on $\langle\sigma_x\rangle_{ss}$, which, on the other hand, can be non-zero only when the phase ϕ is different from 0 and π and, simultaneously, there is a non-zero squeezing at the atomic resonance, as it is the case for the degenerate parametric amplifier.

If the laser field is detuned from the atomic resonance ($\tilde{\Delta}\neq 0$) the dressed states populations are different even for the most frequently discussed cases $\phi = 0, \pi$, for which we have

$$\mathrm{Im}\,\tilde{M}+\delta = \frac{\Delta}{\gamma}+\tilde{\Delta}\,(\delta_N \pm \delta_M)\,, \tag{38}$$

where the upper sign is for $\phi = 0$ and the lower sign for $\phi = \pi$. This means that the $\langle\sigma_x\rangle_{ss}$ component of the Bloch vector changes sign when Δ changes sign, and it is equal to zero only on resonance.

4. Absorption spectrum

The probe absorption spectrum of a two-level atom is given by the Fourier transform of the two-time atomic correlation functions as [30]

$$\mathcal{A}(\omega) = \frac{1}{\pi}\mathrm{Re}\left\{\int_0^\infty \langle[\sigma_-(\tau),\sigma_+(0)]\rangle_{ss}\,\mathrm{e}^{i(\omega-\omega_L)\tau}\,\mathrm{d}\tau\right\}\,, \tag{39}$$

where Re denotes the real part of the integral. The absorption spectrum is defined by the difference of two atomic correlation functions (coming from the commutator in (39)). The evolution of such a difference can be found from the Bloch equations (30) by applying the quantum regression theorem [31]. The equations of motion for the difference of two-time correlation functions can be written as

$$\frac{\partial}{\partial\tau}\begin{pmatrix}\langle[\sigma_-(\tau),\sigma_+(0)]\rangle_{ss}\\ \langle[\sigma_+(\tau),\sigma_+(0)]\rangle_{ss}\\ \langle[\sigma_z(\tau),\sigma_+(0)]\rangle_{ss}\end{pmatrix} = \mathbf{B}\begin{pmatrix}\langle[\sigma_-(\tau),\sigma_+(0)]\rangle_{ss}\\ \langle[\sigma_+(\tau),\sigma_+(0)]\rangle_{ss}\\ \langle[\sigma_z(\tau),\sigma_+(0)]\rangle_{ss}\end{pmatrix} \tag{40}$$

where $\mathbf{B}$ is the 3×3 matrix

$$\mathbf{B} = \begin{pmatrix}-\gamma(\frac{1}{2}+\tilde{N}-i\delta) & -\gamma\tilde{M} & \frac{i}{2}\Omega\\ -\gamma\tilde{M}^* & -\gamma(\frac{1}{2}+\tilde{N}+i\delta) & -\frac{i}{2}\Omega\\ i(\Omega+\beta^*) & -i(\Omega+\beta) & -\gamma(1+2\tilde{N})\end{pmatrix}, \tag{41}$$

and the initial values for the correlation functions are

$$
\begin{aligned}
\langle\sigma_-\sigma_+\rangle_{ss} - \langle\sigma_+\sigma_-\rangle_{ss} &= -\langle\sigma_z\rangle_{ss}\,, \\
\langle\sigma_+\sigma_+\rangle_{ss} &= 0\,, \\
\langle\sigma_z\sigma_+\rangle_{ss} - \langle\sigma_+\sigma_z\rangle_{ss} &= 2\,\langle\sigma_+\rangle_{ss}\,.
\end{aligned}
\tag{42}
$$

Taking the Laplace transform of (40) we obtain the system of algebraic equations for the transformed variables which can be easily solved. The solution gives us the following formula for the Laplace transform of the difference $\langle\sigma_-(\tau)\sigma_+(0)\rangle_{ss} - \langle\sigma_+(0)\sigma_-(\tau)\rangle_{ss}$

$$
\begin{aligned}
A(z) &= \frac{1}{d(z)}\Big\{ i\,\langle\sigma_+\rangle_{ss}\,\Omega\left[\gamma(\frac{1}{2}+\tilde{N}+\tilde{M}+i\delta)+z\right] - \langle\sigma_z\rangle_{ss}\left[\gamma^2(1+2\tilde{N})(\frac{1}{2}+\tilde{N}+i\delta)\right. \\
&\quad \left.+\,\frac{1}{2}\Omega(\Omega+\beta)+\gamma(\frac{3}{2}+3\tilde{N}+i\delta)\,z+z^2\right]\Big\}
\end{aligned}
\tag{43}
$$

where

$$
d(z) = d + \left[\frac{5}{4}+5\tilde{N}(\tilde{N}+1)-|\tilde{M}|^2+\delta^2+\Omega(\Omega+\mathrm{Re}\,\beta)\right] z + 2(1+2\tilde{N})\,z^2 + z^3 \tag{44}
$$

with d given by (34), and

$$
\langle\sigma_+\rangle_{ss} \;=\; \langle\sigma_x\rangle_{ss} - i\,\langle\sigma_y\rangle_{ss} = i\,\frac{\Omega}{2d}\gamma^2\left(\frac{1}{2}+\tilde{N}+\tilde{M}^*-i\delta\right). \tag{45}
$$

From the Laplace transform (43), the probe absorption spectrum defined by (39) is obtained as

$$
\mathcal{A}(\omega) \;=\; \frac{1}{\pi}\,\mathrm{Re}\{A(z)|_{z=-i(\omega-\omega_L)}\} \tag{46}
$$

Formulas (43)-(46) are relatively simple analytical expressions that describe the probe absorption spectrum of the atom driven by the external field with the Rabi frequency Ω, detuned by Δ from the atomic resonance, and damped to the finite bandwidth squeezed vacuum produced by degenerate parametric oscillator.

Let us discuss the simplest case of resonant driving field, $\Delta = 0$, and the squeezed vacuum phase $\phi = 0, \pi$. In this case $\tilde{N}$, $\tilde{M}$, and β are real, $\delta = 0$, and the denominator (44) can be factored into

$$
\begin{aligned}
d(z) &= \left[z+\gamma(\frac{1}{2}+\tilde{N}+\tilde{M})\right]\left[\gamma^2(1+2\tilde{N})(\frac{1}{2}+\tilde{N}-\tilde{M})+\Omega(\Omega+\beta)\right. \\
&\quad \left.+\,\gamma(\frac{3}{2}+3\tilde{N}-\tilde{M})\,z+z^2\right].
\end{aligned}
\tag{47}
$$

Finding the roots of the polynomial $d(z)$, we get

$$
z_0 \;=\; -\gamma_x\,, \qquad z_\pm = -\frac{1}{2}(\gamma_y+\gamma_z) \pm \Omega_R\,, \tag{48}
$$

where

$$\begin{aligned}
\gamma_x &= \gamma\,(\frac{1}{2}+\tilde{N}+\tilde{M})\,, \quad \gamma_y = \gamma\,(\frac{1}{2}+\tilde{N}-\tilde{M})\,, \quad \gamma_z = \gamma_x+\gamma_y\,, &(49)\\
\Omega_R &= \sqrt{\left|\Omega(\Omega+\beta)-\frac{1}{4}\gamma_x^2\right|}\,, &(50)\\
\tilde{N} &= \frac{1}{2}\left\{N(\omega_L)+N(\omega_L+\Omega)\mp(|M(\omega_L)|-|M(\omega_L+\Omega)|)\right\}\,, &(51)\\
\tilde{M} &= \pm\frac{1}{2}\left\{(|M(\omega_L)|+|M(\omega_L+\Omega)|)\mp[N(\omega_L)-N(\omega_L+\Omega)]\right\}\,, &(52)\\
\beta &= \delta_N\pm\delta_M\,. &(53)
\end{aligned}$$

In (51)-(53) the upper sign corresponds to $\phi=0$ and the lower sign to $\phi=\pi$. The roots (48) are all real for $\Omega(\Omega+\beta)-\gamma_x^2/4<0$, and, if $\Omega(\Omega+\beta)-\gamma_x^2/4>0$, $z_\pm$ become a complex conjugate pair with Ω_R replaced by $i\,\Omega_R$. They define the widths of the spectral lines and the effective Rabi frequency. It is clear that $\Omega(\Omega+\beta)-\gamma_x^2/4=0$ is a threshold at which the character of the solution changes.

Below the threshold, $\Omega(\Omega+\beta)-\gamma_x^2/4<0$, and for $\phi=0,\pi$, and $\Delta=0$, the spectrum takes the form

$$\begin{aligned}
\mathcal{A}(\omega) &= \frac{1}{2\pi}\frac{\gamma}{[\,\Omega(\Omega+\beta)+\gamma_y\gamma_z\,]}\left\{\frac{\gamma_x\gamma_y}{(\omega-\omega_L)^2+\gamma_x^2}\right.\\
&+\frac{1}{4\,\Omega_R}\frac{(2\Omega^2-\gamma_x\gamma_y+2\Omega_R\,\gamma_y)\left(\frac{\gamma_y+\gamma_z}{2}+\Omega_R\right)}{(\omega-\omega_L)^2+\left(\frac{\gamma_y+\gamma_z}{2}+\Omega_R\right)^2}\\
&\left.-\frac{1}{4\,\Omega_R}\frac{(2\Omega^2-\gamma_x\gamma_y-2\Omega_R\,\gamma_y)\left(\frac{\gamma_y+\gamma_z}{2}-\Omega_R\right)}{(\omega-\omega_L)^2+\left(\frac{\gamma_y+\gamma_z}{2}-\Omega_R\right)^2}\right\}. \qquad (54)
\end{aligned}$$

Above the threshold, $\Omega(\Omega+\beta)-\gamma_x^2/4>0$, the probe absorption spectrum is given by the following formula

$$\begin{aligned}
\mathcal{A}(\omega) &= \frac{1}{2\pi}\frac{\gamma}{[\,\Omega(\Omega+\beta)+\gamma_y\gamma_z\,]}\left\{\frac{\gamma_x\gamma_y}{(\omega-\omega_L)^2+\gamma_x^2}\right.\\
&+\frac{1}{4\Omega_R}\frac{\Omega_R\,\gamma_y(\gamma_y+\gamma_z)-(2\Omega^2-\gamma_x\gamma_y)\,(\omega-\omega_L+\Omega_R)}{(\omega-\omega_L+\Omega_R)^2+(\frac{\gamma_y+\gamma_z}{2})^2}\\
&\left.+\frac{1}{4\Omega_R}\frac{\Omega_R\,\gamma_y(\gamma_y+\gamma_z)+(2\Omega^2-\gamma_x\gamma_y)\,(\omega-\omega_L-\Omega_R)}{(\omega-\omega_L-\Omega_R)^2+(\frac{\gamma_y+\gamma_z}{2})^2}\right\}. \qquad (55)
\end{aligned}$$

Formulas (54) and (55) are analytical solutions for the probe absorption spectrum for a resonantly driven atom in the finite bandwidth squeezed vacuum. It is clear that the spectrum is symmetric with respect to the laser frequency $\omega_L=\omega_A$. Below the threshold it shows Lorentzian shape contributions with different widths at the laser frequency, and

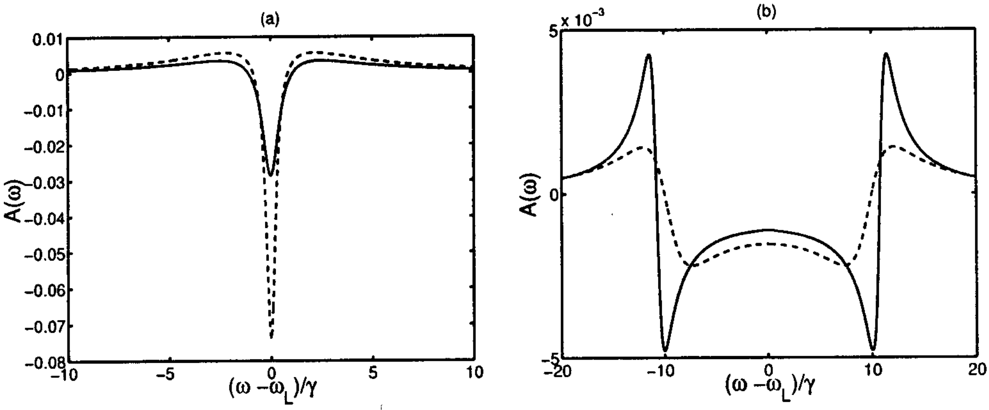

Fig. 1. Absorption spectrum for $\epsilon/\gamma_c = 0.5$ ($N(\omega_A) = 1.78$, $|M(\omega_A)| = 2.22$), $\phi = 0$, solid line — finite bandwidth with $\gamma_c/\gamma = 10$, dashed line — broadband squeezing: (a) below threshold ($\Omega = 1$), and (b) above threshold ($\Omega = 10$).

above the threshold it exhibits a Lorenzian line at the laser frequency and Lorentzian as well as dispersion features at the Rabi sidebands. For finite a bandwidth squeezed vacuum the widths and the amplitudes of the lines are defined by $\tilde{N}$, $\tilde{M}$, and β given by (51)-(53). For broadband squeezing $N(\omega)$ and $M(\omega)$ do not depend on ω, which means that $\tilde{N} = N$ and $\tilde{M} = \pm|M|$ are constants describing the broadband squeezing. In this case the shifts δ_N and δ_M are zero, and consequently $\beta = 0$. The damping parameters γ_x and γ_y depend on squeezing through N and M, as seen from (49). For ordinary vacuum $\gamma_x = \gamma_y = \gamma/2$ and the spectrum simplifies to the standard form [30].

Since $\tilde{N}+\tilde{M} = N(\omega_L+\Omega)+|M(\omega_L+\Omega)|$, it is clear that the width of the central line as well as the effective Rabi frequency are defined by the squeezing properties on the sidebands only, while the widths of the sidebands depend on the squeezing properties at the laser frequency as well as at the sidebands. This feature has been found by Yeoman and Barnett [19] who discussed the resonance fluorescence spectrum.

In Fig. 1 we have plotted examples of the absorption spectrum for both the below threshold (Fig. 1(a)) and the above threshold (Fig. 1(b)) situations. The solid lines represent the spectrum for the finite bandwidth squeezed vacuum calculated according to our formulas, which is compared to that obtained for broadband squeezing. The parameters we used to calculate the spectrum are: $\epsilon/\gamma_c = 0.5$ which gives $N(\omega_A) = 1.78$ and $|M(\omega_A)| = 2.22$, $\phi = 0$, $\Delta = 0$, $\gamma_c/\gamma = 10$ for narrow bandwidth (solid lines), $\gamma_c/\gamma = 100000$ for broadband squeezing (dashed lines), $\Omega = 1$ for figure (a), and $\Omega = 10$ for figure (b). It is evident from Fig. 1 that in real physical situation, when the bandwidth of squeezing is finite the amplification at the central line is diminished, but the dispersion profiles at the sidebands that appear for strong fields become stronger. One can expect better amplification at the sidebands, although in very narrow range of frequencies, if the bandwidth of the squeezed vacuum is narrow. Figure (a) shows a hole burning feature discussed for broadband squeezing by Zhou et al. [32] which exist also for the narrow bandwidth, but is not as deep as for the broad bandwidth. Modification of the Rabi sidebands shown in figure (b) agree qualitatively with that obtained by Bosticky et al.[23] under the secular approximation, which requires sufficiently strong driving fields.

In general case, the simple factorization of the denominator (44) is not possible, and the absorption spectrum cannot be reduced to the form similar to (55). Nevertheless, our analytical formula (43) can still be used to evaluate the spectrum numerically. For nonzero detuning and/or squeezing phase $\phi \neq 0, \pi$, the spectrum is no longer symmetric and exhibits a number of interesting features which appear for the driving fields with the Rabi frequencies comparable to the atomic linewidth. For broadband squeezing such features have recently been discussed by Ficek et al.[33]. For such fields the secular approximation is not valid, but our approach is still applicable and can be used to find the modifications of the spectra when the bandwidth of the squeezed vacuum becomes finite.

5. Conclusion

We have derived simple analytical formulas for the absorption spectrum of a driven two-level atom damped to a squeezed vacuum with finite bandwidth. The derivation is based on the master equation which is valid for the bandwidth of the squeezed vacuum much larger than the natural linewidth of the atom but not necessarily larger than the Rabi frequency of the driving field. This allows us to study the spectra for both weak and strong fields. The formulas obtained in the paper give better insight into the physical origin of the spectral features that appear when atom is damped to a squeezed vacuum with finite bandwidth. If the squeezing bandwidth becomes large our results reproduce the results known for broadband squeezing. We have shown examples of the absorption spectra below and above the threshold for the Rabi oscillations. This threshold depends on the parameters describing squeezed vacuum.

One has to remember, however, that the applicability of the approach is restricted by the Markov approximation used to derive the master equation, which requires the bandwidth of the squeezed vacuum to be much larger than the atomic linewidth. Violating this requirement can even lead to unphysical results.

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References

[1] C.W. Gardiner: *Phys. Rev. Lett.* **56** (1986) 1917

[2] A.S. Parkins: in *Modern Nonlinear Optics,* Part 2, volume LXXXV of *Advances in Chemical Physics,* ed. M. Evans, S. Kielich (Wiley, 1993)

[3] H.J. Carmichael, A.S. Lane, D.F. Walls: *J. Mod. Opt.* **34** (1987) 821

[4] J.M. Courty, S. Reynaud: *Europhys. Lett.* **10** (1989) 237

[5] S. Smart, S.Swain: *Phys. Rev. A* **45** (1992) 6863

[6] S. Smart, S. Swain: *Quantum Optics* **5** (1993) 75
[7] S. Smart, S. Swain: *Phys. Rev. A* **48** (1993) R50
[8] D. Grandclement, G. Grynberg, M. Pinard: *Phys. Rev. Lett.* **59** (1987) 40
[9] R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz, J.F. Valley: *Phys Rev. Lett.* **55** (1985) 2409
[10] R.M. Shelby, M.D. Levenson, S.H. Perlmutter, R.G. DeVoe, D.F. Walls: *Phys. Rev. Lett.* **57** (1986) 691
[11] L.A. Wu, H.J. Kimble, J.L. Hall, H. Wu: *Phys. Rev. Lett.* **57** (1986) 2520
[12] N.P. Georgiades, E.S. Polzik, K. Edamatsu, H.J. Kimble, A.S. Parkins: *Phys. Rev. Lett.* **75** (1995) 3426
[13] C.W. Gardiner, A.S. Parkins, M.J. Collett: *J. Opt. Soc. Am. B* **4** (1987) 1683
[14] M.J. Collett, R. Loudon, C.W. Gardiner: *J. Mod. Opt.* **34** (1987) 881
[15] A.S. Parkins, C.W. Gardiner: *Phys. Rev. A* **37** (1988) 3867
[16] H. Ritsch, P. Zoller: *Phys. Rev. Lett.* **61** (1988) 1097
[17] A.S. Parkins: *Phys. Rev. A* **42** (1990) 4352
[18] A.S. Parkins: *Phys. Rev. A* **42** (1990) 6873
[19] G. Yeoman, S.M. Barnett: *J. Mod. Opt.* **43** (1996) 2037
[20] J.I. Cirac: *Phys. Rev. A* **46** (1992) 4354
[21] Z. Ficek, B.J. Dalton, M.R.B. Wahiddin: *J. Mod. Opt.* **44** (1997) 1005
[22] C. Cohen-Tannoudji, S. Reynaud: *J. Phys. B* **10** (1977) 345
[23] M. Bosticky, Z. Ficek, B.J. Dalton: *Phys. Rev. A* **53** (1996) 4439
[24] R. Tanaś, Z. Ficek, A. Messikh, T. El-Shahat: *J. Mod. Opt.*, to be published (1998)
[25] H. Carmichael, D.F. Walls: *J. Phys. A: Math. Gen.* **6** (1973) 1552
[26] J. Cresser: *J. Mod. Opt.* **39** (1992) 2187
[27] W.H. Louisell: *Quantum Statistical Properties of Radiation* (Wiley, 1973)
[28] G. Yeoman: *Phys. Rev. A* **55** (1997) 710
[29] W.S. Smyth, S. Swain: *Opt. Commun.* **112** (1994) 91
[30] B.R. Mollow: *Phys. Rev. A* **5** (1972) 2217
[31] M. Lax: *Phys. Rev.* **172** (1968) 350
[32] P. Zhou, Z. Ficek, S. Swain: *J. Opt. Soc. Am. B* **13** (1996) 768
[33] Z. Ficek, S. Smyth, S. Swain: *Phys. Rev. A* **52** (1995) 4126