JOURNAL OF MODERN OPTICS, 1997, VOL. 44, NO. 11/12, 2105–2123

Fock states generation in a kicked cavity with a nonlinear medium

W. LEOŃSKI†, S. DYRTING‡ and R. TANAŚ†

† Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, Umultowska 85, 61-615 Poznań, Poland

‡ Department of Physics, University of Queensland, St Lucia 4072, Australia

(*Received 31 October 1996; revision received 17 February 1997*)

Abstract. We discuss a model of a cavity filled with a passive nonlinear 'Kerr' medium and periodically kicked by a series of ultra-short laser pulses. The nonlinear medium is described by the $(2q-1)$th nonlinearity $\chi^{(2q-1)}$. We find analytical formulas describing the field states inside the cavity. We show that such a system can produce, depending on the order of the nonlinearity, superpositions of several Fock states with the small photon numbers $(0,1;0,1,2;$ etc). In particular, the one-photon state can be approached during the evolution of the system with $\chi^{(3)}$ nonlinearity provided the cavity losses are negligible. The purity of states generated in this process, however, can be seriously degraded by the cavity damping. We perform numerical calculations to validate our analytical results.

1. Introduction

Quantum state engineering of the electromagnetic field has been a subject of increasing interest during the past few years. Among a large variety of field states the *n*-photon, or Fock, states are of special importance. They are states that are probably most frequently used in theoretical calculations and, at the same time, are difficult to produce experimentally. A number of papers that have dealt with the problem of generation of such states have already been published. Here, we shall mention only some of them. Stoler and Yurke [1] have studied a possibility of generation of antibunched light. Hong and Mandel [2] have shown that a one-photon Fock state can be produced in the parametric down converter. Another method of producng Fock states, based on the quantum non-demolition (QND) measurement, has been analysed by Brune *et al.* [3, 4]. They have proposed the QND experiment in which detection of the atomic phase by the Ramsey method plays the role of a QND probe gving information on the cavity field energy. They have shown that after a sequence of atomic measurements the cavity field collapses into a Fock state with an unpredictable number of photons. Moreover, Ueda *et al.* [5], who have dealt with the same problem, have also shown that QND measurement can lead to the state reduction towards *n*-photon states. A micromaser system in which two-level atoms are injected into a cavity, the model discussed in detail by Filipowicz *et al.* [6], gives another possibility of generating highly excited Fock states. Micromaser systems as potential sources of Fock states have been proposed by Krause *et al.* [7]. Liebman and Milburn [8] have discussed a system containing a micromaser with feedback. They have shown that such a system can lead to the n-photon states. The method of generation of various quantum states based on the interaction of N two-level atoms injected into a single-mode resonator and interacting with the cavity electromagnetic field has been presented by Vogel *et al.* [9]. Their method is so designed as to produce, at least in principle, arbitrary field states in a single-mode cavity. As Kozierowski and Chumakov [10] have shown, the Fock states can also be generated in the spontaneous emission of the partially inverted Dicke model. Kilin and Horoshko [11] have proposed a scheme based on nonlinear optics that can lead to the generation of n-photon states. A very similar model to the model discussed in this paper has been studied by Kukliński [12], who proposed a system containing a high-Q cavity partially filled by a medium with $\chi^{(3)}$ nonlinear susceptibility and irradiated by a series of coherent pulses. For this system he performed numerical calculations showing that when the size of the cavity is adiabatically varied (i.e. the cavity detuning is varied) Anderson localization takes place, and one can generate a nearly perfect n-photon state in a chosen mode of the electromagnetic field in the cavity. A crucial element of his calculation was the adiabatic change of the cavity volume. He has shown that due to the adiabatic change of the cavity detuning a localized state is generated around the vacuum state and this state is next adiabatically moved to an arbitrarily excited state depending on the parameters used.

In this paper we discuss basically the same system containing a cavity filled with a passive nonlinear 'Kerr' medium and periodically kicked by a series of ultra-short laser pulses. We assume, as in [12], that the cavity contains just one mode of the electromagnetic field and, contrary to [12], that its size does not change during the evolution. Although we shall consider only two cases—$\chi^{(3)}$ and $\chi^{(5)}$ nonlinearities—our model describes media that are characterized by the $(2q-1)$th susceptibility $\chi^{(2q-1)}$, in general. Alternatively, such a system can be considered as a model of appropriate nonlinear oscillator. The model with $\chi^{(3)}$ nonlinearity has been discussed in our earlier paper [13], where we have focused our attention to the problem of generation of one-photon state. In that paper we used a purely numerical approach with the approximate representation for the unitary evolution operator to find the states of the field in the cavity. We have found that for sufficiently weak external pulses the system can evolve to the one-photon state. We would like to emphasize at this point that the mechanism leading to the one-photon state is different from that discussed by Kukliński [12] as should become clear from the present paper. This paper is a continuation and an extension of our earlier paper [13]. We shall deal with the system containing the cavity filled with the medium characterized by $\chi^{(2q-1)}$ nonlinearity. The cavity is irradiated by a series of ultra-short external field pulses. Moreover, we shall assume that the pulses are weak and the cavity field is initially in the vacuum state $|0\rangle$. We shall investigate the dynamics of the system using not only numerical methods but we shall also derive analytical formulas for the probabilities characterizing the field state. Owing to the fact that the external excitation is weak we can apply perturbative methods that enable us to get fully analytical results. Since the system under consideration is irradiated by a periodic series of pulses we shall apply the method of Floquet states [14–17]. The Floquet states are used as a standard tool to solve problems with the periodically time dependent Hamiltonians, like the problem discussed here. They have clear advantages in some numerical calculations, especially in cases where the eigenstates of the free

Hamiltonian must be calculated numerically. In such cases the numerically calculated eigenvalues can be used in further analytical, perturbative calculations that include the time dependent interaction Hamiltonian as a perturbation. In fact, in our case the eigenstates and the eigenvalues of the 'free' Hamiltonian are known (they are n-photon states), so using the Floquet states in this case is not a necessity, although it does not hurt. For this particular case the solution can be obtained without having recourse to the Floquet states (we sketch such a solution in the Appendix). Irrespective of the way of solving the problem, the main physics behind our solutions is the degeneracy of the 'free' Hamiltonian, which for weak perturbations restricts the evolution of the field in the cavity to the first few Fock states. So, $\chi^{(3)}$ medium, for example, corresponds to the problem of a two-level atom driven by an electric field with Rabi oscillations, area theorem, π-pulses, etc. [22]. The mechanism leading to superpositions of the Fock states, and to the one-photon state in particular, is quite simple and different from that discussed by Kukliński [12].

Generally, we consider the system governed by the Hamiltonian

$$\hat{H}(t) = \frac{\chi^{(2q-1)}}{q}\hat{a}^{\dagger q}\hat{a}^{q} + \epsilon(\hat{a}^{\dagger} + \hat{a})f(t), \tag{1}$$

where the first term is the 'free' Hamiltonian and the second term is the perturbation. The 'free' Hamiltonian can be rewritten as

$$\hat{H}(t) = \frac{\chi^{(2q-1)}}{q}\hat{n}(\hat{n} - 1)\ldots(\hat{n} - q + 1), \tag{2}$$

which clearly shows that the Fock states are eigenstates of this Hamiltonian, and that the first q Fock states have their eigenvalues equal to zero, so there is a q-fold degeneracy of the system. For instance, for $q = 2$ we obtain two degenerate states corresponding to $|0\rangle$ and $|1\rangle$, for $q = 3$ three degenerate states $|0\rangle$, $|1\rangle$ and $|2\rangle$, and so forth. This degeneracy is lifted by the perturbation. The standard perturbation procedure for degenerate states [18] indicates that, as far as the perturbation is small ($\epsilon \ll \chi^{(2q-1)}$), the evolution of the system in the lowest order of the perturbation theory will involve only the states from the degenerate subset. This means that for a weak driving external (classical) field the field inside the cavity filled with the nonlinear medium can become a superposition of the first q Fock states with highly non-classical properties.

For more general cases of the systems, described by a not so simple Hamiltonian with no obvious eigenstates which can only be found by numerical methods, a combination of the Floquet states technique and perturbation theory can be an effective method, leading to the solution. For this reason we shall apply the method based on the Floquet states which was proposed by Sambe [14] and adapted by Dyrting *et al.* [17]. For periodic, time-dependent perturbations this method allows one to replace the problem of solving the time-dependent Schrödinger equation by the eigenvalue problem in an extended Hilbert space. The basics of this approach are presented in the next section.

To validate our analytical solutions, we shall apply the numerical method discussed in [13] to solve the problem. We will show that for weak external pulses our analyical solutions agree perfectly with the numerical results.

We shall also discuss shortly the role of damping in the system. As we shall show, the effectiveness of the Fock state preparation can be considerably diminished by the cavity losses. To take into account losses in the system we shall apply the method of Milburn and Holmes [19] based on the master equation. Despite the destructive role of losses in the process of producing non-classical field states, it seems to us important and interesting to draw the attention of the reader to the fact that a cavity with a nonlinear Kerr medium and a field initially in the vacuum state, kicked periodically by a train of classical pulses, can be, to a high accuracy, a source of Fock states.

2. The Floquet states

The Floquet theorem states that when the Hamiltonian $\hat{H}(t)$ of a system is time dependent but periodic then the solution of the Schrödinger equation takes the form

$$|\Phi(t)\rangle = \exp\left(-\mathrm{i}\frac{Et}{\hbar}\right)|\phi(t)\rangle, \tag{3}$$

where $|\phi(t)\rangle = |\phi(t+T)\rangle$ is a periodic function with the period T reflecting the periodicity of the Hamiltonian, and E is called the quasi-energy. The Floquet state has a useful property that the evolution by one period leaves it in the original state multiplied by a phase factor. This means that only the form of the Floquet states at $t = 0$ is needed to find the evolution of the system in fixed steps of a period.

If the state describing the system at time $t = 0$ is equal to $|\Phi(0)\rangle$, the state of time $t = T$ is given by

$$|\Phi(T)\rangle = \hat{U}|\Phi(0)\rangle, \tag{4}$$

where $\hat{U}$ is the unitary evolution operator generated by the Hamiltonian $\hat{H}(t)$. The Floquet states at $t = 0$ are eigenstates of the evolution operator over one period $\hat{U}$, and they are given by

$$\hat{U}|E\rangle = \exp\left(-\mathrm{i}\frac{ET}{\hbar}\right)|E\rangle. \tag{5}$$

The operator $\hat{U}$ is sometimes called the Floquet operator, and the Floquet states $|E\rangle$ are defined in a Hilbert space $\mathcal{H}_1$. Since the Floquet states, which are eigenstates of the Floquet operator can be, in many cases, found effectively using numerical procedures, this method is often used to solve problems with periodic Hamiltonians.

The Schrödinger equation

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Phi(t)\rangle = \hat{H}(t)|\Phi(t)\rangle, \tag{6}$$

after inserting (3) can be written in the form

$$\left[\hat{H}(t) - \mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]|\phi(t)\rangle = E|\phi(t)\rangle, \tag{7}$$

which has the form of an eigenvalue equation with a quasi-Hamiltonian

$$\hat{K} = \hat{h} + \hat{H}(t) \tag{8}$$

where the momentum-like operator $\hat{h} = -\mathrm{i}\hbar(\mathrm{d}/\mathrm{d}t)$ satisfies the commutation relation $[\hat{h}, \hat{t}] = -\mathrm{i}\hbar$ with the position-like operator $\hat{t} = t$ having been introduced.

The operator $\hat{h}$ acts in a Hilbert space $\mathcal{H}_2$ spanned on the states $|l\rangle$ being its eigenstates

$$\hat{h}|l\rangle = l\hbar\Omega|l\rangle, \tag{9}$$

where $l = 0, \pm 1, \pm 2, \ldots, \pm\infty$. The characteristic frequency Ω [20, 21] appearing in (9) is related to the period T and is defined by

$$\Omega = \frac{2\pi}{T} \tag{10}$$

In consequence, the quasi-Hamiltonian $\hat{K}$ is defined in the extended Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ and satisfies the eigenvalue equation

$$\hat{K}|\xi\rangle = \xi|\xi\rangle. \tag{11}$$

The Floquet states $|E\rangle$ that we are interested in, which are defined in the Hilbert space $\mathcal{H}_1$, are obtained from the eigenstates $|\xi\rangle$ of the quasi-Hamiltonian $\hat{K}$ by the projection

$$|E\rangle = \langle t = 0|\xi\rangle, \tag{12}$$

where the state $|t\rangle$ is given by

$$|t\rangle = \sum_{l=-\infty}^{+\infty} \exp(-\mathrm{i}\Omega l t)|l\rangle. \tag{13}$$

If the evolution of the system is governed by a Hamiltonian of the form

$$\hat{H}(t) = \hat{H}_0 + \epsilon\hat{H}_1(t), \tag{14}$$

where $\hat{H}_1(t)$ is a time-dependent, periodic perturbation with period T, the procedure described above allows the application of ordinary perturbation theory to find the solution. This approach was proposed by Sambe [14] and used by Dyrting *et al.* [17] (see also [15, 16]). The numerical methods based on the Floquet states have advantages over some other methods, e.g. the matrix continued fraction technique, because of the simplicity of setting up the calculation on a computer. We shall apply the Floquet states technique and the perturbation theory to find the solution to our problem.

3. The $\chi^{(3)}$ nonlinear medium

Although we are interested in a rather general model with $(2q-1)$th nonlinearity $\chi^{(2q-1)}$, we start our considerations from the simplest case of the $\chi^{(3)}$ nonlinearity. Thus, the system we are interested in now is governed by the following Hamiltonian

$$\hat{H}(t) = \hat{H}_{\mathrm{NL}} + \hat{H}_1(t), \tag{15}$$

where

$$\hat{H}_{\mathrm{NL}} = \frac{\hbar\chi^{(3)}}{2}(\hat{a}^\dagger)^2\hat{a}^2 \tag{16}$$

describes the nonlinear interaction of the cavity field with itself which is mediated by the nonlinear medium (self-phase modulation), whereas

$$\hat{H}_1(t) = \hbar\epsilon(\hat{a}^\dagger + \hat{a})f(t) \tag{17}$$

is a time-dependent Hamiltonian corresponding to the driving of the cavity by the external classical field with a periodic function $f(t)$. The external field is assumed to be periodic, in the form of a series of ultra-short pulses that can be conveniently modelled by the Dirac delta functions. This means that the periodic function $f(t)$ takes the form

$$f(t) = \sum_{n=0}^{\infty} \delta(t - nT). \tag{18}$$

We assume here that the time T between the kicks is much longer than $2\pi/\omega$, where ω is the field frequency. Moreover, we assume that the interaction (17) is weak, i.e. $\epsilon \ll \chi^{(3)}$, and it can thus be treated perturbatively. If the initial state of the system is $|\Phi(0)\rangle$ the state after an arbitrary number k of driving pulses is given by

$$|\Phi(t = kT)\rangle = \hat{U}^k |\Phi(t = 0)\rangle, \tag{19}$$

where the evolution operator $\hat{U}$ is generated by the Hamiltonian (15) which evolves states in time from $t = 0$ to $t = T$. Owing to the periodicity of our system, we can apply the method based on the Floquet technique [14, 17] to find the evolution of the system. We start with the 'free' evolution first, i.e. the parameter ϵ is assumed to be equal to zero. For this case the quasi-Hamiltonian has the form

$$\hat{K}_0 = \hat{h} + \hat{H}_{\mathrm{NL}}. \tag{20}$$

Since the following eigenvalue equations are satisfied

$$\begin{aligned} \hat{H}_{\mathrm{NL}} |n\rangle &= \frac{\hbar\chi^{(3)}}{2} n(n - 1)|n\rangle, \\ \hat{h}|m\rangle &= \hbar\Omega m|m\rangle, \end{aligned} \tag{21}$$

the eigenstates $|n, m\rangle = |n\rangle \otimes |m\rangle$ of the operator $\hat{K}_0$ are labelled by the quantum numbers: $n = (0, 1, 2, \ldots)$ and $m = (0, \pm 1, \pm 2, \ldots)$. Obviously, the eigenenergies of the quasi-Hamiltonan $\hat{K}_0$ are determined by these two numbers and are equal to

$$E_{n,m} = \hbar\left(\frac{\chi^{(3)}}{2} n(n - 1) + m\Omega\right). \tag{22}$$

It is clear from equation (22) that the states $|0, m\rangle$ and $|1, m\rangle$ are degenerate and their energies are equal to $E_{0,m} = E_{1,m} = m\hbar\Omega$. In our further calculations we will only consider characteristic frequencies Ω which guarantee that any other states degenerate with $|0, m\rangle$ and $|1, m\rangle$ are $|n', m'\rangle$, where $n' \geqslant 3$. Since the interaction $\hat{H}_1$ is linear in the cavity field operators, to diagonalize $\hat{K}$ it is sufficient to perform perturbation calculations on the states $|0, m\rangle$ and $|1, m\rangle$ to the first order in ϵ omitting other degenerate states.

When we include the interaction with the external field ($\epsilon \neq 0$) the quasi-Hamiltonian becomes

$$\hat{K} = \hat{K}_0 + \epsilon\hat{K}_1, \tag{23}$$

where

$$\hat{K}_1 = \hbar(\hat{a}^\dagger + \hat{a}) f(\hat{t}) \tag{24}$$

Since the function $f(t)$ is periodic it has a Fourier representation

$$f(t) = \frac{1}{T}\sum c_l \exp(\mathrm{i}\Omega l t), \quad c_l = \int_0^T f(t) \exp(-\mathrm{i}\Omega l t)\, \mathrm{d}t, \tag{25}$$

and the operator $\hat{K}_1$ can be expressed in the following form

$$\hat{K}_1 = \frac{\hbar}{T}(\hat{a}^\dagger + \hat{a}) \sum_{l=-\infty}^{+\infty} c_l \exp(\mathrm{i}\Omega l\hat{t}). \tag{26}$$

For the driving field in the form of the periodic train of delta pulses given by (18) the coefficients c_l are equal to

$$\begin{aligned} c_l &= \int_0^T \delta(t-\tau)\exp(-\mathrm{i}l\Omega t)\mathrm{d}t = \Big|_{\tau\to 0} \\ &= \exp(-\mathrm{i}\Omega l\tau)\big|_{\tau\to 0}, \end{aligned} \tag{27}$$

The small interval of time τ has been introduced to ensure that a single kick is located inside the period T. Of course, the series of pulses discussed in this paper has the simple form of the Dirac delta functions. The Floquet technique allows us to discuss systems with arbitrary pulse shape, the only limitation is the periodic character of the series. However, one should keep in mind that for some specific pulse envelopes special integration methods should be applied to find the appropriate Fourier transforms appearing in equation (25).

Due to the degeneracy, the evolution of the system, to the lowest order in ϵ, will be restricted to the states $|0,m\rangle$ and $|1,m\rangle$. Now, we are in a position to apply the standard perturbation theory for systems involving degenerate states [18]. First, we expand the Floquet state $|\xi\rangle$ defined in the extended Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ and the eigenvalue ξ into a power series in ϵ and insert them into equation (11). Next, after comparing the coefficients at the same powers of ϵ, we get a set of equations for finding the eigenstates and eigenvalues in a given order of perturbation. For zeroth order the state $|\xi^{(0)}\rangle$ is given by

$$|\xi^{(0)}\rangle = a|0,m\rangle + b|1,m\rangle. \tag{28}$$

To find the first order contributions we have to solve the following eigenvalue equation

$$\frac{\hbar}{T}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \xi^{(1)}\begin{pmatrix} a \\ b \end{pmatrix}, \tag{29}$$

which has the solution

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{2^{1/2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}. \tag{30}$$

The quantity $\epsilon\xi^{(1)}$ is the first order correction to the energy, and we have $\xi^{(1)} = \pm\hbar/T$. The resulting states labelled by $|\xi_\pm^{(0)}\rangle$ correspond to the Floquet states $|E_\pm^{(0)}\rangle$. According to (12) and (13), the latter are given by

$$|E_\pm^{(0)}\rangle = \langle t = 0|\xi_\pm^{(0)}\rangle = \frac{1}{2^{1/2}}(|0\rangle \pm |1\rangle). \tag{31}$$

After continuing the perturbation procedure we get the first order solutions for the states in the extended Hilbert space and, in consequence, the first order Floquet states and their energies. These states are of the form

$$|E_\pm^{(1)}\rangle = (1 \pm \epsilon A)|E_\pm^{(0)}\rangle \pm \epsilon B|2\rangle + \mathcal{O}(\epsilon^2), \tag{32}$$

where

$$A = \frac{1}{2\pi} \sum_{\substack{l=-\infty \\ l \neq 0}}^{+\infty} \frac{c_l}{l},$$
$$B = \frac{1}{2\pi} \sum_{l=-\infty}^{+\infty} \frac{c_l}{l + \frac{\chi^{(3)}}{\Omega}} \tag{33}$$

and the energies $E_\pm^{(1)}$ are

$$E_\pm^{(1)} = \pm \frac{\epsilon \hbar}{T} + \mathcal{O}(\epsilon^2). \tag{34}$$

Since we are interested in the generation of Fock states from the system with the cavity field being initially in the vacuum state, we can express $|0\rangle$ in terms of the first order solutions for the Floquet state (32).

$$|0\rangle = \frac{1}{2^{1/2}} \left(\frac{|E_+^{(1)}\rangle}{1 + \epsilon A} + \frac{|E_-^{(1)}\rangle}{1 - \epsilon A} \right) + \mathcal{O}(\epsilon^2). \tag{35}$$

Then, applying to both sides of (35) the operator $\hat{U}^k$ and using (5) with the eigenvalues (34) of the Floquet states, after straightforward algebra, we obtain the probability amplitudes for the Fock states, which for the times t just after the kth pulse are given by

$$\begin{aligned} a_0(k) &= \langle 0|\hat{U}^k|0\rangle = \cos(k\epsilon) + \mathcal{O}(\epsilon^2), \\ a_1(k) &= \langle 1|\hat{U}^k|0\rangle = -\mathrm{i}\sin(k\epsilon) + \mathcal{O}(\epsilon^2), \\ a_2(k) &= \langle 2|\hat{U}^k|0\rangle = \frac{-\mathrm{i}\epsilon B}{2^{1/2}} \sin(k\epsilon) + \mathcal{O}(\epsilon^2), \end{aligned} \tag{36}$$

where B is calculated according to (33) and is equal to

$$B = \frac{\exp(-\mathrm{i}\chi^{(3)} T/2)}{\sin(\chi^{(3)} T/2)}. \tag{37}$$

The occupation probabilities are thus given by

$$\begin{aligned} P_0(k) &= \cos^2(k\epsilon) + \mathcal{O}(\epsilon^2), \\ P_1(k) &= \sin^2(k\epsilon) + \mathcal{O}(\epsilon^2), \\ P_2(k) &= \frac{2\epsilon^2}{\sin^2(\chi^{(3)} T/2)} \sin^2(k\epsilon) + \mathcal{O}(\epsilon^3). \end{aligned} \tag{38}$$

It is seen that the system starts to evolve from the vacuum state $|0\rangle$, and after subsequent kicks the probability $P_0(k)$ decreases, whereas the probability corresponding to the one-photon state $P_1(k)$ increases. In consequence, after $k = \pi/2\epsilon$ pulses the system evolves to the pure one-photon state. The influence of the higher n-photon states on the system is proportional to ϵ^2 and for weak external excitations can be neglected. For instance, the probability for the two-photon state $P_2(k)$ has the amplitude equal to $\epsilon^2/(2\sin^2(\chi^{(3)} T/2))$. This approach enables us to get the closed form analytical results describing the evolution of the system. The situation discussed here resembles that for the two-level atom in an external field [22]. The degeneracy of the Hamiltonian $\hat{H}_{\mathrm{NL}}$ restricts the evolution in the lowest (zeroth) order of the perturbation theory to the first two Fock states. The degeneracy is lifted by the perturbation in the first order giving corrections to the energy proportional to ϵ (see also the Appendix).

To verify our analytical results we perform numerical calculations and compare their results with those based on formulas (38). This will be done on the basis of the unitary evolution operator $\hat{U}$, according to equation (19), similarly as in [13]. Owing to the fact that the ultra-short pulses are modelled by the Dirac delta functions the time-evolution of the system can be divided into two different stages. The first stage is a 'free' evolution determined by the Hamiltonian $\hat{H}_{\mathrm{NL}}$ during the time T between two subsequent pulses. This evolution is described by the following unitary operator

$$\hat{U}_0 = \exp\left[-\mathrm{i}\frac{\chi^{(3)}T}{2}(\hat{a}^\dagger)^2\hat{a}^2\right]. \tag{39}$$

The second stage of the time-evolution of the system is caused by its interaction with an infinitely short pulse. This part of the evolution is described by the Hamiltonian $\hat{H}_1$, given by equation (17). Thus, the evolution operator corresponding to the interaction during a single pulse can be written as

$$\hat{U}_1 = \exp\left[-\mathrm{i}\epsilon(\hat{a}^\dagger + \hat{a})\right]. \tag{40}$$

The overall evolution of the state of the system can thus be described as a subsequent action of the operators $\hat{U}_0$ and $\hat{U}_1$ on the initial state. Assuming that for time $t = 0$ the system was in a vacuum state we express the state $|\Phi_k\rangle$ just after the kth kick as

$$|\Phi_k\rangle = (\hat{U}_1\hat{U}_0)^k|0\rangle. \tag{41}$$

With this procedure we are able to obtain the state after an arbitrary kth pulse.

In figure 1 we plot the probabilities $P_0(k)$ and $P_1(k)$ corresponding to the vacuum $|0\rangle$ and one-photon $|1\rangle$ states, respectively. For the plots we use units such that $\chi^{(3)} = 1$. The probabilities are plotted as functions of the number of pulses k. The lines illustrate the evolution of the system (vacuum and one-photon states) according to the analytical formulas (38), whereas the marks correspond to the results obtained from the numerical calculations based on equation (41). We see that the system starts its evolution from the vacuum state $|0\rangle$. Then the probability for the one-photon state increases, and after the number of pulses $k = \pi/2\epsilon$, the system ends up in an almost perfect one-photon Fock state. Of course, this is only true with an accuracy up to terms $\sim \epsilon^2$. For small ϵ we observe very good agreement between the results of numerical calculations and those obtained from the analytical formulas.

Figure 2 corresponds to the same situation as in figure 1 but we have plotted the probability P_2 for the two-photon state $|2\rangle$. We see that it oscillates as a sine squared function again. However, the amplitude of these oscillations is much smaller than that for the probabilities P_0 and P_1 shown in figure 1. This amplitude is proportional to $\epsilon^2 \sim 0{\cdot}002$, and it is negligible in the scale of figure 1. Similarly, as for figure 1, the results of our numerical calculations are also marked, and they agree almost perfectly with those obtained from the analytical formula.

However, as the external and the cavity field coupling increases the situation changes considerably. This situation is illustrated in figure 3, where we have

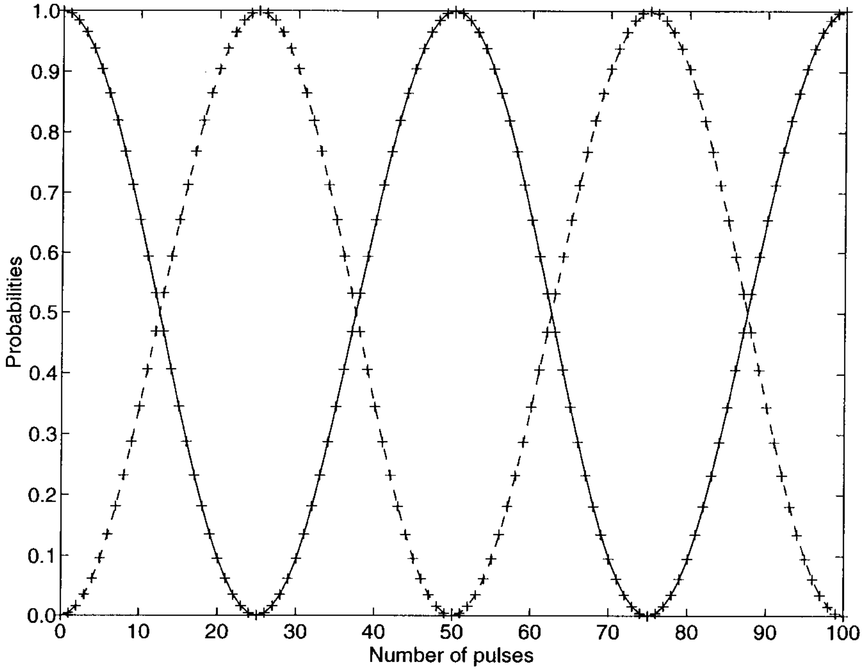

Figure 1. Time evolution of the probabilities for the vacuum $|0\rangle$ (solid line) and one-photon $|1\rangle$ states. The time $T = \pi$, and the kick strength $\epsilon = \pi/50$. The cross marks correspond to the numerical results.

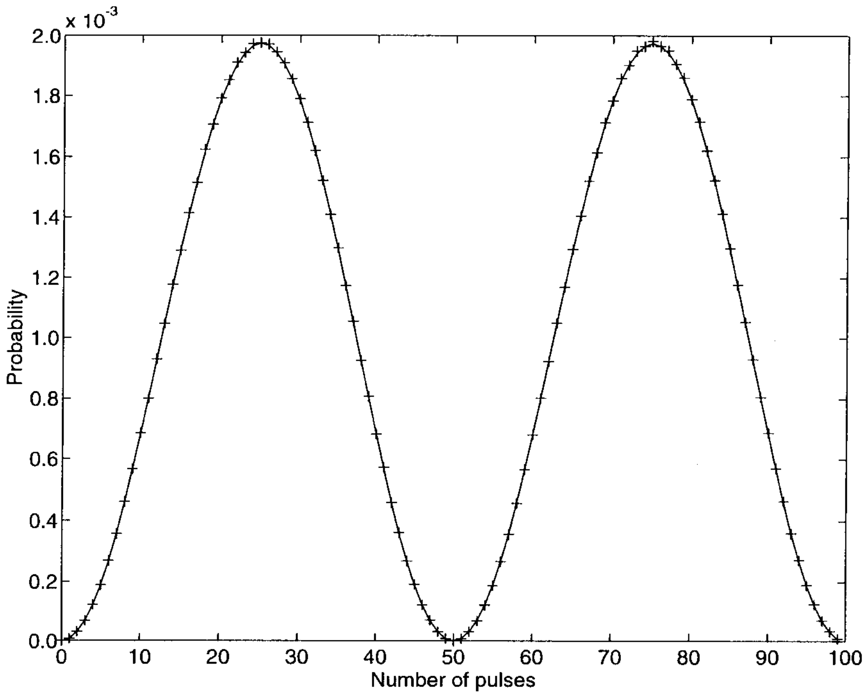

Figure 2. The same as for figure 1, but for the probability P_2 corresponding to the two-photon state $|2\rangle$. The cross marks correspond to the numerical results.

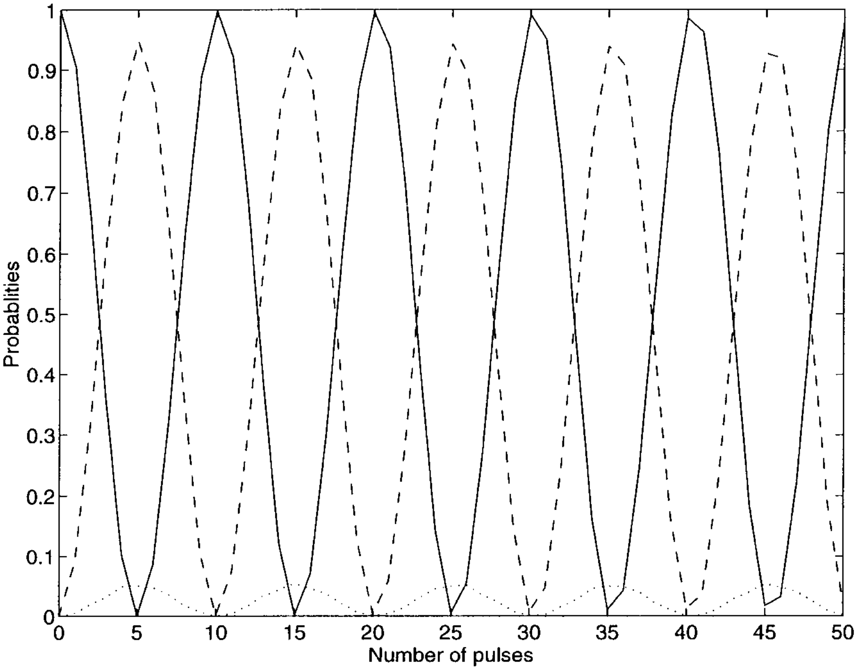

Figure 3. The probabilities P_0 (solid line), P_1 (dashed line), P_2 (dotted line) found numerically. The parameters are the same as for figure 1, but for stronger external field coupling ($\epsilon = \pi/10$).

plotted results for $\epsilon = \pi/10$. We see that the probability P_2 corresponding to the two-photon state starts to influence the dynamics of the system. Obviously, this influence becomes more and more pronounced as ϵ increases. For this case the validity of our analytical perturbative approach breaks down and only the numerical approach can be applied. Figure 4 shows the situation when the value of $\epsilon = \pi/2$ is greater than the value of $\chi^{(3)} = 1$. For this case the dynamics of the system are quite obscured by the influence of more and more states.

4. Higher nonlinearities

In this section we shall shortly discuss the case of higher nonlinearities. Let us assume that the medium is characterized by the nonlinearity $\chi^{(5)}$, which is the next nonlinearity affecting the propagaton of the field at frequency ω in the nonlinear medium. Similarly as for the case discussed in the previous section, we shall apply the Floquet states and find analytical perturbative solutions for the probabilities corresponding to the first Fock states. Thus, we start from the following 'free' quasi-Hamiltonian

$$\hat{K}_0 = \hat{h} - \frac{\hbar\chi^{(5)}}{3}(\hat{a}^{\dagger})^3\hat{a}^3, \tag{42}$$

where $\hat{h}$ is the same momentum-like operator as that introduced in the previous

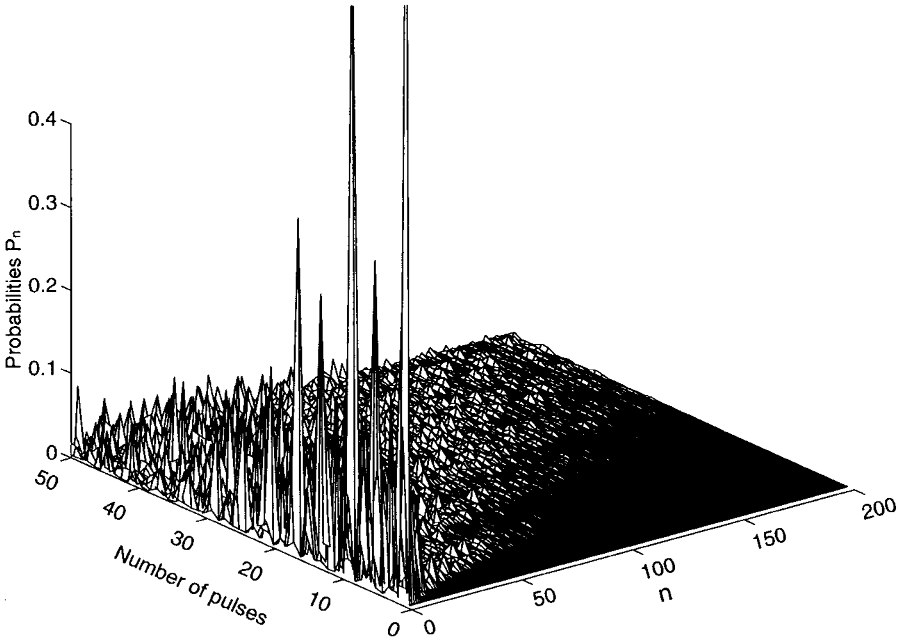

Figure 4. The probabilities for subsequent n-photon states found numerically. The parameters are the same as for figure 3, but for $\epsilon = \pi/2$.

section. The quasi-energies generated by this quasi-Hamiltonian are

$$E_{n,m} = \hbar\left(\frac{\chi^{(5)}}{3}n(n-1)(n-2) + m\Omega\right). \tag{43}$$

This time the energies are triply degenerate, so we can expect that the dynamics of the system will now involve the lowest three Fock states. The dynamics of the system are now determined by the quasi-Hamiltonian $\hat{K} = \hat{K}_0 + \hat{K}_1$, where $\hat{K}_1$ is defined in (26). Similarly, as in the previous section we can write the state corresponding to the zeroth order solution

$$|\xi^{(0)}\rangle = a|0,m\rangle + b|1,m\rangle + c|2,m\rangle, \tag{44}$$

Applying the same perturbative procedure as for the case of the $\chi^{(3)}$ medium we solve the eigenvalue problem and get a set of three solutions for the eigenenergies. As a result, we can write the three zeroth order solutions to the Floquet states. They are of the following form:

$$\begin{aligned} |E_0^{(0)}\rangle &= \frac{1}{3^{1/2}}(2^{1/2}|0\rangle - |2\rangle), \\ |E_\pm^{(0)}\rangle &= \frac{1}{6^{1/2}}(|0\rangle \pm 3^{1/2}|1\rangle + |2\rangle). \end{aligned} \tag{45}$$

In the next step we write the first order solutions for the Floquet states. They can be expressed as:

$$\begin{aligned}\left|E_0^{(1)}\right\rangle &= \frac{1}{3^{1/2}}\left(2^{1/2}|0\rangle - |2\rangle\right) - \epsilon B|3\rangle + \mathcal{O}(\epsilon^2),\\ \left|E_\pm^{(1)}\right\rangle &= \frac{1}{6^{1/2}}\left(|0\rangle \pm 3^{1/2}|1\rangle + 2^{1/2}|2\rangle\right) + \epsilon B|3\rangle + \mathcal{O}(\epsilon^3),\end{aligned}\tag{46}$$

where the coefficient B is identical to that defined in equation (33) with the replacement $\chi^{(3)} \to \chi^{(5)}$. In the same way as before we derive analytical formulas for the probability amplitudes corresponding to the n-photon states $|n\rangle$, $(n = 0, 1, 2, 3)$. They are defined as:

$$\begin{aligned}a_0(k) &= \langle 0|\hat{U}^k|0\rangle = \tfrac{1}{3}\left[2 + \cos(3^{1/2}k\epsilon)\right] + \mathcal{O}(\epsilon^2),\\ a_1(k) &= \langle 1|\hat{U}^k|0\rangle = \frac{-\mathrm{i}}{3^{1/2}}\sin(3^{1/2}k\epsilon) + \mathcal{O}(\epsilon^2),\\ a_2(k) &= \langle 2|\hat{U}^k|0\rangle = \frac{2^{1/2}}{3}\left[\cos(3^{1/2}k\epsilon) - 1\right] + \mathcal{O}(\epsilon^2),\\ a_3(k) &= \langle 3|\hat{U}^k|0\rangle = \frac{2^{1/2}}{3^{1/2}}\epsilon B\left[\cos(3^{1/2}k\epsilon) - 1\right] + \mathcal{O}(\epsilon^2),\end{aligned}\tag{47}$$

and the probabilities are

$$\begin{aligned}P_0(k) &= \tfrac{1}{9}(2 + \cos(3^{1/2}k\epsilon))^2 + \mathcal{O}(\epsilon^2),\\ P_1(k) &= \tfrac{1}{3}\sin^2(3^{1/2}k\epsilon) + \mathcal{O}(\epsilon^2),\\ P_2(k) &= \tfrac{2}{9}(\cos(3^{1/2}k\epsilon) - 1)^2 + \mathcal{O}(\epsilon^2),\\ p3(k) &= \frac{2\epsilon^2}{3\sin^2(T\chi^{(5)}/2)}(\cos(3^{1/2}k\epsilon) - 1)^2 + \mathcal{O}(\epsilon^3).\end{aligned}\tag{48}$$

We see for a sufficiently weak external field excitation the dynamics of the system involve the subspace of the lowest three n-photon states with $n = 0, 1, 2$. The probabilities for the higher states are proportional to ϵ^z, where z increases as n becomes higher and higher. As an example, figure 5 depicts the situation, when the parameter $\epsilon = \pi/50$ (we use units of $\chi^{(5)} = 1$). Lines appearing in figure 5 correspond to our analytical results, whereas the marks are plotted from the data obtained numerically. The numerical procedure is identical to that described in the previous section and is based on equations (39)–(41). We assume again that the evolution of our system starts from the pure vacuum state. However, after a series of subsequent pulses the probability for the vacuum state decreases, whereas the states $|1\rangle$ and $|2\rangle$ become significant. In consequence, after ~ 30 pulses the state of the system becomes a superposition of ~ 90%of the two-photon state and of ~ 10% of the vacuum state. In this case the resulting state is a superposition of three Fock states, and the evolution never leads to just one of them, but we can get the state that is very close to $|2\rangle$. Similarly as for the case of $\chi^{(3)}$, for the parameters chosen here, the contribution of higher Fock states is negligible, and again we observe very good agreement between our analytical and numerical results.

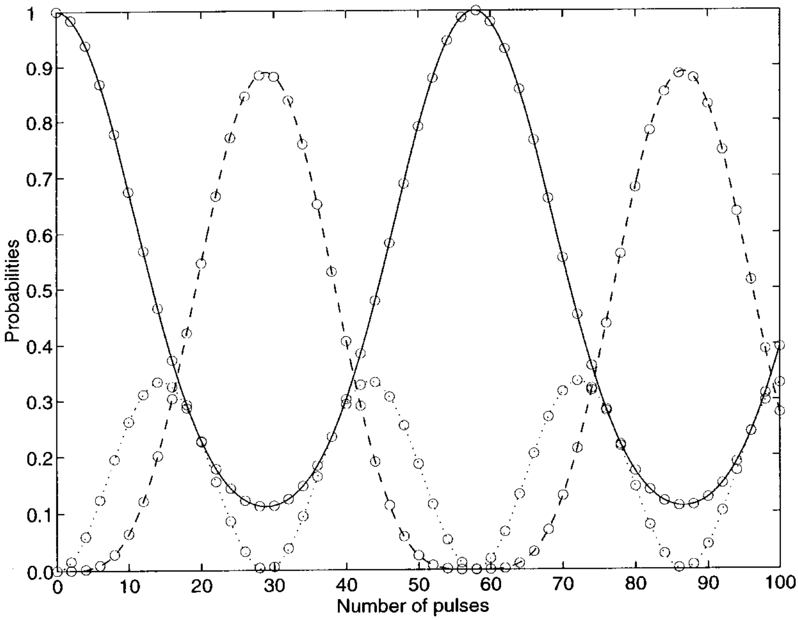

Figure 5. Analytical solutions for the probabilities of the vacuum (solid line), one-photon (dotted line) and two-photon (dashed line) states. The parameter $\epsilon = \pi/50$ (all parameters are measured in units of $\chi^{(5)} = 1$). The circle marks correspond to the probabilties found numerically.

5. Cavity damping

In real physical situations we cannot avoid dissipation, which means that we cannot take ϵ too small because the cavity field could be completely damped out during the 'free' evolution between two pulses. Moreover, the dissipation in the system leads to a mixture of the quantum states instead of their coherent superpositions. Hence, the pure state description of the field evolution presented above cannot be applied. Therefore, to describe the influence of damping on the system evolution one should use, for instance, the quantum jumps method [23] (and the references quoted therein) or the density matrix approach [19, 24]. In this paper we shall concentrate on the latter method, similarly as in [13]. As we have discussed earlier, the time evolution of our system can be divided into two stages—the evolution during an infinitely short pulse and the evolution between two subsequent pulses. As the pulses are very short we can neglect the dissipation during their action. Therefore, the time-evolution can be described by the unitary operator $\hat{U}_1$ defined in equation (40). We should, however, take into account losses during the time evolution between two subsequent pulses, which means that we should solve the appropriate master equation. Thus, for the case of the nonlinearity $\chi^{(3)}$ this equation has the following form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\frac{\chi^{(3)}}{2}\left[(a^{\dagger})^2 a^2, \rho\right] + \frac{\gamma}{2}(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a). \tag{49}$$

If $\rho(t)$ is the density matrix for the time just after the pulse at the moment of time t, the matrix elements in the Fock number state basis for the time just before the next laser pulse is given by [19, 24]:

$$\langle p|\rho(t+T)|q\rangle = \exp\left[\mathrm{i}\frac{\Theta}{2}(p-q)\right] f(T)^{(p+q)/2}(p!q!)^{-1/2}\sum_{n=p}^{\infty}\langle n|\rho(t)|n-(p-q)\rangle$$
$$\times\left[n!\left[n-(p-q)\right]!\right]^{1/2}\frac{(1+\mathrm{i}\delta)^{-(n-p)}}{(n-p)!}\left[1-f(T)\right]^{(n-p)}, \tag{50}$$

where

$$\delta = \frac{(p-q)}{k}, \tag{51}$$

$$f(T) = \exp\left[-\kappa\Theta - \mathrm{i}\Theta(p-q)\right], \tag{52}$$

$$\kappa = \frac{\gamma}{\chi^{(3)}}, \tag{53}$$

and

$$\Theta = \chi^{(3)}T. \tag{54}$$

The quantity γ appearing here is a damping constant responsible for the cavity losses, and T is the time between two subsequent pulses. Thus, solving the master equation (49) we can determine the probabilities of finding the system in an arbitrary n-photon state. The only limitations of this method are our computational abilities.

Figure 6 shows the probabilities of the vacuum and the one-photon state for $\epsilon = \pi/50$ and $T = \pi$. We have chosen two values for the damping constant γ: 0·1 (figure 6(*a*)) and 0·01 (figure 6(*b*)). It is seen that for weak damping ($\gamma = 0{\cdot}01$) over

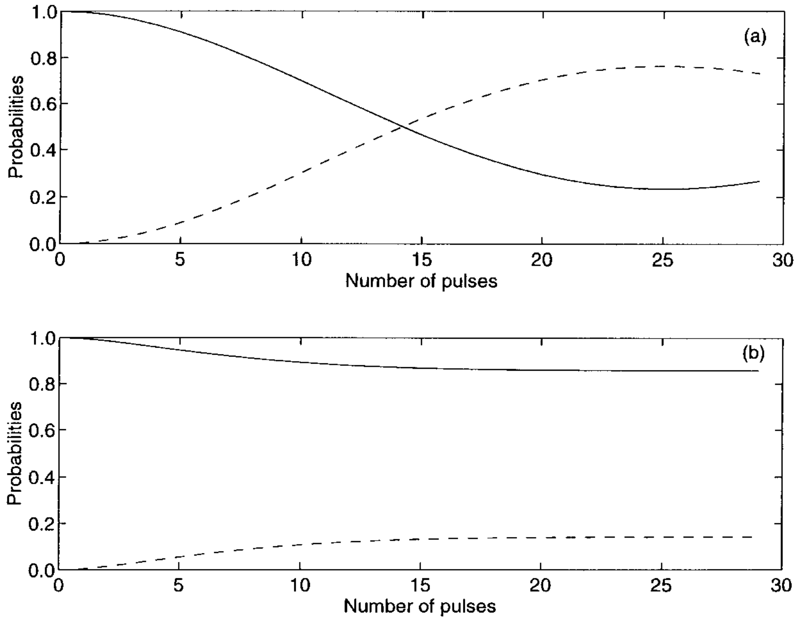

Figure 6. The probabilities for the vacuum $|0\rangle$ (solid lines) and one-photon $|1\rangle$ states (dashed lines). The damping constant $\gamma = 0{\cdot}01$ (figure 6(*a*)) and $\gamma = 0{\cdot}1$ (figure 6(*b*)). The kick strength $\epsilon = \pi/50$ and the time $T = \pi$.

75 per cent of the population is still found in state $|1\rangle$. As a consequence, it is possible to get the field in a cavity being very close to the one-photon state. However, as the damping increases the situation changes considerably. For $\gamma = 0{\cdot}1$ the probability for the one-photon state starts its evolution from zero and tends to its steady state value equal to $\sim 0{\cdot}15$. We see that for this case our system is very far from the pure one-photon state and almost all the population remains in the vacuum state $|0\rangle$. Thus, the dissipation in the system drastically lowers the effectiveness of producing the one-photon state. Nonetheless, we hope that for sufficiently weak damping our system can evolve to a state which is very close to the one-photon state.

6. Conclusions

We have discussed the possibility of generation of the Fock states by systems containing nonlinear media inside a kicked cavity. We have studied two cases—one medium with the nonlinearity $\chi^{(3)}$, and the other with the higher nonlinearity $\chi^{(5)}$. We have assumed that the cavity field was initially in the vacuum state $|0\rangle$. In addition, the interaction between the external and cavity fields has been assumed to be weak for both models (those involving the nonlinearity $\chi^{(3)}$ as well as $\chi^{(5)}$). Using the Floquet states and diagonalizing the quasi-Hamiltonian in the extended Hilbert space perturbatively we have derived the analytical formulas for the amplitudes and probabilities corresponding to the n-photon states for the two cases. We have shown that for weak external excitations our system evolves within a closed set of number states. Moreover, we have performed numerical calculations to prove the validity of our analytical results. For the cases of weak external excitation the two approaches lead to almost identical results. However, we would like to stress that the main result of this paper is a new mechanism giving a possibility of producing pure Fock states. We have shown that for a sufficiently weak external excitation the systems discussed here can lead, with a high accuracy, to the n-photon states. For the $\chi^{(3)}$ medium we have obtained an almost 100% one-photon state, whereas for the case of $\chi^{(5)}$ up to 90% of the two-photon state has been generated. Unfortunately, as we have also shown here, the cavity losses can substantially reduce or even destroy the nice picture of generation of pure Fock states in the model discussed. To obtain the pure Fock states we must assume that damping is sufficiently weak, i.e. $\gamma \ll \chi$. It would require a high Q cavity to store the field. We realize that this is a very strong requirement for the experimental realization of our models. Nevertheless, various experiments, for instance those where the very subtle effect of 'vacuum Rabi splitting' has been measured [25, 26], give us some hope for practical application of our theoretical considerations. Finally, we would like to emphasize once more the fact that the model considered here with a cavity filled with a passive nonlinear medium and pumped by a classical field allows, at least theoretically, the production of such a quantum field state as the one-photon state.

Acknowledgment

This work was supported by the KBN Research Grant 2 P03B 73 13.

Appendix

In this Appendix we briefly sketch another method of solving the problem discussed in this paper. The Hamiltonian (16) is degenerate and the eigenvalues for the Fock states $|0\rangle$ and $|1\rangle$ are equal to zero. The perturbation (17) has non-zero elements between the two degenerate states, the degeneracy will thus be lifted in the first order of the perturbation theory. For weak perturbations it is reasonable to assume that the evolution of the system will be, in the lowest order of the perturbation theory, restricted to the subset of degenerate states, ie. to the first two Fock states. Making such a two-state approximation, it is possible to reduce the problem of the field in the cavity to an equivalent problem of a two-level atom in a field with the envelope $f(t)$.

In the two-state approximation the resulting field state will be a superposition of the two degenerate states

$$|\Phi(t)\rangle = a_0(t)|0\rangle + a_1(t)|1\rangle. \tag{A 1}$$

Inserting the state $|\Phi(t)\rangle$ into the Schrödinger equation (6), we can easily find the following set of differential equations for the state amplitudes $a_0(t)$ and $a_1(t)$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}a_0(t) &= -\mathrm{i}\epsilon f(t)a_1(t), \\ \frac{\mathrm{d}}{\mathrm{d}t}a_1(t) &= -\mathrm{i}\epsilon f(t)a_0(t). \end{aligned} \tag{A 2}$$

Adding and subtracting the two equations, we get

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left[a_0(t) + a_1(t)\right] &= -\mathrm{i}\epsilon f(t)\left[a_0(t) + a_1(t)\right], \\ \frac{\mathrm{d}}{\mathrm{d}t}\left[a_0(t) - a_1(t)\right] &= \mathrm{i}\epsilon f(t)\left[a_0(t) - a_1(t)\right], \end{aligned} \tag{A 3}$$

and the solutions are

$$\begin{aligned} a_0(t) + a_1(t) &= \left[a_0(t) + a_1(t)\right]\exp\left[-\mathrm{i}\Theta(t)\right], \\ a_0(t) - a_1(t) &= \left[a_0(t) - a_1(t)\right]\exp\left[+\mathrm{i}\Theta(t)\right], \end{aligned} \tag{A 4}$$

where we have introduced the pulse area

$$\Theta(t) = \epsilon\int_0^t f(t')\,\mathrm{d}t'. \tag{A 5}$$

If the system is initially in the vacuum state, $a_0(0) = 1$ and $a_1(0) = 0$, we obtain the solutions

$$\begin{aligned} a_0(t) &= \cos\Theta(t), \\ a_1(t) &= -\mathrm{i}\sin\Theta(t). \end{aligned} \tag{A 6}$$

Solutions (A 6) are equivalent to our solutions (36), but this time they are valid for any envelope $f(t)$ of the pulse, and the periodicity of the excitation is not required. This is exactly the situation for the two-level atom excited by an external field [22].

Similar calculations can be performed for the $\chi^{(5)}$ nonlinearity. In this case the 'free' Hamiltonian is three-fold degenerate, so the evolution will involve the first three Fock states, and the state can be written as

$$|\Phi\rangle = a_0(t)|0\rangle + a_1(t)|1\rangle + a_2(t)t|2\rangle. \tag{A 7}$$

The procedure, analogous to the previous case, leads to the set of equations

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}a_0 &= -\mathrm{i}\epsilon f(t)a_1(t),\\ \frac{\mathrm{d}}{\mathrm{d}t}a_1 &= -\mathrm{i}\epsilon f(t)\left[a_0(t) + 2^{1/2}a_2(t)\right],\\ \frac{\mathrm{d}}{\mathrm{d}t}a_2 &= -\mathrm{i}\epsilon f(t)2^{1/2}a_1(t).\end{aligned} \tag{A 8}$$

After simple algebraic manipulations we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}X_1(t) &= -\mathrm{i}3^{1/2}\epsilon f(t)X_1(t),\\ \frac{\mathrm{d}}{\mathrm{d}t}X_2(t) &= \mathrm{i}3^{1/2}\epsilon f(t)X_2(t),\\ \frac{\mathrm{d}}{\mathrm{d}t}X_3(t) &= 0,\end{aligned} \tag{A 9}$$

where

$$\begin{aligned}X_1(t) &= \frac{1}{3^{1/2}}\left[a_0(t) + 2^{1/2}a_2(t)\right] + a_1(t),\\ X_2(t) &= \frac{1}{3^{1/2}}\left[a_0(t) + 2^{1/2}a_2(t)\right] - a_1(t),\\ X_3(t) &= 2^{1/2}a_0(t) - a_2(t).\end{aligned} \tag{A 10}$$

The solutions are

$$\begin{aligned}X_1(t) &= X_1(0)\exp\left[-\mathrm{i}3^{1/2}\Theta(t)\right],\\ X_2(t) &= X_2(0)\exp\left[+\mathrm{i}3^{1/2}\Theta(t)\right],\\ X_3(t) &= X_3(0).\end{aligned} \tag{A 11}$$

Fimally, assuming that initially the field is in the vacuum state, $a_0(0) = 1$ and $a_1(0) = a_2(0) = 0$, we arrive at the solutions for the amplitudes of the first three Fock states

$$\begin{aligned}a_0(t) &= \frac{1}{3}\left[2 + \cos\left[3^{1/2}\Theta(t)\right]\right]\\ a_1(t) &= -\frac{\mathrm{i}}{3}\sin\left[3^{1/2}\Theta(t)\right]\\ a_2(t) &= \frac{2^{1/2}}{3}\left[\cos\left[3^{1/2}\Theta(t)\right] - 1\right].\end{aligned} \tag{A 12}$$

Again, solutions (A 12) are equivalent to the solutions (47) obtained earlier, but this time without any restriction as to the pulse shape. The method presented in this Appendix sheds more light on the physics leading to superpositions of the Fock states discussed in this paper.

References

[1] Stoler, D., and Yurke, B., 1986, *Phys. Rev.* A, **34,** 3143.
[2] Hong, C. K., and Mandel, L., 1986, *Phys. Rev. Lett.*, **56,** 58.
[3] Brune, M., Haroche, S., Lefevre, V., Raimond, J. M., and Zagury, N., 1990, *Phys. Rev. Lett.*, **65,** 976.
[4] Brune, M., Haroche, S., Raimond, J. M., Davidovich, L., and Zagury, N., 1992, *Phys. Rev.* A, **45,** 5193.
[5] Ueda, M., Imoto, N., Nagaoka, H., and Ogawa, T., 1992, *Phys. Rev.* A, **46,** 2859.
[6] Filipowicz, P., Javanainen, J., and Meystre, P., 1986, *J. opt. Soc. Am.* B, **3,** 906.
[7] Krause, J., Scully, M. O., Walther, T., and Walther, H., 1989, *Phys. Rev.* A, **39,** 1915.
[8] Liebman, A., and Milburn, G. J., 1995, *Phys. Rev.* A, **51,** 736.
[9] Vogel, K., Akulin, V. M., and Schleich, W. P., 1993, *Phys. Rev. Lett.*, **71,** 1816.
[10] Kozierowski, M., and Chumakov, S. M., 1995, *Phys. Rev.* A, **52,** 4194.
[11] Kilin, S. Y., and Horoshko, D. B., 1995, *Phys. Rev. Lett.*, **74,** 5206.
[12] Kukliński, J. R., 1990, *Phys. Rev. Lett.*, **64,** 2507.
[13] Leoński, W., and Tanaś, R., 1994, *Phys. Rev.* A, **49,** R20.
[14] Sambe, H., 1973, *Phys. Rev.* A, **7,** 2203.
[15] Manakov, N. L., Ovsiannikov, V. D., and Rapoport, L. P., 1986, *Phys. Rep.*, **141,** 319.
[16] Millack, T., 1990, *J. Phys.* B: *At. Mol. Opt. Phys.*, **23,** 1693.
[17] Dyrting, S., Holmes, C. A., and Milburn, G., 1993, *Phys. Rev.* A, **48,** 969.
[18] Schiff, L. I., 1968, *Quantum Mechanics* (New York: McGraw-Hill, Inc.).
[19] Milburn, G. J., and Holmes, C. A., 1991, *Phys. Rev.* A, **44,** 4704.
[20] Rzążewski, K., and Mostowski, J., 1987, *Phys. Rev.* A, **35,** 4414.
[21] Swiatłowski, J, and Leoński, W., 1992, *J. mod. Optics*, **39,** 1651.
[22] Allen, L., and Eberly, J. H., 1975, *Optical Resonance and Two-Level Atoms* (New York: John Wiley and Sons, Inc.).
[23] Garraway, B. M., and Knight, P. L., 1994, *Phys. Rev.* A, **49,** 1266.
[24] Milburn, G. J., and Holmes, C. A., 1986, *Phys. Rev. Lett.*, **56,** 2237.
[25] Bernardot, F., Nussenzveig, P., Brune, M., Raimond, J. M., and Haroche, S., 1992, *Europhys. Lett.*, **17,** 33.
[26] Thompson, R. J., Rempe, G., and Kimble, H. J., 1992, *Phys. Rev. lett.*, **68,** 1132.