ON THE PHASE PROPERTIES OF BINOMIAL AND NEGATIVE BINOMIAL STATES

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INTRODUCTION

The binomial and negative binomial states are well studied in recent years [1-4]. The binomial state describes the state of the field having a binomial photon distribution with the mean photon number $\bar{n} = Np$, and the variance $(\Delta n)^2 = Np(1-p)$ such that $(\Delta n)^2$ is always less than $\bar{n}$, where N and p $(0 \leq p \leq 1)$ are the parameters specifying the binomial distribution. The binomial state can display antibunching, sub-Poissonian statistics as well as squeezing. The binomial state is 'intermediate' between a pure number state and a pure coherent state: for $p = 1$ it becomes the number state $|N\rangle$ and for $p \to 0$ and $N \to \infty$ but with $Np = n =$ const, it becomes a coherent state with mean number of photons $\bar{n}$. The interaction of binomial field with matter has been well documented. The statistical properties of the binomial states and possibilities of their generation in experiments are also discussed. On the other hand, negative binomial state is 'intermediate' between a pure thermal state and a coherent state and displays super-Poissonian statistics and squeezing. For $w \to 0$, the negative binomial state reduces to a pure thermal state with the mean number of photons $\bar{n} = (1-p)/p$ and for $w \to \infty$, $p \to 1$, but with $\bar{n} = (1+w)(1-p)/p =$ const it becomes a coherent state, where w $(w \geq 0)$ and p $(0 < p < 1)$ are the parameters specifying the negative binomial state. Some interesting properties of these states and possibilities of their production in certain quantum optical interactions are reported recently. The interaction of this state with a two-level atom has also been reported. Since both binomial as well as negative binomial states can exhibit squeezing which is a phase sensitive effect so it is interesting

Coherence and Quantum Optics VII
Edited by Eberly, Mandel, and Wolf, Plenum Press, New York, 1996

to study their quantum phase properties.

RESULTS AND DISCUSSION

To describe quantum phase properties of these states we applied the hermitian phase formalism introduced by Pegg and Barnett [5]. We have explicitly shown [6] that the phase properties of the binomial states interpolate, as one could expect, between the number states with complete random phase and the coherent states. For given N the phase variance approaches a minimum with $p = 0.5$ meaning that the binomial state with $p = 0.5$ has the best defined phase for the given N. On the other hand, for given p, the phase variance decreases as N increases and asymptotically tends to zero as N tends to infinity. We have also shown that the photon number fluctuations and phase fluctuations for such states exhibit opposite behaviour, and there is a possibility to find the binomial state for which the photon number variance is equal to the phase variance. This behaviour is convincingly seen when plotting the number and phase squeezing curves, which cross when the two fluctuations are equal. This happens in the vicinity of the maximum of the number-phase uncertainty product. Interestingly, for the negative binomial state for a particular value of its parameter the peak of the phase distribution is sharper than for the coherent state, but because the wings of the distribution are more pronounced, the phase variance is still greater than that for a coherent state with the same number of photons. The number-phase uncertainty product and the number and the phase squeezing are also studied for these states.

Acknowledgement This work has been partially supported by the Alexander von Humboldt Stiftung.

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