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THREE-LEVEL ATOM IN A SQUEEZED VACUUM[1]

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A three-level lambda system with a non-zero coherence transfer rate between the two allowed transitions, damped to a squeezed vacuum, and driven by coherent external fields is studied. It is shown that driving such a system on one of the transitions leads to a non-zero steady-state dipole moment on the other transition. This can be considered as a process of 'coherent' four wave mixing with the participation of squeezed vacuum.

1. Introduction

Three-level atoms in a squeezed vacuum exhibit a number of interesting differences with respect to the atoms placed in ordinary vacuum. Some of them have been studied in papers [1-6]. There are, for example, spectacular qualitative changes in the steady-state populations such as two-photon population inversions [1-3]. The results for three-level systems driven by two independent laser beams [4-6] show that the relative heights and widths of the peaks in the fluorescence spectra can be subnatural or supernatural depending on the relative phase between the driving fields and squeezed vacuum, similarly as in two-level systems. The well known, for a three-level atom, effect of population trapping is reduced in squeezed vacuum.

Recently, Ferguson et al. [7] have examined the effect of a single broad-band squeezed vacuum on the stationary populations and coherences in a three-level atom of lambda configuration that is driven by two independent laser fields. They also included the coherence transfer rates that couple the two allowed atomic transitions. In this paper we follow the approach used in [7], and we consider a three-level atom in lambda configuration that is damped to a single broad-band squeezed vacuum, with the bandwidth that is much larger than the distance between the two lower levels, but instead of two fields driving two allowed transitions, we assume that only one atomic transition is driven. Nevertheless, we have found that there is a non-zero steady-state atomic coherence associated with the other transition.

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2. The model and equations of motion

The system we consider is schematicaly shown in Fig. 1. A non-degenerate three-level atom with the allowed transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ is coherently driven by a resonant field on the transition $1 \leftrightarrow 3$ and damped into a broad-band squeezed vacuum the carrying frequency of which is equal to ω_s. We assume that the distance (frequency) between the two lower states is bigger than the Rabi frequency Ω of the driving field and the atomic damping rates, but the squeezed vacuum is considered as a single broad-band squeezed vacuum, i.e., we assume that its bandwidth is larger than this distance, so both of the lower states are subjected to the same vacuum. We also assume that the atomic system has non-zero coherence transfer rate γ_c that couples the two allowed transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$.

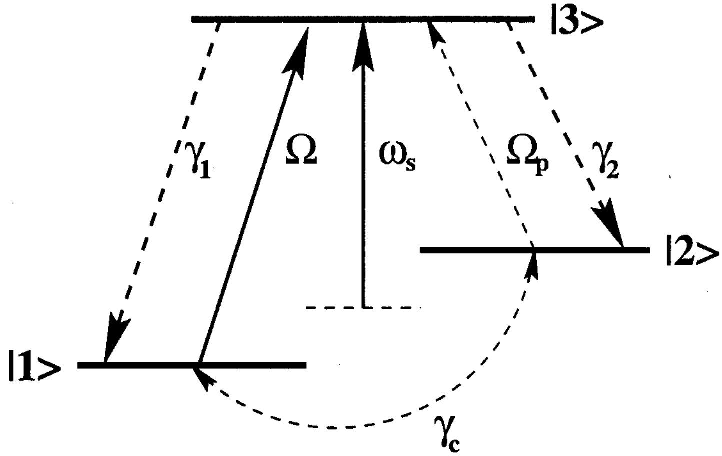

Fig. 1. Schematic diagram of the system

Evolution of the system is described by the master equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho, \tag{1}$$

where the Hamiltonian of the system is given by

$$H = \hbar\,\omega_{31}\,|3\rangle\langle 3| + \hbar\,\omega_{21}\,|2\rangle\langle 2| + \{\hbar\,\Omega e^{-i\omega_1 t}\,|3\rangle\langle 1| + \hbar\,\Omega_p e^{-i\omega_2 t}|3\rangle\langle 2| + H.c.\}, \tag{2}$$

and the irreversible part of the master equation is given by

$$\begin{aligned}
\mathcal{L}\rho \;=\; & -\frac{1}{2}(N+1)\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^+ S_j^- + S_i^+ S_j^-\,\rho - 2S_j^-\,\rho\, S_i^+) \\
& -\frac{1}{2}\,N\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^- S_j^+ + S_i^- S_j^+\rho - 2S_j^+\,\rho\, S_i^-) \\
& -\frac{1}{2}\,M\sum_{i,j}\Gamma_{ij}\,(\rho\, S_i^+ S_j^+ + S_i^+ S_j^+\rho - 2S_j^+\,\rho\, S_i^+)e^{-i2\omega_s t}
\end{aligned}$$

$$-\frac{1}{2}M^{*}\sum_{i,j}\Gamma_{ij}\left(\rho S_i^- S_j^- + S_i^- S_j^- \rho - 2S_j^- \rho S_i^-\right)e^{i2\omega_s t}, \tag{3}$$

with $S_1^- = |3\rangle\langle 1|$, $S_2^- = |3\rangle\langle 2|$. The sum over i, j is with $i, j = 1, 2$ only. The parameters N and $M = |M|\exp(i\phi)$ characterize squeezed vacuum, where $|M|^2 \leq N(N+1)$, and ω_s is the carrier frequency of the squeezed vacuum, and ϕ is its phase. We have assumed that N and M do not depend on the frequency. The damping rates Γ_{ij} are the spontaneous emission rates $\Gamma_{11} = \gamma_1$, $\Gamma_{22} = \gamma_2$ for the two allowed transitions, and $\Gamma_{12} = \Gamma_{21} = \gamma_c$ is the coherence transfer rate given by [7]

$$\gamma_c = \frac{\boldsymbol{\mu}_{13}\cdot\boldsymbol{\mu}_{23}}{12\pi\epsilon_0\hbar c^3}(\omega_{31}^3 + \omega_{32}^3) \tag{4}$$

The laser driving field is described by the Rabi frequency Ω, and the probe field by the Rabi frequency Ω_p (actually our Ω's are twice the Rabi frequencies). We assume here that the relation $2\omega_s - \omega_1 - \omega_2 = 0$ is satisfied, where ω_1 is the frequency of the driving field, and ω_2 is the frequency of the probe field, or in our case this is the frequency of the atomic dipole moment on the $2 \leftrightarrow 3$ transition. On neglecting all rapidly oscillating terms, and assuming that there is no probe field ($\Omega_p = 0$) and the driving field is resonant, the master equation (1) leads to the following equations

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11} &= -N\gamma_1\rho_{11} + (N+1)\gamma_1\rho_{33} + i\Omega(\rho_{13} - \rho_{31}) \\
\frac{\partial}{\partial t}\rho_{12} &= -\gamma_{12}\rho_{12} - i\Omega\rho_{23} \\
\frac{\partial}{\partial t}\rho_{13} &= -\gamma_{13}\rho_{13} + M^{*}\gamma_c\rho_{32} + i\Omega(\rho_{11} - \rho_{33}) \\
\frac{\partial}{\partial t}\rho_{22} &= -N\gamma_2\rho_{22} + (N+1)\gamma_2\rho_{33} \\
\frac{\partial}{\partial t}\rho_{23} &= -\gamma_{23}\rho_{23} + M^{*}\gamma_c\rho_{31} + i\Omega\rho_{21} \\
\frac{\partial}{\partial t}\rho_{33} &= N\gamma_1\rho_{11} + N\gamma_2\rho_{22} - (N+1)(\gamma_1+\gamma_2)\rho_{33} - i\Omega(\rho_{13} - \rho_{31}),
\end{aligned} \tag{5}$$

where the damping parameters γ_{ij} are given by

$$\begin{aligned}
\gamma_{12} &= \frac{1}{2}(\gamma_1 + \gamma_2)N \\
\gamma_{13} &= \left(\gamma_1 + \frac{1}{2}\gamma_2\right)N + \frac{1}{2}(\gamma_1 + \gamma_2) \\
\gamma_{23} &= \left(\gamma_2 + \frac{1}{2}\gamma_1\right)N + \frac{1}{2}(\gamma_1 + \gamma_2).
\end{aligned} \tag{6}$$

The relations $\rho_{ij} = \rho_{ji}^{*}$, and $\rho_{11} + \rho_{22} + \rho_{33} = 1$ hold for the atomic density matrix elements. It is interesting to note that due to the non-zero coherence transfer rate γ_c and non-zero squeezing parameter M, there are in equations (5) terms that couple the two atomic coherences ρ_{13} and ρ_{32}, which in consequence leads to non-zero steady-state values of them. This feature of the system is our main interest in this paper.

3. Steady-state solutions

The steady-state solutions of equations (5) give for the atomic coherences the following formulas

$$
\begin{aligned}
\rho_{12} &= \frac{\Omega^2 M \gamma_c}{\tilde{\gamma}_{13}(\gamma_{12}\gamma_{23}+\Omega^2)}(\rho_{11}-\rho_{33}) \\
\rho_{13} &= \frac{i\Omega}{\tilde{\gamma}_{13}}(\rho_{11}-\rho_{33}) \\
\rho_{23} &= \frac{-i\Omega M^* \gamma_c \gamma_{12}}{\tilde{\gamma}_{13}(\gamma_{12}\gamma_{23}+\Omega^2)}(\rho_{11}-\rho_{33})
\end{aligned}
\tag{7}
$$

where the parameter $\tilde{\gamma}_{13}$ is given by

$$
\tilde{\gamma}_{13} = \gamma_{13} - \frac{\gamma_{12} \mid M \mid^2 \gamma_c^2}{\gamma_{12}\gamma_{23}+\Omega^2}, \tag{8}
$$

and the steady-state population inversion is equal to

$$
\rho_{11}-\rho_{33} = \frac{N\gamma_1\tilde{\gamma}_{13}}{N(3N+2)\gamma_1\tilde{\gamma}_{13}+2\Omega^2(3N+1)} \tag{9}
$$

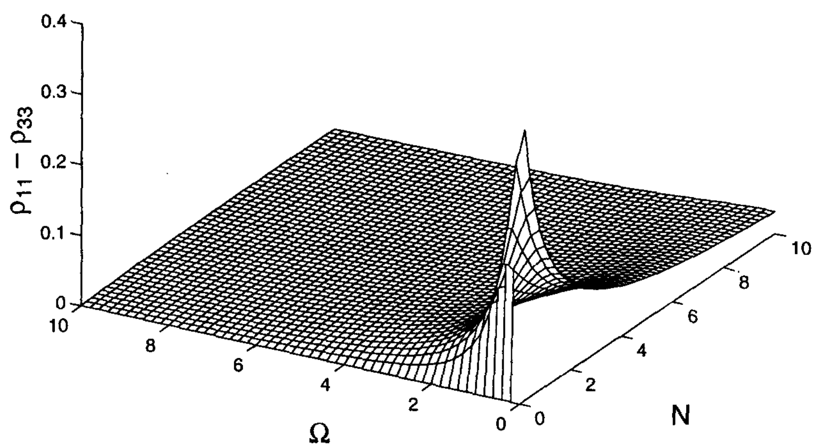

Fig. 2. Plot of $\rho_{11}-\rho_{33}$ as a function of N and Ω, with $\gamma_1=\gamma_2=\gamma_c=1$ and $|M|=\sqrt{N(N+1)}$

It is seen from formulas (7) that the squeezed vacuum, $M \neq 0$, in the presence of the non-zero coherence transfer rate, $\gamma_c \neq 0$, have a dramatic effect on the atomic coherences. The atomic coherences ρ_{12} and ρ_{23} that are not directly driven by the pump field acquire non-zero steady-state values that are proportional to the product of the value of squeezing M and the coherence transfer rate γ_c as well as to the atomic inversion $\rho_{11} - \rho_{33}$. To illustrate the steady-state solutions(7) and (9) we plot them as functions of N and Ω, assuming $\gamma_1 = \gamma_2 = \gamma_c = 1$ and that the squeezed vacuum is the ideal squeezed state with $|M| = \sqrt{N(N+1)}$. The phase of squeezing ϕ is chosen so as to have matrix elements real and positive, in other words, we plot the absolute values of the coherences. In Fig. 2 we plot the atomic inversion $\rho_{11} - \rho_{33}$, which is always positive, so there is no population inversion between the levels 1 and 3. It exhibits a peak for moderate values of Ω and N and it falls down for larger values. In Fig. 3

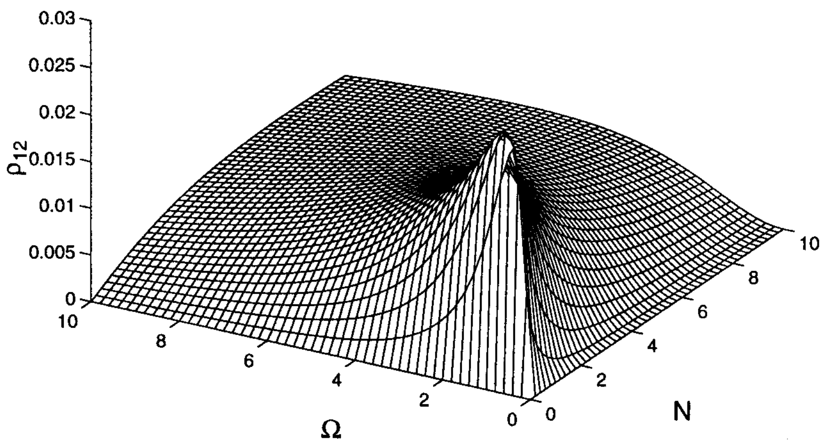

Fig. 3. Plot of ρ_{12} as a function of N and Ω with $\gamma_1 = \gamma_2 = \gamma_c = 1$ and $|M| = \sqrt{N(N+1)}$

the atomic coherence ρ_{12} associated with the forbiden transition $1 \leftrightarrow 2$ is plotted. It shows a peak that can be atributed to the peak of the inversion, its absolute values are not very high, but it is obvious that the squeezed vacuum together with the non-zero coherence transfer rate introduces coherence between the two lower levels of the lambda system. Such coherences can play a role in amplification without inversion, the subject that we shall not discuss here. Especially interesting is the coherence ρ_{23}, i.e., the coherence associated with the allowed transition $2 \leftrightarrow 3$, which is illustrated in Fig. 4. The non-zero value of this coherence means a non-zero atomic dipole moment oscillating at frequency $\omega_2 = \omega_{32}$ related to this transition, and consequently, a field radiated by the atom . This can be seen as a sort of 'coherent' four wave mixing process with the participation of the squeezed vacuum field, and the relation $2\omega_s - \omega_1 - \omega_2 = 0$ indicates that two photons of the squeezed vacuum and one photon of the driving field

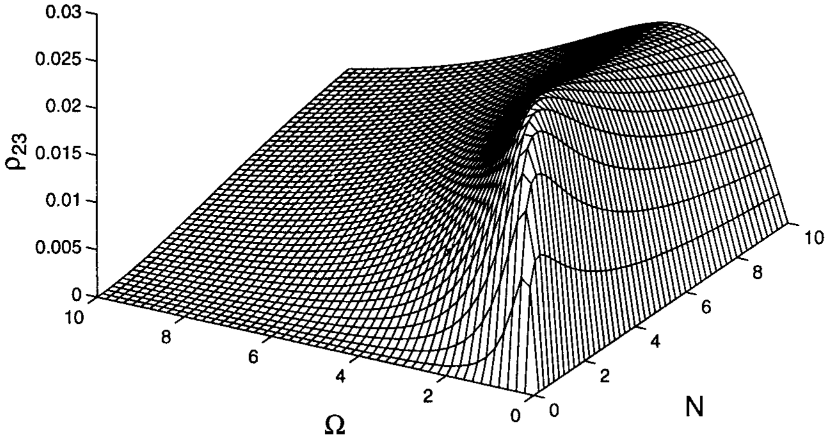

Fig. 4. Plot of ρ_{23} as a function of N and Ω with $\gamma_1 = \gamma_2 = \gamma_c = 1$ and $|M| = \sqrt{N(N+1)}$

combine into a photon of the resulting field. The squeezed vacuum behaves in this case, in a sense, like a coherent field, despite the fact that its mean value is zero. However, the mean value of the square of the squeezed vacuum field is different from zero (it is proportional to M) and this is the quantity that plays a role similar to a coherent field in the process of wave mixing with the pump field. To have a non-zero dipole moment on the undriven transition it is crucial that the two atomic transitions are correlated via the coherence transfer rate. The effect does not appear for independent transitions.

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