

Squeezing in two-atom resonance fluorescence induced by two-photon coherences

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Received 12 October 1993

Abstract. We examine squeezing properties of resonance fluorescence from two interacting atoms. Previous theoretical analyses of this model have predicted squeezing on one-photon resonance. Here we show that it is possible to obtain squeezing in two-atom resonance fluorescence through two-photon transitions induced by a cooperative atomic interaction. The squeezing occurs near a two-photon resonance and is not sensitive to the duration of the detection time in comparison with the decay time of the subradiant state. Moreover, squeezing near a two-photon resonance is completely independent of the direction of observation in respect to the atomic axis and can be significantly larger than that previously found on one-photon resonance.

1. Introduction

Cooperative atomic emission and absorption has been a subject of continuous interest since it was first proposed by Dicke [1]. The question of to what extent the radiation scattered by a cooperative atomic system is altered, compared with the single atom case, has become of interest as it contains information about the inter-atomic interactions. It has become common to describe this situation using the approximation of many two-level atoms confined to a region much smaller than the wavelength of the driving field and coupled identically to a single radiation mode. While this description does not include physical features such as spatial separations and dipole–dipole interactions between the atoms, it is nevertheless of interest, as the simplest model of a group of atoms with a cooperative decay.

Because it is difficult to treat N atoms with the dipole–dipole interaction and a driving laser field, some previous work has been devoted to study collective effects in the case of several atoms. These include superradiance [2, 3], two-photon absorption [4–6], and non-classical effects such as photon antibunching [7–9], squeezing [10, 11], and quantum jumps [12–14]. Although the several (two or three) atom system is admittedly an elementary model, it offers some advantages over the multiatom problem. Almost exact analytical solutions for a two-atom system interacting cooperatively and driven by a coherent laser field have been obtained, in order to understand possible deviations from the single atoms. Many of these results are analogous to phenomena that one would expect in multiatom systems.

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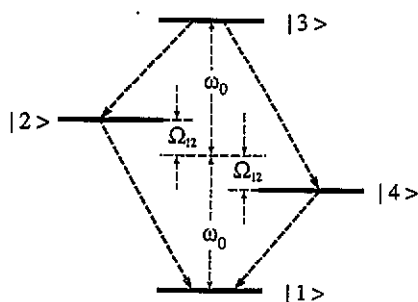


Figure 1. Energy-level diagram for two identical atoms represented as a single four-level system. The frequency shift of the intermediate levels $|2\rangle$ and $|4\rangle$ is due to the dipole-dipole interactions Ω_{12} . The ground state $|1\rangle$ and the most excited state $|3\rangle$ remain in two-photon resonance $2\omega_0$, where ω_0 is the resonant frequency of a single atom.

The earliest analytical and numerical treatments of the two-atom model with the dipole-dipole interaction assumed a constant interatomic separation during the radiation process. This is equivalent to a study of the collective effects for an idealized atomic beam consisting of a random distribution of atoms moving with the same velocity. For a real beam, whose atoms move with non-uniform velocities the dipole-dipole interaction is important only for large atomic densities. However, for more dense atomic beams three-and-more atom effects should be included into the calculations, which makes the algebra exceedingly lengthy.

Recently, there has been a renewal of interest in the system of two or three atoms due to the experimental progress made in confining a few atoms or ions at small interatomic separations in ion and neutral-atom traps [15]. The trapped atoms are essentially motionless and lie at a known and controllable distance from one another, permitting qualitatively new studies of interatomic interactions not accessible in a gas cell or an atomic beam. Moreover, this enables the single-atom effects to be separated from those arising from the correlations between the atoms. For example, the phenomena of superradiance and subradiance have recently been observed in an experiment involving a microscopic planar Pauli trap [16].

In view of the above it is useful to consider the squeezing properties of the fluorescence field emitted from two trapped atoms and to incorporate the modification in the fluctuations due to the cooperative decay as well as the dipole-dipole interaction. It is now well established that interactions between a pair of identical atoms give rise to new resonances in the fluorescence and weak-probe absorption spectra [17–21], especially the dipole-dipole interaction induces novel two-photon resonances on the double frequency of the atomic transitions [4–6]. These resonances are induced by the simultaneous transitions in the two-atom system, where the atoms jump simultaneously from their ground to excited states. A correct interpretation of the origin of these simultaneous transitions is possible by referring to the collective states of the two-atom system [1, 22]. In the collective level scheme, the two-atom system behaves as a single four-level system (figure 1), in which energies of the intermediate $|2\rangle$ and $|4\rangle$ levels depend on the dipole-dipole interaction Ω_{12} between the atoms. For large interatomic separations r_{12} , the level shift due to the dipole-dipole interaction is very small, $\Omega_{12} \approx 0$, and the excitation of the atoms proceeds by two degenerate transitions, with resonant intermediate states. These intermediate states have a normal spontaneous lifetime so that the double excitation from $|1\rangle$ to $|3\rangle$

proceeds by sequential process. For small r_{12} , the intermediate states suffer a large shift due to the dipole-dipole interaction Ω_{12} , and remove themselves from one-photon resonance. Nevertheless the double excitation between the ground state $|1\rangle$ and the most excited state $|3\rangle$ remains in two-photon resonance via a virtual state which lives only for a time of order of the reciprocal of the one-photon detuning given by the dipole-dipole interaction shift Ω_{12} . In this way the large r_{12} sequential transitions are replaced at small r_{12} by effectively simultaneous two-photon transitions.

Photon antibunching and squeezing properties of the fluorescence field emitted from a two-atom system have also been shown to yield information about the interatomic interactions [10, 11]. In particular, if the laser field is tuned to resonance with the transition $|1\rangle \rightarrow |2\rangle$ and its intensity is not too large, a pronounced photon antibunching can be obtained in such a two-atom system, well known from single-atom resonance fluorescence [23, 24]. Squeezing, which for single-atom resonance fluorescence has its maximum for the laser field on resonance with the atomic frequency [25, 26], shifts to the region of finite detuning Δ (see figure 2). In other words, this means that the laser frequency is tuned to resonance with a particular pair of energy levels of the two-atom system that are shifted by the dipole-dipole interaction. In this case, however, the two-atom system behaves like an individual two-level system with transition frequency characteristics of the $|1\rangle \rightarrow |2\rangle$ transition.

The fluctuations of the fluorescence field, apart from the large squeezing at finite detuning Δ , exhibit also a small dispersion-like structure near $\Delta = 0$ (see figure 2). In contrast to the resonance occurring at finite Δ , which has a clear physical interpretation, the dispersive structure centred on $\Delta = 0$ is not so easy to understand. To our

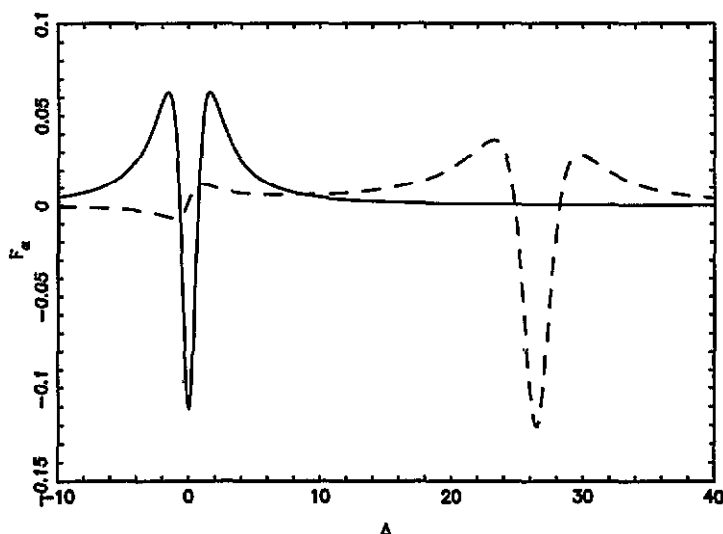


Figure 2. Normalized quadrature component F_a of the fluorescence field as a function of the detuning Δ between the atomic and laser field frequencies for two independent atoms (full curve) and two interacting atoms (broken curve). The dipole-dipole interaction between the atoms shifts squeezing (negative values of F_a) to the region of finite detuning. For interacting atoms F_a exhibits also a small dispersion-like structure near $\Delta \approx 0$, the interpretation of which is the subject of this paper.

knowledge no explanation has been offered so far for the appearance of the dispersive structure at $\Delta = 0$.

In this paper we reconsider squeezing properties of the fluorescence field emitted from two two-level atoms. In particular, we make a detailed study of the physical origin of the dispersive structure in the fluctuations of the fluorescence field. Previous work [10, 11] has emphasized squeezing at finite detuning which is of a two-level-atom character. The effect that we are concerned with occurs when the laser field is tuned to the atomic resonance. In order to explore this we extend the previous studies [10] to the case of large intensities of the driving laser field and small interatomic separations. We show that the dispersive structure at $\Delta = 0$ becomes more dominant as the intensity of the driving field increases. In addition, a large squeezing is generated near $\Delta = 0$, which can be as much as twice the squeezing found previously at one-photon resonance. The origin of the dispersive structure and squeezing at $\Delta = 0$ is attributed to the two-photon coherences, which are significant for small interatomic separations and large intensities of the driving field. This is inherent collective effect arising from the presence of the interactions between the atoms.

2. Master equation and evolution of the density matrix elements

Consider a system of two identical two-level atoms separated by a distance r_{12} and driven by a coherent laser field of constant amplitude \mathcal{E}_0 , and of frequency ω_L tuned close to the atomic transition frequency ω_0 . Simultaneously the atoms are coupled to all other modes of the electromagnetic field being initially in their vacuum states. The ground state $|g_i\rangle$ and the excited state $|e_i\rangle$ ($i=1, 2$) of each atom are assumed to be connected by an electric dipole transition, with a dipole matrix element μ .

The equations of motion used to describe the system can be equally derived from the master equation approach, or from the Heisenberg equation of motion. Here we choose to use the master equation for the reduced density operator describing the atomic dynamics, which in a frame rotating with the laser frequency ω_L and in the rotating-wave approximation is [22, 27]

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i \sum_{l=1}^2 \{ \delta [S_l^{\dagger}, \rho] - \frac{1}{2} \Omega [(S_l^{\dagger} e^{-i\varphi_L} + S_l^{-} e^{i\varphi_L}), \rho] \} \\ & - i \sum_{l \neq k}^2 \Omega_{lk} [S_l^{\dagger} S_k^{-}, \rho] - \sum_{l,k} \gamma_{lk} (S_l^{\dagger} S_k^{-} \rho + \rho S_l^{\dagger} S_k^{-} - 2 S_k^{-} \rho S_l^{\dagger}). \end{aligned} \quad (1)$$

Here $\Omega = |\mu \cdot \mathcal{E}_0|/\hbar$ is the Rabi frequency associated with the driving field of amplitude \mathcal{E}_0 and phase φ_L , $\delta = (\omega_0 - \omega_L)$, $2\gamma_{ii} = 2\gamma_{kk} = 2\gamma$ is the Einstein A coefficient, and S_i^{\dagger} are the atomic raising and lowering operators, which together with the population inversion operator S_i^z fulfil the well known commutation relations:

$$[S_i^{\dagger}, S_k^{-}] = 2S_i^z \delta_{ik} \quad [S_i^z, S_k^{\pm}] = \pm S_i^{\pm} \delta_{ik}. \quad (2)$$

The parameters γ_{lk} and Ω_{lk} ($l \neq k$), which appear in equation (1) describe the collective damping and the collective frequency shift, respectively, and are produced by the mutual interaction of the atoms through their own electromagnetic field. These parameters depend on the interatomic distance r_{12} , and on the geometrical configur-

ation of the atomic dipoles, and are given by [22, 27, 28]

$$\gamma_{lk} = \frac{3}{2}\gamma \left[[1 - (\hat{\mu} \cdot \hat{r}_{lk})^2] \frac{\sin(k_0 r_{lk})}{k_0 r_{lk}} + [1 - 3(\hat{\mu} \cdot \hat{r}_{lk})^2] \left(\frac{\cos(k_0 r_{lk})}{(k_0 r_{lk})^2} - \frac{\sin(k_0 r_{lk})}{(k_0 r_{lk})^3} \right) \right] \quad (3)$$

$$\Omega_{lk} = \frac{3}{2}\gamma \left[-[1 - (\hat{\mu} \cdot \hat{r}_{lk})^2] \frac{\cos(k_0 r_{lk})}{k_0 r_{lk}} + [1 - 3(\hat{\mu} \cdot \hat{r}_{lk})^2] \left(\frac{\sin(k_0 r_{lk})}{(k_0 r_{lk})^2} + \frac{\cos(k_0 r_{lk})}{(k_0 r_{lk})^3} \right) \right] \quad (4)$$

where $\hat{\mu}$ and \hat{r}_{lk} are unit vectors along the transition electric dipole moment and the vector $\mathbf{r}_{lk} = \mathbf{r}_k - \mathbf{r}_l$, respectively, and $k_0 = \omega_0/c$.

In equation (1), we have specified the orientation of the interatomic axis to be perpendicular to the direction of propagation of the laser beam. This is not an essential feature as we are interested in effects which occur at very small interatomic separations where the orientation of the atomic axis is not important. Some interesting effects, which are sensitive to the orientation of the interatomic axis relative to the direction of propagation of the laser beam, have been recently reported [29–31]. However, these effects are significant for large interatomic separations.

For the purpose of the following development it is convenient to employ the collective (Dicke) atomic states [1, 22]

$$\begin{aligned} |1\rangle &= |g_1\rangle|g_2\rangle & |2\rangle &= 2^{-1/2}(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle) \\ |3\rangle &= |e_1\rangle|e_2\rangle & |4\rangle &= 2^{-1/2}(|e_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle) \end{aligned} \quad (5)$$

in which the density matrix elements are

$$\rho_{ij} = \langle i|\rho|j\rangle \quad i, j = 1, 2, 3, 4. \quad (6)$$

The elements ρ_{ii} are the populations of the collective states $|i\rangle$, and $\rho_{ij} (i \neq j)$ the coherences. From equation (1) we find that the density matrix elements satisfy the equations of motion

$$\begin{aligned} \dot{\rho}_{22} &= (1+a)(\rho_{33} - \rho_{22}) + i\beta[(\rho_{12} - \rho_{21}) + (\rho_{32} - \rho_{23})], \\ \dot{\rho}_{33} &= -2\rho_{33} - i\beta(\rho_{32} - \rho_{23}), \\ \dot{\rho}_{44} &= (1-a)(\rho_{33} - \rho_{44}), \\ \dot{\rho}_{23} &= -\frac{1}{2}[(3+a) - i(\Delta - b)]\rho_{23} + i\beta[\rho_{13} + (\rho_{33} - \rho_{22})], \\ \dot{\rho}_{12} &= -i\beta - \frac{1}{2}[(1+a) - i(\Delta + b)]\rho_{12} + (1+a)\rho_{23} + i\beta[2\rho_{22} + \rho_{33} + \rho_{44} - \rho_{13}], \\ \rho_{13} &= -(1-i\Delta)\rho_{13} + i\beta(\rho_{23} - \rho_{12}), \end{aligned} \quad (7a)$$

and

$$\begin{aligned} \dot{\rho}_{14} &= -\frac{1}{2}[(1-a) - i(\Delta - b)]\rho_{14} - (1-a)\rho_{43} + i\beta\rho_{24}, \\ \dot{\rho}_{43} &= -\frac{1}{2}[(3-a) - i(\Delta + b)]\rho_{43} - i\beta\rho_{42}, \\ \dot{\rho}_{24} &= -(1+i\beta)\rho_{24} + i\beta(\rho_{14} + \rho_{34}), \end{aligned} \quad (7b)$$

where the dot indicates differentiation with respect to $\tau = 2\gamma t$. The other parameters have also been made dimensionless: $\Delta = \delta/\gamma$, $b = \Omega_{12}/\gamma$, $a = \gamma_{12}/\gamma$, and $\beta = \Omega/2\sqrt{2}\gamma$. In obtaining equation (7), use has been made of the relation

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1 \quad (8)$$

stemming from the trace condition. The Hermiticity of ρ further implies that ρ_{22}, ρ_{33}

and ρ_{44} are real functions and $\rho_{21} = \rho_{12}^*$, $\rho_{23} = \rho_{32}^*$, $\rho_{14} = \rho_{41}^*$, $\rho_{43} = \rho_{34}^*$, $\rho_{13} = \rho_{31}^*$, $\rho_{24} = \rho_{42}^*$, $\rho_{43} = \rho_{34}^*$.

It is evident from equation (7) that the system of equations decouples into two groups. One consists of nine equations for the matrix elements describing transitions via the superradiant state $|2\rangle$, and the other consists of six equations for the matrix elements describing transitions via the subradiant state $|4\rangle$. However, as long as $r_{12} \neq 0$ the matrix element ρ_{44} is coupled to the coherences involving the superradiant state $|2\rangle$ rather than the coherences involving the subradiant state $|4\rangle$. Assuming that the atoms are in their ground states just before the laser is turned on, the matrix elements (7b) retain their initial zero values, and only the set of nine equations (7a) can have non-zero steady-state solutions.

Setting $\dot{\rho}_{ij} = 0$ in equation (7) gives the steady-state values of density matrix elements

$$\begin{aligned}\rho_{33} &= \rho_{44} = 4\beta^4/D \\ \rho_{22} &= 4\beta^2(1 + \Delta^2 + \beta^2)/D \\ \rho_{12} &= 2i\beta\{2\beta(\beta + i\Delta) + (1 + \Delta^2)[(1 + a) + i(\Delta + b)]\}/D \\ \rho_{23} &= -4i\beta^3(1 + i\Delta)/D \\ \rho_{13} &= -2\beta^2(1 + i\Delta)[(1 + a) + i(\Delta + b)]/D\end{aligned}\quad (9)$$

where

$$D = 16\beta^4 + 8(1 + \Delta^2)\beta^2 + (1 + \Delta^2)[(1 + a)^2 + (\Delta + b)^2]. \quad (10)$$

The steady-state solutions (9) are identical to that obtained previously [10, 11], and have been used to discuss photon antibunching and squeezing in the two-atom resonance fluorescence. However, previous calculations have been specifically oriented towards studying the effects near one-photon resonance $|1\rangle \rightarrow |2\rangle$. It has been shown [10, 11] that a pronounced photon antibunching and squeezing can be obtained in resonance fluorescence from a two-atom system, when the laser driving field is tuned to the one-photon transitions $|1\rangle \rightarrow |2\rangle$. What happens near two-photon resonance $|1\rangle \rightarrow |3\rangle$ has not been discussed previously. It is seen from equations (9) and (10) that the populations of the collective atomic levels as well as the coherences exhibit resonant behaviours not only on the one-photon resonance $\Delta = -b$, but also on the two-photon resonance $\Delta = 0$. The existence of the resonance at $\Delta = 0$ illustrates the occurrence of multi-photon processes in the two-atom system. We will use equation (9) to discuss squeezing properties of the fluorescence field near two-photon resonance, and show that for this effect two-photon coherences are important.

3. Fluctuations of the fluorescence field

In order to calculate squeezing properties of the fluorescence field let us introduce the quadrature components at frequency ω and wave vector k

$$E_\theta = E^{(+)}(R, t) \exp[i(\omega t - k \cdot R + \theta)] + E^{(-)}(R, t) \exp[-i(\omega t - k \cdot R + \theta)], \quad (11)$$

$$E_{\theta-\pi/2} = -i\{E^{(+)}(R, t) \exp[i(\omega t - k \cdot R + \theta)] - E^{(-)}(R, t) \exp[-i(\omega t - k \cdot R + \theta)]\}$$

satisfying the commutation relation

$$[E_\theta, E_{\theta-\pi/2}] = 2iC \quad (12)$$

where $E^{(+)}(\mathbf{R}, t)$ ($E^{(-)}(\mathbf{R}, t)$) is the positive (negative)-frequency part of the fluorescence field and C is a positive c number.

Introducing the fluctuation operator

$$\Delta E_\alpha = E_\alpha - \langle E_\alpha \rangle \quad \alpha = \theta, \theta - \pi/2 \quad (13)$$

we can write the variance of E_α as

$$\langle (\Delta E_\alpha)^2 \rangle = \langle :(\Delta E_\alpha)^2: \rangle + C \quad (14)$$

where the colon stands for normal ordering of the operators. Since squeezing is defined by the requirement that the variance of one of two non-commuting observables shall be less than half of the absolute value of the expectation value of their commutator, the variance $\langle (\Delta E_\alpha)^2 \rangle$ has to be less than C to meet this requirement. According to (14), squeezing of the field is characterized by the condition that either $\langle :(\Delta E_\theta)^2: \rangle$ or $\langle :(\Delta E_{\theta-\pi/2})^2: \rangle$ is negative.

To analyse squeezing in the fluorescence field we use the following relation between the scattered field and atomic operators in the far-field limit [22, 27]

$$E^{(+)}(\mathbf{R}, t) = -(\omega_0/c)^2 \sum_{i=1}^2 \frac{\hat{\mathbf{R}} \times (\hat{\mathbf{R}} \times \boldsymbol{\mu})}{R} S_i^-(t - R/c) \exp(-ik\hat{\mathbf{R}} \cdot \mathbf{r}_i) \quad (15)$$

where $k = \omega_0/c$, $\hat{\mathbf{R}}$ is the unit vector in the direction $\mathbf{R} = R\hat{\mathbf{R}}$ of the observation point, \mathbf{r}_i is the position vector of the i th atom and S_i^- are the atom operators.

Having available the fluorescence field operators expressed by the atomic operators and by using the identities [22]

$$\begin{aligned} \langle S_1^+ \rangle &= 2^{1/2}(\rho_{23} - \rho_{43} + \rho_{12} + \rho_{14}) \\ \langle S_2^+ \rangle &= 2^{1/2}(\rho_{23} + \rho_{43} + \rho_{12} - \rho_{14}) \\ \langle S_1^+ S_2^+ \rangle &= \rho_{13} \\ \langle S_1^+ S_1^- \rangle &= \langle S_2^+ S_2^- \rangle = \rho_{33} + \frac{1}{2}(\rho_{22} + \rho_{44}) \\ \langle S_1^+ S_2^- \rangle &= \frac{1}{2}(\rho_{22} - \rho_{44}) - \frac{1}{2}(\rho_{42} - \rho_{24}) \\ \langle S_1^+ S_2^+ S_1^- S_2^- \rangle &= \rho_{33} \end{aligned} \quad (16)$$

we can directly apply the steady-state solutions (9) to calculate squeezing in resonance fluorescence.

From equations (14)–(16) we find that the normalized normally ordered variance $\langle :(\Delta E_\alpha)^2: \rangle$ is given by

$$\begin{aligned} F_\alpha &= \langle :(\Delta E_\alpha)^2: \rangle / U^2(\mathbf{R}) \\ &= 2\rho_{33} + \rho_{22} + \rho_{44} + (\rho_{22} - \rho_{44}) \cos(k\hat{\mathbf{R}} \cdot \mathbf{r}_{12}) + \rho_{31} \exp\{2i[(\omega - \omega_L)t + \alpha]\} \\ &\quad + \rho_{13} \exp\{-2i[(\omega - \omega_L)t + \alpha]\} + i(\rho_{42} - \rho_{24}) \sin(k\hat{\mathbf{R}} \cdot \mathbf{r}_{12}) \\ &\quad - \left[(\rho_{32} + \rho_{21}) \cos(\frac{1}{2}k\hat{\mathbf{R}} \cdot \mathbf{r}_{12}) - i(\rho_{34} - \rho_{41}) \sin(\frac{1}{2}k\hat{\mathbf{R}} \cdot \mathbf{r}_{12}) \right] \\ &\quad \times \exp\{i[(\omega - \omega_L)t + \alpha]\} + [(\rho_{23} + \rho_{12}) \cos(\frac{1}{2}k\hat{\mathbf{R}} \cdot \mathbf{r}_{12}) \\ &\quad + i(\rho_{43} - \rho_{14}) \sin(\frac{1}{2}k\hat{\mathbf{R}} \cdot \mathbf{r}_{12})] \exp\{-i[(\omega - \omega_L)t + \alpha]\} \end{aligned} \quad (7)$$

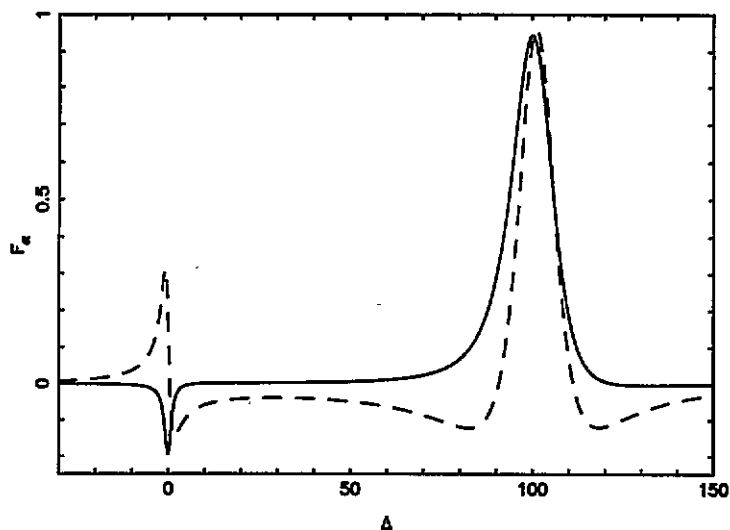


Figure 3. F_α as a function of the detuning Δ for $kr_{12}=0.1\pi$, $\beta=2.5$, $\mathbf{R} \perp \mathbf{r}_{12}$, and different phases α : $\alpha=\pi/4$ (full curve), $\alpha=\pi/2$ (broken curve).

where $\alpha = \theta - \mathbf{k} \cdot \mathbf{R}$, and $U^2(\mathbf{R}) = (4k^4\mu^2/R^2) \sin^2 \psi_0$, with ψ_0 the angle between the observation direction \mathbf{R} and the atomic transition dipole moment $\boldsymbol{\mu}$.

Equation (17) shows that the normalized normally ordered variance F_α depends on phase α not only through the one-photon atomic coherences but also through the two-photon atomic coherences. This dependence suggests that there are two different processes that can lead to squeezing in two-atom resonance fluorescence. This is illustrated in figure 3, where the variance F_α calculated at the frequency $\omega = \omega_L$, is plotted against the detuning Δ for $kr_{12}=0.1\pi$, $\beta=2.5$, $\mathbf{R} \perp \mathbf{r}_{12}$ and different phases α . The variances F_α show a strong dependence on phase α near both one- and two-photon resonance. Moreover, a considerable amount of squeezing is found at these resonances. It is seen from figure 3 that near a two-photon resonance a change by $\pi/4$ of the initial phase of the driving field changes a dispersion-like structure of the variance F_α into an absorption-like type. According to equations (9) and (17), the normalized normally ordered variance F_α for $b \gg \beta \gg 1$ can be written as

$$F_\alpha = \frac{4\beta^2}{b} \left[\frac{\Delta}{(1+\Delta^2)} \cos 2\alpha - \frac{1}{(1+\Delta^2)} \sin 2\alpha \right] \quad (18)$$

where we have retained only those terms which contribute near two-photon resonance. Equation (18) predicts a dispersion-like structure for $\alpha=0$ or $\pi/2$, and an absorption-like structure for $\alpha=\pi/4$. Moreover equation (18) shows that the presence of the dipole-dipole interaction is essential to obtain squeezing near two-photon resonance. The emergence of an additional dipole-dipole interaction induced squeezing is a clear indication of a totally different process, which can appear in two-atom resonance fluorescence. The dipole-dipole interaction induces two-photon transitions $|1\rangle \rightarrow |3\rangle$, which are responsible for the origin of the two-photon coherences. These coherences cause squeezing near two-photon resonance, whereas near one-photon resonance squeezing is caused by the one-photon coherence $|1\rangle \rightarrow |2\rangle$. This is shown in figure 4, where we plot the one-photon coherences $|\rho_{12} + \rho_{23}|$ and the two-photon

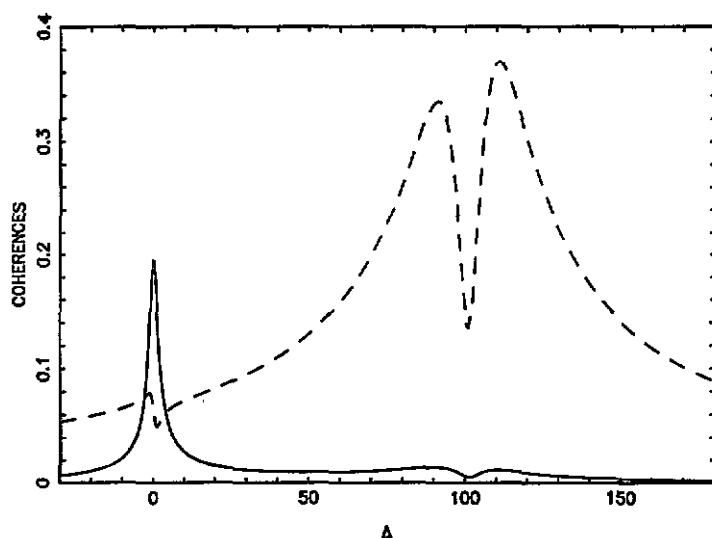


Figure 4. Two-photon coherence $|\rho_{13}|$ (full curve) and one-photon coherences $|\rho_{12} + \rho_{23}|$ (broken curve) as a function of the detuning Δ for $k\tau_{12} = 0.1\pi$, and $\beta = 2.5$.

coherence $|\rho_{13}|$ in function of the detuning Δ for the same parameters as in figure 3. It is seen that near one-photon resonance the two-photon coherence is zero and only the one-photon coherences $|\rho_{12} + \rho_{23}|$ are large, but near two-photon resonance only the two-photon coherence $|\rho_{13}|$ is significant.

It is interesting to note from equation (17) that the angular distribution of the squeezing properties of the fluorescence field about the line joining the atoms (proportional to $\cos(\frac{1}{2}k\hat{R} \cdot r_{12})$ and $\sin(\frac{1}{2}k\hat{R} \cdot r_{12})$) depends only on the one-photon coherences, whereas the contribution from the two-photon coherences is completely independent of the direction of observation.

In figure 2 we have chosen the parameters in such a way that at $\Delta = -b$ the variance F_a becomes minimal and reaches its minimum value -0.125 , which corresponds to maximum squeezing of the steady-state resonance fluorescence produced by a two level atom [10, 11, 25]. Two-atom resonance fluorescence, however, can produce significantly larger squeezing near two-photon resonance $\Delta = 0$, provided that the Rabi frequency of the driving field is large. This is shown in figure 5, where the variance F_a is plotted near $\Delta = 0$ for $\alpha = 0$, $k\tau_{12} = 0.02\pi$, $\hat{R} \perp r_{12}$, $\omega = \omega_L$ and different β . As the Rabi frequency increases the squeezing increases and attains its maximum value -0.21 for very large Rabi frequencies. This value, compared with the -0.125 derived above, is almost two times larger than that obtained near a one-photon resonance. The result shows that the squeezing in two-atom resonance fluorescence can be enhanced if the interatomic separation is small and the laser frequency is tuned to the two-photon resonance.

Until now we have calculated the squeezing properties of the steady-state fluorescence field, which is reached after the atoms have interacted with the driving field for several natural lifetimes. For a strong driving field ($\beta \gg 1$) we find from equation (9) that the collective atomic states saturate at identical values

$$\rho_{11} = \rho_{22} = \rho_{33} = \rho_{44} = \frac{1}{4}. \quad (19)$$

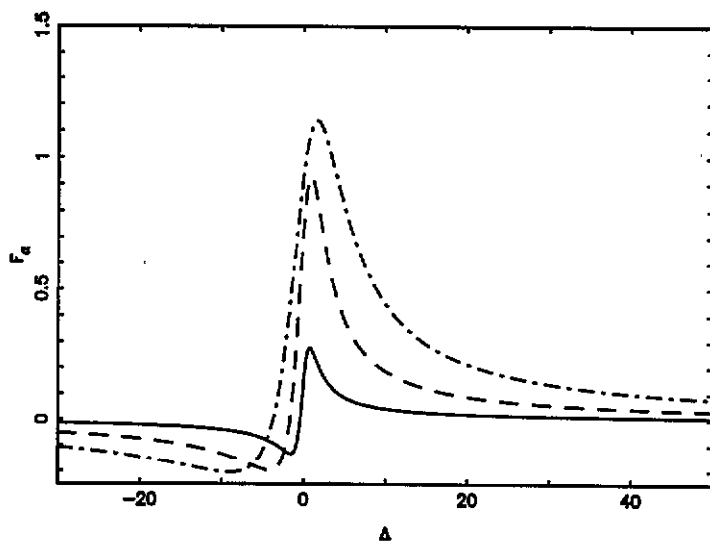


Figure 5. F_a as a function of Δ for $\alpha=0$, $kr_{12}=0.02\pi$, $\hat{K} \perp r_{12}$ and different β : $\beta=25$ (full curve), $\beta=50$ (broken curve), $\beta=75$ (chain curve).

It is seen from equation (7a) that the subradiant state $|4\rangle$ experiences a variation on a time scale of order $(\gamma - \gamma_{12})^{-1}$, which time must be small compared to typical observation times ($\sim 50\gamma^{-1}$) in order that the steady state be reached, with the subradiant state participating fully in the interaction. For $t < (\gamma - \gamma_{12})^{-1}$ the subradiant state does not participate in the interaction, and the system reaches a steady state between the symmetric states $|1\rangle$, $|2\rangle$ and $|3\rangle$. The question then arises as to whether the observed fluorescence is from atoms which have been interacting with the laser field for less than, or greater than, the characteristic time $(\gamma - \gamma_{12})^{-1}$. This can be estimated as follows. Suppose that atoms are typically in the interaction region for a time $T \sim K\gamma^{-1}$. If $kr_{12} \ll 0.1$, then $(1 - a)^{-1} \gg 100$ and the values of K should be sufficiently large to ensure that the subradiant state is important. For values of $K \sim 50$ the interaction time T is smaller than $(\gamma - \gamma_{12})^{-1}$ implying that the fluorescence is taken from the symmetric states only. In this case a change of the steady-state solutions for the density matrix elements is required. Assuming that $kr_{12} \ll 1$ we can approximate γ_{12} by γ . In this case the subradiant state $|4\rangle$ remains unpopulated ($\rho_{44}=0$) during the time $T < (\gamma - \gamma_{12})^{-1}$, and the steady-state values of the density matrix elements are

$$\begin{aligned}
 \rho_{33} &= 4\beta^4/D' \\
 \rho_{22} &= 4\beta^2(1 + \Delta^2 + \beta^2)/D' \\
 \rho_{12} &= 2i\beta\{2\beta(\beta + i\Delta) + (1 + \Delta^2)[2 + i(\Delta + b)]\}/D' \\
 \rho_{23} &= -4i\beta^3(1 + i\Delta)/D' \\
 \rho_{13} &= -2\beta^2(1 + i\Delta)[2 + i(\Delta + b)]/D'
 \end{aligned} \tag{20}$$

where

$$D' = 12\beta^4 + 8(1 + \Delta^2)\beta^2 + (1 + \Delta^2)[4 + (\Delta + b)^2]. \tag{21}$$

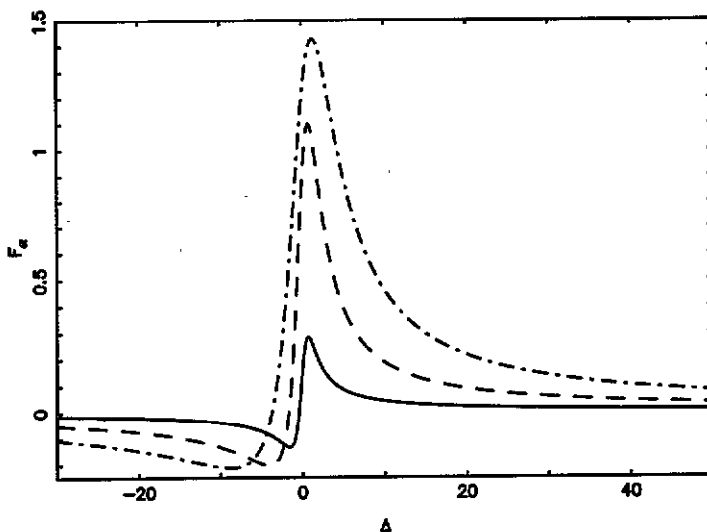


Figure 6. Same as in figure 5 but for the detection time short compared to the decay time of the subradiant state $|4\rangle$.

For a weak driving field ($\beta \ll 1$) the steady-state solutions (9) and (20) are essentially equal if we set $\alpha = 1$ in equations (9). However for a strong driving field we find that the steady-state populations of the collective atomic states are

$$\rho_{11} = \rho_{22} = \rho_{33} = \frac{1}{3}. \quad (22)$$

This result is significantly different from that given in equation (19), where the subradiant state was coupled to the symmetric states. It is worth repeating that if the detection time is short compared to the decay time of the subradiant state, then the steady-state populations of the symmetric states are enhanced by a factor $\frac{4}{3}$. Despite this, the variance F_α , near $\Delta = 0$, is not changed significantly from the values given for the detection time long compared to the decay time of the subradiant state, leading to the same qualitative behaviour of squeezing in the fluorescence field. This is shown in figure 6, where we plot the variance F_α as a function of the detuning Δ , for $\alpha = 0$, $kr_{12} = 0.02\pi$ and different Rabi frequencies. It is obvious from figures 5 and 6 that the variances of the fluorescence field are almost identical independent of the relation between the detection time and the decay time of the subradiant state.

4. Conclusions

We have shown that it is possible to obtain squeezing in two-atom resonance fluorescence through two-photon transitions induced by a cooperative atomic interaction. The squeezing occurs near a two-photon resonance and is not sensitive to the duration of the detection time. In contrast to squeezing appearing near a one-photon resonance, which strongly depends on the direction of observation about the line joining the atoms, the squeezing near a two-photon resonance is completely independent of the direction of observation. Moreover, the maximum of squeezing near a two-photon resonance can be almost twice as large as for one-photon transitions. In

order to explore this, small interatomic separations and large Rabi frequencies of the driving field are needed. Current technology can produce an ion crystal with atom-atom separations comparable to the wavelength of the driving field [15, 16]. To explore effects near two-photon resonance separations much smaller than the resonant wavelength are required. Despite this, in this paper we have demonstrated that the fluorescence field from two identical atoms can show a large squeezing near two-photon resonance, as a result of the two-photon coherences induced by the interatomic interactions.

Acknowledgment

The research was supported by the Australian Research Council, to whom the authors extend their thanks.

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