

QUANTUM RESONANCE FLUORESCENCE FROM MUTUALLY CORRELATED ATOMS

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I. INTRODUCTION

Photon antibunching and squeezing are two unique phenomena that reveal the quantum properties of the radiation field. These effects are just two examples of nonclassical light, that is, light with properties that are not predicted by the classical wave theory of light. According to quantum mechanics, electrons in atoms can occupy only certain energy levels. An electron can jump from its lowest energy level—the ground state—to a second higher energy level by absorbing light of definite frequency from a pumping beam. Next, the electron can fall back to the ground state,

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emitting a photon. It has been predicted from the statistics of quantum theory that once the electron had returned to the ground state, there would be a delay before the electron would be re-excited by the pumping beam. This delay would result in an intermittent emission of light called photon antibunching. This intermittent emission produces a radiation in which the variance of the number of photons is less than the mean number of photons. The classical theory of electromagnetic radiation, which does not quantize energy, does not predict antibunching.

Yuen [1] has predicted the possibility of another nonclassical phenomenon, squeezed light. The Heisenberg uncertainty principle predicts that it is never possible to be absolutely precise in measuring one of two noncommuting observables. The product of the fluctuations of the two noncommuting observables must be greater than or equal to one-half of the absolute value of their commutator. For all field states that have classical analog the field quadrature variances are also greater than or equal to this commutator. For the vacuum state and coherent states the noise in the two noncommuting field quadratures is distributed symmetrically between the two quadratures and the variance of the field quadrature is equal to the commutator establishing the level of quantum noise (vacuum fluctuations). There are, however, quantum states of the field such that the variance of one of two noncommuting field observables is smaller than the vacuum fluctuations. Such a field is referred to as squeezed light. In squeezed light the quantum fluctuations in one quadrature component are reduced below their vacuum values at the expense of increased fluctuations in the other component, such that the uncertainty relation is not violated.

Photon antibunching has been predicted theoretically for the first time in resonance fluorescence of a two-level atom [2, 3]. Since then, a number of papers have appeared analyzing the possibilities of obtaining photon antibunching in various processes offered by nonlinear optics [4–11]. Significant contribution to these studies has been given by Kielich and co-workers [12–17]. The possibility of obtaining squeezed light has been extensively studied since the first theoretical papers by Walls and Zoller [18] and Mandel [19] on reduction of noise and photon statistics in resonance fluorescence of a two-level atom. Many linear and nonlinear processes have predicted a large amount of reduction of noise below the classical limit. Almost a complete reduction of noise (98% below the vacuum limit) was found by Tanaś and Kielich [20] in a self-squeezed light produced by a propagation of a coherent laser beam in a nonlinear medium.

Several experimental groups have been successful in producing nonclassical light. However, photon antibunching has been observed only in

fluorescing sodium atoms [21]. This was the first experiment in which the nonclassical effect was observed in optics. A number of groups have been actively involved in the actual generation of squeezed light. Slusher et al. [22] generated for the first time a squeezed light in which a 7% noise reduction below the vacuum limit was observed. Wu et al. [23] reported more than 50% reduction of noise below the vacuum limit in an optical oscillator. Heidmann et al. [24] used a two-mode optical parametric oscillator operating above threshold to generate two highly correlated beams of light. The measured noise in the intensity difference of the two beams was 30% below the classical limit. In an improved experiment, Debuisschert et al. [25] observed a 69% noise reduction in the intensity difference. Yamamoto et al. [26] developed semiconductor lasers with intensity fluctuations reduced by 95% below the noise level of usual lasers.

The interest in the investigation and generation of squeezed light is due not only to the reduction of the natural noise of light but also to the possibilities of practical applications. For example, squeezed light may be useful in detecting gravitational waves, which would require a very sensitive detector operating at a very low noise level. Squeezed light could also be useful in optical communications, where it might be important to cut down noise, and in making sensitive spectroscopic measurements, for example, in biological samples.

As mentioned above, photon antibunching and squeezing are two nonclassical effects predicted in resonance fluorescence of two-level atoms. In this paper we review the past work and the present status of photon antibunching and squeezed light produced in resonance fluorescence. In Section II, we derive the master equation for two-level atoms interacting with a quantized electromagnetic field. In Section III, we give the definitions of photon antibunching and squeezing. Section IV deals with photon antibunching and squeezing in spontaneous emission and resonance fluorescence of a single two-level atom. In Section V, we discuss the effect of the interatomic interactions on the two nonclassical effects. In Section VI, we present new results on squeezing in two-atom spontaneous emission. These results show that the interatomic interactions can create squeezed light in spontaneous emission if the atoms were initially prepared in a linear superposition of their ground and excited states. Finally, in Section VII, we summarize our results.

II. MASTER EQUATION

We consider a collection of N identical nonoverlapping atoms, separated by distances r_{ij} ($i \neq j$) and interacting with a quantized multimode electromagnetic field. Each atom is modeled as a two-level system with

the ground state $|g_i\rangle$ ($i = 1, 2, \dots, N$) and the excited state $|e_i\rangle$. In the electric dipole approximation the Hamiltonian of this system has the following form:

$$H = H_0 + H_{\text{int}} \quad (1)$$

with

$$H_0 = \hbar\omega_0 \sum_{i=1}^N S_i^z + \hbar \sum_{\mathbf{k}, s} \omega_{\mathbf{k}} a_{\mathbf{k}s}^\dagger a_{\mathbf{k}s} \quad (2)$$

and

$$H_{\text{int}} = i\hbar \sum_{\mathbf{k}, s} \sum_{i=1}^N [\boldsymbol{\mu}_i \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i) a_{\mathbf{k}s} (S_i^+ + S_i^-) - \text{h.c.}] \quad (3)$$

where ω_0 is the atomic transition frequency, s is the polarization index ($s = 1, 2$), $S_i^+ = |e_i\rangle\langle g_i|$ and $S_i^- = |g_i\rangle\langle e_i|$ are operators raising and lowering the energy of i th atom, and S_i^z describes its energy. These operators fulfill the well-known commutation relations

$$[S_i^+, S_j^-] = 2S_i^z \delta_{ij} \quad [S_i^z, S_j^\pm] = \pm S_i^\pm \delta_{ij} \quad (4)$$

In Eq. (3), $\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)$ is the coupling constant between the quantized electromagnetic field and the electric dipole moments $\boldsymbol{\mu}_i = \langle e_i | \boldsymbol{\mu} | g_i \rangle$, and is given by

$$\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i) = \left(\frac{2\pi\omega_{\mathbf{k}}}{\hbar V} \right)^{1/2} \hat{\mathbf{e}}_{\mathbf{k}s} e^{i\mathbf{k} \cdot \mathbf{r}_i} \quad (5)$$

where $\hat{\mathbf{e}}_{\mathbf{k}s}$ is the unit polarization vector, \mathbf{r}_i is a coordinate of the i th atom.

A master equation for the reduced density operator ρ of the N -atom system interacting with the quantized electromagnetic field is derived from the Hamiltonian (1). It can be derived using any of a number of traditional techniques [27].

We apply a Born-Markov method [28] adapted to the situation of a stationary reservoir. The time evolution of the density operator $W(t)$ of the atoms-field system in the interacting picture obeys the equation

$$i\hbar \frac{\partial}{\partial t} W^I(t) = [H_{\text{int}}^I(t), W^I(t)] \quad (6)$$

where H_{int} is given by Eq. (3), and the superscript I stands for operators in the interacting picture.

Formally integrating Eq. (6) gives

$$W^I(t) = W^I(0) + \frac{1}{i\hbar} \int_0^t dt' [H_{\text{int}}^I(t'), W^I(t')] \quad (7)$$

Substituting this solution into the right side of Eq. (6), and taking the trace over the reservoir states of each side of Eq. (6), we get

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \rho^I(t) &= \text{Tr}_R [H_{\text{int}}^I(t), W^I(0)] \\ &+ \frac{1}{i\hbar} \int_0^t dt' \text{Tr}_R \{ [H_{\text{int}}^I(t), [H_{\text{int}}^I(t'), W^I(t')]] \} \end{aligned} \quad (8)$$

where $\rho^I(t) = \text{Tr}_R W^I(t)$ is the reduced density operator of the atomic system.

We choose an initial state with no correlations between the atomic system and the quantized electromagnetic field, i.e., $W^I(0) = \rho^I(0)\rho_R(0)$, where $\rho_R(0)$ is the density operator for the field reservoir. We also assume that the interaction Hamiltonian satisfies the condition [29, 30]

$$\text{Tr}_R [H_{\text{int}}^I(t), \rho_R(0)] = 0 \quad (9)$$

This can easily be arranged. The left side of Eq. (9) is a system operator. If the left side of Eq. (9) is nonzero, the system Hamiltonian can be altered to include any part in H_{int} so that when added to the left side of Eq. (9) zero occurs. On the basis of these assumptions Eq. (8) reduces to

$$\frac{\partial}{\partial t} \rho^I(t) + \frac{1}{\hbar^2} \int_0^t dt' \text{Tr}_R \{ [H_{\text{int}}^I(t), [H_{\text{int}}^I(t'), W^I(t')]] \} = 0 \quad (10)$$

We now employ the Born approximation in which the atom-field interaction is supposed to be weak, and there is no effect of the atoms on the reservoir. With this approximation we can write

$$W^I(t') = \rho^I(t')\rho_R(0) \quad (11)$$

and after changing time variable to $t' = t - \tau$, Eq. 10 simplifies to

$$\frac{\partial}{\partial t} \rho^I(t) + \frac{1}{\hbar^2} \int_0^t d\tau \operatorname{Tr}_R \{ [H_{\text{int}}^I(t), [H_{\text{int}}^I(t - \tau), \rho_R(0) \rho^I(t - \tau)]] \} = 0 \quad (12)$$

After a Laplace transform over time t , with Eq. (3) and assuming that all modes of the quantized electromagnetic field are in a vacuum state defined by

$$\begin{aligned} \operatorname{Tr}_R [\rho_R(0) a_{\mathbf{k}s}^\dagger a_{\mathbf{k}'s'}] &= 0 & \operatorname{Tr}_R [\rho_R(0) a_{\mathbf{k}s} a_{\mathbf{k}'s'}^\dagger] &= \delta^3(\mathbf{k} - \mathbf{k}') \delta_{ss'} \\ \operatorname{Tr}_R [\rho_R(0) a_{\mathbf{k}s} a_{\mathbf{k}'s'}] &= \operatorname{Tr}_R [\rho_R(0) a_{\mathbf{k}s}^\dagger a_{\mathbf{k}'s'}^\dagger] = 0 \end{aligned} \quad (13)$$

we obtain (ignoring the superscript I)

$$\begin{aligned} \rho(0) - z\rho(z) &= - \sum_{i,j} \gamma_{ij}(z) [\rho(z) S_i^+ S_j^- + S_i^+ S_j^- \rho(z) - 2S_j^- \rho(z) S_i^+] \\ &\quad - i \sum_i \Omega_{ii}(z) [S_i^+ S_i^-, \rho(z)] \\ &\quad - i \sum_{i \neq j} \Omega_{ij}(z) [S_i^+ S_j^-, \rho(z)] \end{aligned} \quad (14)$$

Here $\rho(z)$ is the Laplace transform of $\rho(t)$, and the parameters are given by

$$\begin{aligned} \gamma_{ij}(z) &= \frac{1}{c} \sum_s \int [\boldsymbol{\mu} \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)] [\boldsymbol{\mu}^* \cdot \mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_j)] \frac{z/c}{(z/c)^2 + (k_0 - k)^2} d^3\mathbf{k}, \\ \Omega_{ii}(z) &= \frac{1}{c} \sum_s \int |\boldsymbol{\mu} \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)|^2 \left[\frac{k - k_0}{(z/c)^2 + (k - k_0)^2} \right. \\ &\quad \left. - \frac{k + k_0}{(z/c)^2 + (k + k_0)^2} \right] d^3\mathbf{k}, \\ \Omega_{ij}(z) &= \frac{1}{c} \sum_s \int [\boldsymbol{\mu}^* \cdot \mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_i)] [\boldsymbol{\mu} \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_j)] \\ &\quad \times \left[\frac{k - k_0}{(z/c)^2 + (k - k_0)^2} + \frac{k + k_0}{(z/c)^2 + (k + k_0)^2} \right] d^3\mathbf{k}, \end{aligned} \quad (15)$$

where z is the complex Laplace transform parameter, and $\mu = \mu_1 = \mu_2$. To obtain Eq. (15) we have used the commutation relations (4) and made the rotating-wave approximation [31]; i.e., we neglected rapidly oscillating terms with frequency $2\omega_0$ (the so-called counter rotating terms).

Now we employ the Markov approximation. This neglects retardation effects [32] and is valid in the long-time limit $t \gg \omega_0^{-1}$, providing this is short compared with the typical relaxation times of the system, and is small in comparison with the time required for appreciable changes in population of the atomic levels, i.e.,

$$(r_{ij})_{\max} \ll c \Delta t \quad (16)$$

With these approximations we can replace the $\gamma_{ij}(z)$, $\Omega_{ii}(z)$, and $\Omega_{ij}(z)$ parameters by their limiting values as $z \rightarrow 0^+$. After this, the inverse Laplace transform of Eq. (14) leads to the master equation

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & - \sum_{i,j} \gamma_{ij} (\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2 S_j^- \rho S_i^+) \\ & - i \sum_i \Omega_{ii} [S_i^+ S_i^-, \rho] - i \sum_{i \neq j} \Omega_{ij} [S_i^+ S_j^-, \rho] \end{aligned} \quad (17)$$

where the coefficients in the equation are

$$\begin{aligned} \gamma_{ij} &= \frac{\pi k_0^2}{c} \int d\Omega_k \sum_s [\mu^* \cdot \mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_i)] [\mu \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_j)] \\ \Omega_{ii} &= \frac{2k_0}{c} \int dk \frac{k^2}{(k - k_0)^2} \int d\Omega_k \sum_s |\mu \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)|^2 \\ \Omega_{ij} &= \frac{1}{c} \int dk \frac{2k^3}{(k - k_0)^2} \int d\Omega_k \sum_s [\mu^* \cdot \mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_i)] [\mu \cdot \mathbf{g}_{\mathbf{k}s}(\mathbf{r}_j)] \end{aligned} \quad (18)$$

and $\Omega_k = (\theta_k, \varphi_k)$ is a solid angle over which the quantized electromagnetic field is distributed.

We now examine the values of the coefficients that appear in Eq. (18). On substituting Eq. (5) and on integrating over the total solid angle 4π we

get

$$\gamma_{ij} = \frac{3}{2}\gamma \left\{ \left[1 - (\hat{\mu} \cdot \hat{r}_{ij})^2 \right] \frac{\sin(k_0 r_{ij})}{k_0 r_{ij}} + \left[1 - 3(\hat{\mu} \cdot \hat{r}_{ij})^2 \right] \left[\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\} \quad (19)$$

$$\Omega_{ij} = \frac{3}{2}\gamma \left\{ - \left[1 - (\hat{\mu} \cdot \hat{r}_{ij})^2 \right] \frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} + \left[1 - 3(\hat{\mu} \cdot \hat{r}_{ij})^2 \right] \left[\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\} \quad (20)$$

where $2\gamma = 4k_0^3\mu^2/3\hbar$ is the Einstein A coefficient for spontaneous emission, $\hat{\mu}$ and \hat{r}_{ij} are unit vectors along the transition electric dipole moment and the vector \mathbf{r}_{ij} , respectively. Moreover, $r_{ij} = |\mathbf{r}_{ij}|$ and $k_0 = \omega_0/c = 2\pi/\lambda$, where λ is the resonant wavelength.

The evaluation of Ω_{ii} is an involved problem. The term Ω_{ii} represents the part of the Lamb shift induced by the first-order coupling in the interaction Hamiltonian (3). After performing integrations, Ω_{ii} takes the following form:

$$\Omega_{ii} = -\frac{2\gamma}{\pi} \ln \left\{ \left| \frac{\omega_c}{\omega_0} - 1 \right| \left(\frac{\omega_c}{\omega_0} + 1 \right) \right\} \quad (21)$$

where ω_c is the cutoff frequency. It is well known that to obtain a complete calculation of the Lamb shift, it is necessary to include a second-order, multilevel Hamiltonian including electron mass renormalization [33]. If these are included, the standard nonrelativistic vacuum-Lamb-shift result is obtained.

With the parameters (19)–(21) and on transforming Eq. (17) to the Schrödinger picture, the master equation reduces to

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i\omega_0 \sum_i [S_i^z, \rho] - i \sum_{i \neq j} \Omega_{ij} [S_i^+ S_j^-, \rho] \\ & - \sum_{ij} \gamma_{ij} (\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+) \end{aligned} \quad (22)$$

where ω_0 is the renormalized frequency which is equal to the sum of the atomic frequency ω_0 and Ω_{ii} . The above master equation has been derived assuming that the atoms are coupled to the vacuum modes of the quantized electromagnetic field with no interaction and coupling to external fields. The derivation of the master equation is easily extended to take such interactions into account. With the external coherent laser field the master equation (22) takes the form [27, 34, 35]

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i\omega_0 \sum_i [S_i^z, \rho] - i \sum_{i \neq j} \Omega_{ij} [S_i^+ S_j^-, \rho] \\ & - \sum_{ij} \gamma_{ij} (\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+) \\ & - \frac{i}{2} \sum_i \{ [\rho, S_i^+] \Omega - [S_i^-, \rho] \Omega^* \} \end{aligned} \quad (23)$$

where $\Omega = \mu \cdot \mathcal{E}_0 / \hbar$ is the Rabi frequency describing a strength of interaction between the atoms and an external coherent field \mathcal{E}_0 . The coefficient γ_{ij} is given by Eq. (19), and Ω_{ij} is given by Eq. (20). For $i \neq j$, they depend on the interatomic separation r_{ij} and describe collective properties of the multiatom system. For large interatomic separations $k_0 r_{ij}$ goes to infinity, and then γ_{ij} and Ω_{ij} go to zero; i.e., there is no coupling between the atoms. For small interatomic separation, $k_0 r_{ij} \ll 1$, and then γ_{ij} reduces to γ , and Ω_{ij} reduces to the static dipole-dipole potential [36] which, for $k_0 r_{ij} \rightarrow 0$, tends to infinity.

Equation (23) is the final form of the master equation and will play a basic role in our calculations of photon antibunching and squeezing in interaction of the atomic systems with the quantized electromagnetic field.

III. NONCLASSICAL STATES OF LIGHT

To determine the nonclassical states of light we define the normalized second-order correlation function and variances of the electromagnetic field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^{(+)}(\mathbf{r}, t) + \mathbf{E}^{(-)}(\mathbf{r}, t)$. The normalized second-order two-times correlation function is determined by the relation [37, 38]

$$\begin{aligned} g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) \\ = \frac{G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)}{G^{(1)}(\mathbf{R}_1, t) G^{(1)}(\mathbf{R}_2, t + \tau)} \end{aligned} \quad (24)$$

where

$$G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) = \langle E^{(-)}(\mathbf{R}_1, t) E^{(-)}(\mathbf{R}_2, t + \tau) E^{(+)}(\mathbf{R}_2, t + \tau) E^{(+)}(\mathbf{R}_1, t) \rangle \quad (25)$$

$$G^{(1)}(\mathbf{R}, t) = \langle E^{(-)}(\mathbf{R}, t) E^{(+)}(\mathbf{R}, t) \rangle \quad (26)$$

The correlation function $G^{(1)}(\mathbf{R}, t)$ is proportional to a probability of finding one photon around the direction \mathbf{R} at time t , whereas $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$ is proportional to a joint probability of finding one photon around the direction \mathbf{R}_1 at time t and another photon around the direction \mathbf{R}_2 at the moment of time $t + \tau$. For a coherent light the probability of finding a photon around the direction \mathbf{R}_1 at time t is independent of the probability of finding another photon around the direction \mathbf{R}_2 at time $t + \tau$ and the correlation function $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$ simply factorizes on $G^{(1)}(\mathbf{R}_1, t)G^{(1)}(\mathbf{R}_2, t + \tau)$, giving $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) = 1$ for all τ . For a chaotic field the correlation function $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$ for $\tau = 0$ is greater than for $\tau > 0$ giving $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t) > g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$. This is a manifestation of the tendency of photons to be emitted by a chaotic light source in correlated pairs, and is called photon bunching. Photon antibunching, as the name implies, is the opposite of bunching, and describes a situation in which fewer photons appear close together than further apart. The condition for photon antibunching is $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t) < g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$ and implies that the probability of detecting two photons at the same time t is smaller than the probability of detecting two photons at different times t and $t + \tau$. Moreover, the fact that there is a small probability of detecting photon pairs with zero time separation indicates that the one time correlation function $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t)$ is smaller than one. This effect is called photon anticorrelation. The normalized correlation function (24) for $\tau = 0$ may be written as

$$g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t) = 1 + \frac{\int P(\epsilon) \{ |\epsilon|^2 - \langle |\epsilon|^2 \rangle \}^2 d^2\epsilon}{\langle |\epsilon|^2 \rangle^2} \quad (27)$$

where $P(\epsilon)$ is the Glauber P representation for the electromagnetic field with the complex amplitude ϵ . Hence, we see that photon antibunching has no classical analog in the sense that its diagonal coherent-state representation cannot be nonnegative.

Another nonclassical effect, which is very promising for further application in science and technology, is the squeezed state of light. To define a squeezed state of light let us introduce the quadrature components $E_\theta, E_{\theta-\pi/2}$ at frequency ω , wave vector \mathbf{k} , and given in terms of the positive, negative frequency components $E^{(+)}(\mathbf{R}, t), E^{(-)}(\mathbf{R}, t)$ of the electromagnetic field as

$$E_\theta = E^{(+)}(\mathbf{R}, t) e^{i(\omega t - \mathbf{k} \cdot \mathbf{R} + \theta)} + E^{(-)}(\mathbf{R}, t) e^{-i(\omega t - \mathbf{k} \cdot \mathbf{R} + \theta)} \quad (28)$$

$$E_{\theta-\pi/2} = -i \left[E^{(+)}(\mathbf{R}, t) e^{i(\omega t - \mathbf{k} \cdot \mathbf{R} + \theta)} - E^{(-)}(\mathbf{R}, t) e^{-i(\omega t - \mathbf{k} \cdot \mathbf{R} + \theta)} \right] \quad (29)$$

and satisfying the commutation relation

$$[E_\theta, E_{\theta-\pi/2}] = 2iC \quad (30)$$

where C is a positive c-number.

The fluctuations $\Delta E_\theta, \Delta E_{\theta-\pi/2}$ of the quadrature operators then satisfy an uncertainty relation

$$(\Delta E_\theta)^2 (\Delta E_{\theta-\pi/2})^2 \geq |C|^2 \quad (31)$$

Thus, a large fluctuation in one quadrature component is accompanied by a small fluctuation in the other. The situation for equality in (31) is called a minimum uncertainty state.

Introducing the fluctuation operator

$$\Delta E_\alpha = E_\alpha - \langle E_\alpha \rangle \quad \alpha = \theta, \theta - \pi/2 \quad (32)$$

and using Eqs. (28) and (29), we can write

$$(\Delta E_\alpha)^2 = \langle (\Delta E_\alpha)^2 \rangle = \langle :(\Delta E_\alpha)^2: \rangle + C \quad (33)$$

where the form $:E_\alpha:$ is referred to as the normal ordering of E_α in which all annihilation operators are placed to the right of all certain operators.

For a coherent state of field $\langle :(\Delta E_\alpha)^2: \rangle = 0$ and from Eq. (31) we have

$$\langle (\Delta E_\alpha)^2 \rangle = C \quad \alpha = \theta, \theta - \pi/2 \quad (34)$$

Hence, the coherent state is a minimum uncertainty state with equal fluctuations for both quadrature components. For a chaotic field, both quadrature components $\langle (\Delta E_\theta)^2 \rangle$ and $\langle (\Delta E_{\theta-\pi/2})^2 \rangle$ are greater than C and we call this a chaotic state. It is possible to generate states for which

$\langle (\Delta E_\alpha)^2 \rangle$ is less than C for one of the quadrature components. These states are called squeezed states. According to (33), a squeezed state of the field is characterized by the condition that either $\langle :(\Delta E_\theta)^2: \rangle$ or $\langle :(\Delta E_{\theta-\pi/2})^2: \rangle$ is negative. This condition can be written as

$$\langle :(\Delta E_\theta)^2: \rangle = \int (\Delta \operatorname{Re} \varepsilon_\theta)^2 P(\varepsilon_\theta) d^2 \varepsilon_\theta \quad (35)$$

Hence, we see that squeezed states, similar to photon antibunching, have no classical analog in the sense that their diagonal coherent-state representation cannot be nonnegative.

In the next sections we consider the possibility of obtaining both photon antibunching and squeezed states in resonance fluorescence of two-level atoms.

IV. SINGLE-ATOM RESONANCE FLUORESCENCE

The interaction of the electromagnetic field with the atoms leads to the phenomenon of resonance fluorescence. This phenomenon has attracted the attention of many researchers in recent years in that photon antibunching and squeezing were first discovered in resonance fluorescence.

To analyze photon antibunching and squeezed states in resonance fluorescence we use the master equation (23) and the following relation between the radiation field and atomic operators in the far-field limit [27, 34]:

$$\mathbf{E}^{(+)}(\mathbf{R}, t) = \mathbf{E}_0^{(+)}(\mathbf{R}, t) - k^2 \sum_{i=1}^N \frac{\hat{R} \times (\hat{R} \times \boldsymbol{\mu})}{R} S_i^- \left(t - \frac{R}{c} \right) e^{-i\mathbf{k} \cdot \mathbf{r}_i} \quad (36)$$

where \hat{R} is the unit vector in the direction $\mathbf{R} = \hat{R}R$ of the observation point, \mathbf{r}_i is the position vector of the i th atom, and $\mathbf{E}_0^{(+)}(\mathbf{R}, t)$ denotes the positive frequency part of the vacuum field. Insertion of (36) into (25), (26), and (33) leads to

$$\begin{aligned} G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) \\ = \psi^2(\mathbf{R}_1) \psi^2(\mathbf{R}_2) \sum_{i,j,k,l} \langle S_i^+(t) S_j^+(t + \tau) S_k^-(t + \tau) S_l^-(t) \rangle \\ \times \exp \left[i\mathbf{k} \left(\mathbf{r}_{il} \cdot \hat{R}_1 + \mathbf{r}_{jk} \cdot \hat{R}_2 \right) \right] \end{aligned} \quad (37)$$

$$G^{(1)}(\mathbf{R}, t) = \psi^2(\mathbf{R}) \sum_{i,j} \langle S_i^+(t) S_j^-(t) \rangle \exp(i\mathbf{k} \mathbf{r}_{ij} \cdot \mathbf{R}) \quad (38)$$

$$\langle :(\Delta E_\alpha)^2: \rangle = \psi^2(\mathbf{R}) \left[\langle (\Delta R_\alpha)^2 \rangle + \frac{1}{2} \langle R_3 \rangle \right] \quad (39)$$

with $\psi^2(\mathbf{R}) = (2k^4\mu^2/R^2)\sin^2\psi_0$, where ψ_0 is the angle between the observation direction \mathbf{R} and the atomic transition dipole moment $\boldsymbol{\mu}$, and $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ is the distance between atoms i and j .

In Eq. (39), R_α ($\alpha = \theta, \theta - \pi/2$) and R_3 are Dicke's spin variables [39], which can be expressed in terms of the atomic operators S_i^+ and S_i^- as

$$\begin{aligned} R_\theta &= \frac{1}{2}(S_\theta^+ + S_\theta^-) \\ R_{\theta-\pi/2} &= \frac{1}{2i}(S_\theta^+ - S_\theta^-) \end{aligned} \quad (40)$$

and

$$R_3 = \frac{1}{2}[S_\theta^+, S_\theta^-]$$

where

$$S_\theta^\pm = \sum_i S_i^\pm \exp[\pm i(k\hat{R} \cdot \mathbf{r}_i - \theta)] \quad (41)$$

Having available the fluorescent field correlation functions expressed by the atomic correlation functions according to Eqs. (37)–(39), we can directly apply our master equation (23) to calculate photon antibunching and squeezed states in resonance fluorescence.

First, we consider the simplest process of spontaneous emission from a single two-level atom. In this case Eqs. (37)–(39) simplify to

$$G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) \quad (42a)$$

$$= \psi^2(\mathbf{R}_1)\psi^2(\mathbf{R}_2)\langle S_1^+(t)S_1^+(t + \tau)S_1^-(t + \tau)S_1^-(t) \rangle \quad (42b)$$

$$G^{(1)}(\mathbf{R}, t) = \psi^2(\mathbf{R})\langle S_1^+(t)S_1^-(t) \rangle \quad (42b)$$

$$\langle :(\Delta E_\alpha)^2: \rangle = \psi^2(\mathbf{R})\left[\frac{1}{2}\langle S_\theta^+(t)S_\theta^-(t) \rangle - \frac{1}{4}\langle S_\theta^+(t) + S_\theta^-(t) \rangle^2\right] \quad (42c)$$

where

$$S_\theta^\pm = S_1^\pm \exp[\pm i(k\hat{R} \cdot \mathbf{r}_1 - \theta)] \quad (42d)$$

From the master equation (27) it is easy to find that

$$\begin{aligned} \langle S_i^+(t)S_i^+(t + \tau)S_i^-(t + \tau)S_i^-(t) \rangle &= \langle S_i^+(t)S_i^+(t)S_i^-(t)S_i^-(t) \rangle e^{-2\gamma\tau} \\ \langle S_i^+(t)S_i^-(t) \rangle &= \langle S_i^+(0)S_i^-(0) \rangle e^{-2\gamma t} \\ \langle S_\theta^\pm(t) \rangle &= \langle S_1^\pm(0) \rangle e^{\mp i\theta} e^{-\gamma t} \end{aligned} \quad (43)$$

Since $[S_1^\pm(t)]^2 \equiv 0$, we have that $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) = 0$ for all times t and τ , and there is no photon antibunching in spontaneous emission from a single atom. This result has a simple physical interpretation. In a single-atom spontaneous emission we have only one photon, and a joint probability of detecting of two photons is always zero for all t and τ .

To calculate squeezed states in a single-atom spontaneous emission we have to know one-time correlation functions $\langle S_\theta^+(t) S_\theta^-(t) \rangle$ and $\langle S_\theta^\pm(t) \rangle$. From Eqs. (42) and (43) we have that

$$\begin{aligned} F_\theta(t) &= \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R}) \\ &= \frac{1}{2} \left[\langle S_1^+(0) S_1^-(0) \rangle - \frac{1}{2} \langle S_1^+(0) e^{i\theta} + S_1^-(0) e^{-i\theta} \rangle^2 \right] e^{-2\gamma t} \end{aligned} \quad (44)$$

The fluctuations in the quadrature component E_θ depend on the initial values of the expectation values $\langle S_1^+(0) S_1^-(0) \rangle$ and $\langle S_1^\pm(0) \rangle$. It is seen from Eq. (44) that the fluctuations $F(t)$ can be negative (squeezing) only if the single atom has the nonvanishing dipole moment $\langle S_1^\pm(t) \rangle$. If the atom is initially in the ground $|g_1\rangle$ or excited $|e_1\rangle$ state then $\langle S_1^\pm(0) \rangle = 0$ and there is no squeezing in the spontaneous emission from a single atom. To obtain negative values of $F_\theta(t)$ we have to prepare the atom in a linear superposition of its ground and excited states. Consider the initial ($t = 0$) superposition state [40]:

$$|\psi_A\rangle = \cos \frac{1}{2} \theta_0 |e_1\rangle + \sin \frac{1}{2} \theta_0 e^{i\varphi_0} |g_1\rangle \quad (45)$$

where $0 \leq \theta_0 \leq \pi$ and $0 \leq \varphi_0 \leq 2\pi$. In this state

$$\begin{aligned} \langle S_1^+(0) S_1^-(0) \rangle &= \cos^2 \frac{1}{2} \theta_0 \\ \langle S_1^\pm(0) \rangle &= \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta_0 e^{\pm i\varphi_0} \end{aligned} \quad (46)$$

and

$$F_\theta(t) = \frac{1}{2} \left[\cos^2 \left(\frac{1}{2} \theta_0 \right) - \frac{1}{2} \sin^2(\theta_0) \cos^2(\varphi_0 - \theta) \right] e^{-2\gamma t} \quad (47)$$

Figure 1 shows the time evolution of $F_\theta(t)$, given by Eq. (47), for $(\varphi_0 - \theta) = 0$ and different θ_0 . It is evident from Fig. 1 that for some values of θ_0 the atom radiates field that is squeezed. However, squeezing decreases during spontaneous emission as the atom decays toward its ground state and disappears in the steady state. The minimum value of $F_\theta(t = 0)$ corre-

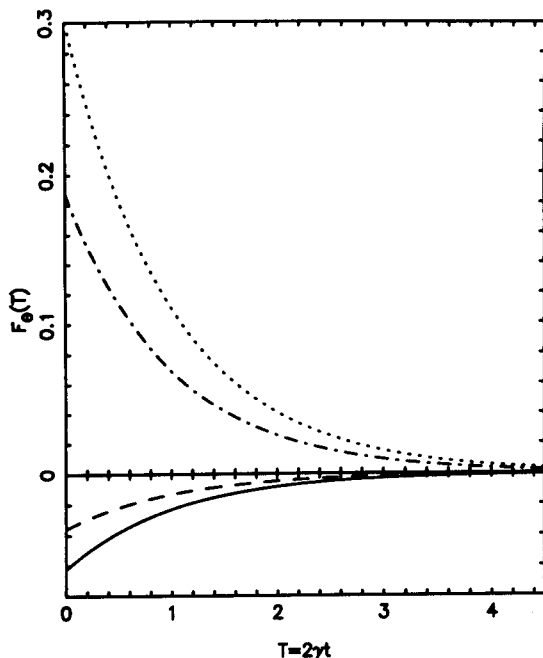


Figure 1. Time dependence of $F_\theta(T) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ for $\varphi_0 - \theta = 0$ and different values of θ_0 : $\theta_0 = 120^\circ$ (solid line), $\theta_0 = 100^\circ$ (dashed line), $\theta_0 = 60^\circ$ (dash-dotted line), $\theta_0 = 45^\circ$ (dotted line).

sponding to optimum squeezing occurs for $\theta_0 = 2\pi/3$ and $(\varphi_0 - \theta) = 0$ when $F(0) = -\frac{1}{16}$. The results presented in Fig. 1 have a simple physical interpretation: Squeezing is sensitive to the phase θ . Spontaneous emission does not introduce any phase information. Therefore, the essential condition of squeezing in the spontaneous emission is to prepare the initial state of the system which includes phase information.

We have shown here that the spontaneous emission of a single atom does not show photon antibunching, but shows squeezing if the atom is initially in a suitable superposition of its states. The situation is different when the atom interacts with an external coherent laser field. In resonance fluorescence the atom is re-excited by the laser field after emitting a fluorescent photon. This excitation allows the atom to re-establish a dipole moment and to radiate more fluorescent photons. In this case photon antibunching can appear and the squeezing can persist in the steady state. To show this more quantitatively we start from the master equation (23), which for the atomic correlation functions with $\Omega \neq 0$ leads to the

following equation of motion [31]:

$$\begin{aligned}\frac{d}{dt}\langle S_1^-(t) \rangle &= \frac{1}{2}i\Omega - \gamma\langle S_1^-(t) \rangle - i\Omega\langle S_1^+(t)S_1^-(t) \rangle \\ \frac{d}{dt}\langle S_1^+(t) \rangle &= -\frac{1}{2}i\Omega - \gamma\langle S_1^+(t) \rangle + i\Omega\langle S_1^+(t)S_1^-(t) \rangle \quad (48) \\ \frac{d}{dt}\langle S_1^+(t)S_1^-(t) \rangle &= -2\gamma\langle S_1^+(t)S_1^-(t) \rangle - i\Omega(\langle S_1^+(t) \rangle - \langle S_1^-(t) \rangle)\end{aligned}$$

where we have assumed that the Rabi frequency is real and the laser frequency ω_L is exactly equal to the atomic transition frequency ω_0 ; i.e., detuning is zero.

The system of Eq. (48) can be easily solved by Laplace transform techniques. The time evolution of the atomic correlation functions for the atom initially in the ground state $|g_1\rangle$ is given by [3]

$$\begin{aligned}\langle S_1^\pm(t) \rangle &= \frac{\mp 2i\beta}{1 + 8\beta^2} \\ &+ \frac{2\beta}{u(1 + 8\beta^2)} \left[\left(\frac{1}{4} - 4\beta^2 \right) \cos(ut) - u \sin(ut) \right] \\ &\times \exp\left(-\frac{3}{4}t\right) \quad (49) \\ \langle S_1^+(t)S_1^-(t) \rangle &= \frac{4\beta^2}{1 + 8\beta^2} \left\{ 1 - \left[\cos(ut) + \frac{3}{4u} \sin(ut) \right] \exp\left(-\frac{3}{4}t\right) \right\} \quad (50)\end{aligned}$$

where, for simplicity, we have introduced the notation

$$t = 2\gamma t \quad \beta = \frac{\Omega}{4\gamma} \quad u = \left(4\beta^2 - \frac{1}{16} \right)^{1/2} \quad (51)$$

To study the normalized second-order correlation function $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$, we have to find the correlation function

$$\langle S_1^+(t)S_1^+(t + \tau)S_1^-(t + \tau)S_1^-(t) \rangle.$$

From the quantum regression theorem [41], it is well known that for $\tau > 0$

the two-time average

$$\langle S_1^+(t)S_1^+(t+\tau)S_1^-(t+\tau)S_1^-(t) \rangle$$

satisfies the same equation of motion as the one-time average $\langle S_1^+(t)S_1^-(t) \rangle$. By Eqs. (24), (42), and (50), we find that in the steady state ($t \rightarrow \infty$) the normalized second-order correlation function $g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau)$ takes the form [2, 3]

$$\begin{aligned} g^{(2)}(\tau) &= \lim_{t \rightarrow \infty} g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t + \tau) \\ &= 1 - \left[\cos(u\tau) + \frac{3}{4u} \sin(u\tau) \right] \exp\left(-\frac{3}{4}\tau\right) \end{aligned} \quad (52)$$

For $\tau = 0$, the correlation function $g^{(2)}(0) = 0$, showing a complete photon anticorrelation between emitted photons. As τ increases ($\tau > 0$), the correlation function $g^{(2)}(\tau)$ increases. This effect reflects the existence of photon antibunching in a single-atom resonance fluorescence.

Using the time-dependent solutions (49) and (50) we can discuss squeezing in resonance fluorescence. In the steady-state ($t \rightarrow \infty$) the fluctuations in the quadrature component E_θ are [18, 42]

$$F_\theta(\infty) = \frac{2\beta^2}{(1 + 8\beta^2)^2} (8\beta^2 + \cos 2\theta) \quad (53)$$

Resonance fluorescence will exhibit squeezed fluctuations if $8\beta^2 + \cos 2\theta < 0$, i.e., for a weak driving field. Maximum squeezing in steady-state resonance fluorescence occurs for $\theta = \pi/2$ and $\beta^2 = \frac{1}{24}$, when $F_\theta(\infty) = -\frac{1}{32}$. This value, compared with the $-\frac{1}{16}$ derived above, is one-half of that obtained for the spontaneous emission. The mechanism responsible for the generation of squeezing in resonance fluorescence differs from that in spontaneous emission. When the initial phase information is introduced to the single atom, then the spontaneous emission produces squeezing. Resonance fluorescence produces squeezing through phase information introduced by a coherent field. Resonance fluorescence, however, can produce optimum squeezing obtained in the spontaneous emission providing that the resonance is time dependent [42]. This is shown in Figs. 2 and 3, where we plot $F_\theta(T) = \langle (\Delta E_\theta)^2 \rangle / \psi^2(\mathbf{R})$, as given by Eq. 42c, versus the time $T = 2\gamma t$, for $\theta = \varphi_0 = 0$ and various values of the parameters β and θ_0 . It is seen from Fig. 2 that as the intensity of the laser field increases, the optimum squeezing in $F_\theta(T)$ shifts to the region of shorter

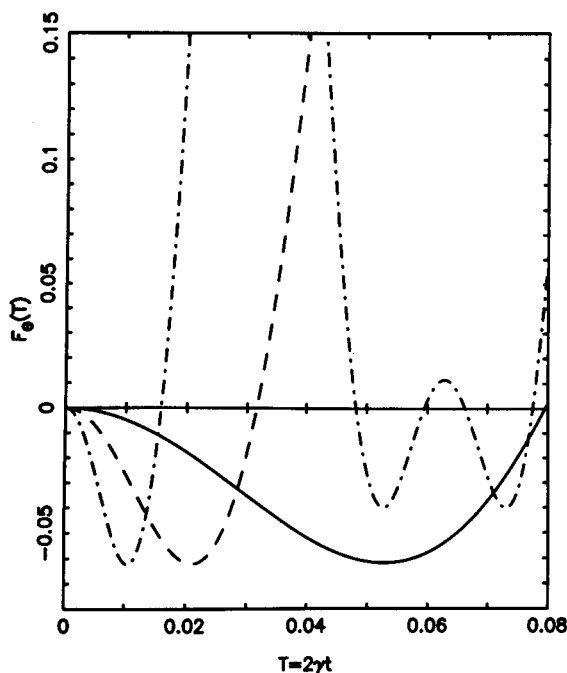


Figure 2. Time dependence of $F_\theta(T) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ for $\theta = \theta_0 = \varphi_0 = 0$ and for different field strengths $\beta = 10$ (solid line), $\beta = 25$ (dashed line), $\beta = 50$ (dash-dotted line).

times, and $F_\theta(T)$ itself shows an oscillatory behavior reflecting the Rabi oscillations. The optimum squeezing reaches a value of $-\frac{1}{16}$ at a very short time t and for $\beta = 50$. Figure 3 shows that the optimum squeezing can be obtained at different times t , depending on the initial state of the atom. If the atom is initially prepared in an equal superposition of its ground and excited states then the optimum squeezing appears at a time shorter than for the atom prepared initially in its ground state.

The simple model presented here provides the underlying mechanism for obtaining photon antibunching and squeezing in resonance fluorescence. In this model two-level atoms, independent of each other, interact with the electromagnetic field. Steady-state resonance fluorescence shows photon antibunching for an arbitrary intensity of the exciting field, whereas squeezing occurs only for a weak intensity of the exciting field. For a strong exciting field, squeezing occurs in the transient regime of resonance fluorescence.

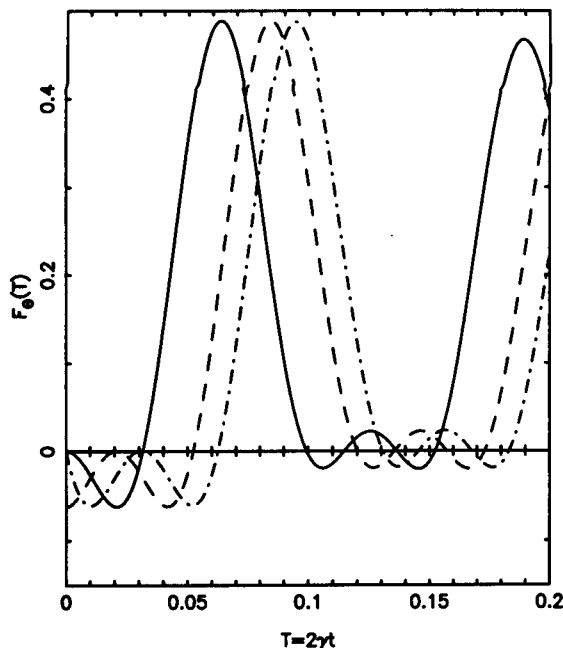


Figure 3. Time dependence of $F_\theta(T) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ for $\beta = 25$, $\varphi_0 = 0$ and different values of θ_0 : $\theta_0 = \pi$ (solid line), $\theta_0 = \pi/2$ (dashed line), $\theta_0 = 2\pi/3$ (dash-dotted line).

V. MULTIATOM RESONANCE FLUORESCENCE

The remainder of this chapter is concerned with the interaction between two-level atoms and the quantized electromagnetic field. Our analysis so far has been concerned with a simple model in which the electromagnetic field interacts with a single two-level atom. However, photon antibunching and squeezing can be considerably modified when more atoms interact with the electromagnetic field. We are particularly interested in the role of the interatomic interactions in modifying the nonclassical effects. We focus our attention on photon antibunching and squeezing in a two-atom resonance fluorescence. Although a two-atom system is admittedly an elementary model, it offers some advantages over the multiatom problem. Because of its simplicity, one obtains detailed and almost exact dynamical solutions with a variety of initial conditions.

The simplest formulation of the problem of multiatom resonance fluorescence is associated with consideration of two atoms in the Dicke model.

In this model it is assumed that the interatomic separations are much smaller than the resonant wavelength, and level shifts associated with the presence of the dipole–dipole interaction between the atoms are ignored. With these assumptions the collective parameters γ_{ij} ($i \neq j$), which appear in the master equation (23), reduce to $\gamma = \gamma_{ii}$, whereas the parameter Ω_{ij} is equal to zero. For the two-atom Dicke model the master equation (23) reduces to [27]

$$\frac{\partial \rho}{\partial t} = \frac{1}{2} i \Omega [S^+ + S^-, \rho] - \gamma (S^+ S^- \rho + \rho S^+ S^- - 2 S^- \rho S^+) \quad (54)$$

where $S^\pm = S_1^\pm + S_2^\pm$ and $S^z = S_1^z + S_2^z$ are the collective atomic dipole operators. For simplicity, the laser frequency ω_L is assumed to be exactly equal to the atomic transition frequency ω_0 .

For a strongly driven system, $\Omega \gg 2\gamma$, an approximation technique has been suggested by Agarwal et al. [43] and Kilin [44], which greatly simplifies the master equation (54). This technique transforms (54) to new collective operators R^\pm and R^z as follows:

$$\begin{aligned} S^\pm &= \pm \frac{1}{2} i (R^+ + R^-) + R^z \\ S^z &= -\frac{1}{2} i (R^+ - R^-) \end{aligned} \quad (55)$$

The operators R are a rotation of the operators S . For a strong field, the R^\pm vary with time approximately as $\exp(\pm i \Omega t)$, while R^z varies slowly. Substituting S^\pm and S^z from Eq. (55) into the master equation (54) and dropping rapidly oscillating terms such as $R^\pm R^z$, $R^+ R^+$, we find the approximate master equation [43–45]:

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= i \Omega [R^z, \rho] - \gamma \{ (R^z R^z \rho + \rho R^z R^z - 2 R^z \rho R^z) \\ &\quad + \frac{1}{4} [(R^+ R^- \rho + \rho R^+ R^- - 2 R^- \rho R^+) \\ &\quad + (R^- R^+ \rho + \rho R^- R^+ - 2 R^+ \rho R^-)] \} \end{aligned} \quad (56)$$

Equation (56) enables us to obtain the equation of motion for the expectation value of an arbitrary operator Q as $\langle \dot{Q} \rangle = \text{tr}(\dot{\rho} Q)$. In particu-

lar, the equations of motion for the transformed dipole operators are

$$\begin{aligned}\frac{d}{dt}\langle R^z \rangle &= -\gamma \langle R^z \rangle \\ \frac{d}{dt}\langle R^\pm \rangle &= -\left(\frac{3}{2}\gamma \pm i\Omega\right)\langle R^\pm \rangle \\ \frac{d}{dt}\langle R^+ R^+ \rangle &= -(5\gamma + 2i\Omega)\langle R^+ R^+ \rangle\end{aligned}\quad (57)$$

Equation (57) is simple in form and can be solved exactly. Performing the inverse transformation from R to the S operators, and using the quantum regression theorem [41], we obtain from Eqs. (55) and (57) the following solutions for the normalized second-order correlation function [43, 46]:

$$\begin{aligned}g^{(2)}(\tau) &= 1 + \frac{1}{32}\exp(-3\gamma\tau) + \frac{3}{32}\exp(-5\gamma\tau)\cos(2\Omega\tau) \\ &\quad - \frac{3}{8}\exp\left(-\frac{3}{2}\gamma\tau\right)\cos(\Omega\tau)\end{aligned}\quad (58)$$

And for the fluctuation in the quadrature components $E_{\theta=0}$ and $E_{\theta=\pi/2}$ of two atoms starting from their ground states [45], we obtain

$$\begin{aligned}\langle :(\Delta E_{\theta=0})^2: \rangle / 2\psi^2(\mathbf{R}) \\ = \frac{2}{3} + \frac{1}{12}\exp(-3\gamma t) - \frac{1}{4}\exp(-5\gamma t)\cos(2\Omega t) \\ - \exp(-3\gamma t)\sin^2(\Omega t) - \frac{1}{2}\exp\left(-\frac{3}{2}\gamma t\right)\cos(\Omega t)\end{aligned}\quad (59)$$

and

$$\langle :(\Delta E_{\theta=\pi/2})^2: \rangle / 2\psi^2(\mathbf{R}) = \frac{2}{3} - \frac{1}{6}\exp(-3\gamma t) - \frac{1}{2}\exp\left(-\frac{3}{2}\gamma t\right)\cos(\Omega t)\quad (60)$$

Figure 4 shows that the photon antibunching ($g^{(2)}(\tau) > g^{(2)}(0)$) is preserved when resonance fluorescence is from two interacting atoms. However, the photon anticorrelation effect ($g^{(2)}(0) < 1$) is reduced compared to that for a single atom. The time dependence of fluctuations in the quadrature component $E_{\theta=0}$ is shown in Fig. 5. As for a single atom, squeezing appears only in the transient regime of resonance fluorescence.

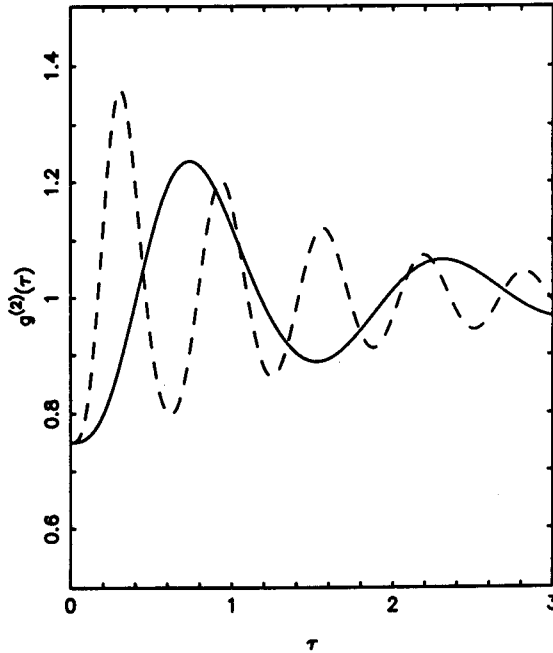


Figure 4. Time dependence of the normalized second-order correlation function $g^{(2)}(\tau)$ for two atoms and for various β : $\beta = 1$ (solid line), $\beta = 3$ (dashed line).

Its maximum value, however, is reduced compared to that for a single atom. These results indicate that cooperative effects reduce photon anti-correlations and squeezing in resonance fluorescence. Moreover, as for single atoms, there is no squeezing in the steady-state resonance fluorescence when the atoms are excited by a strong laser field.

In the Dicke model the dipole-dipole interaction between the atoms is ignored. This approximation has no justification, since for a small interatomic separation the parameter Ω_{ij} , which appears in the master equation (23), is very large and goes to infinity as the interatomic separation r_{12} goes to zero. Therefore, it seems natural to study in some detail what happens when the dipole-dipole interaction terms are included in the two-atom Dicke model. We will also assume an arbitrary separation r_{12} between the atoms and nonzero detuning between the laser frequency ω_L and the atomic transition frequency ω_0 . With the dipole-dipole interaction included, the master equation (23) leads to a closed set of nine equations of motion for the atomic correlation functions. This set of equations can be solved exactly in the steady-state limit, and the solution

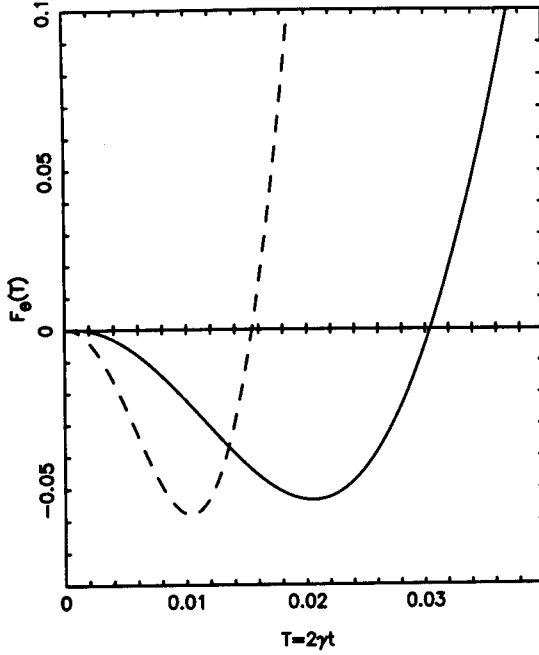


Figure 5. Time dependent of $F_{\theta}(T) = \langle :(\Delta E_{\theta=0})^2: \rangle / 2\psi^2(\mathbf{R})$ for two atoms and different β : $\beta = 25$ (solid line), $\beta = 50$ (dashed line).

is [47, 48]

$$\begin{aligned}
 \langle X_1 \rangle &= -8\beta[8\beta^2 + (1+a)(1+\Delta^2)]/D \\
 \langle X_2 \rangle &= 8\beta^2[8\beta^2 + (1+\Delta^2)]/D \\
 \langle X_3 \rangle &= 8(1+\Delta^2)\beta^2/D \\
 \langle X_4 \rangle &= 8\beta^2[(1+a) - \Delta(\Delta+b)]/D \\
 \langle X_5 \rangle &= -32\beta^3/D \\
 \langle X_6 \rangle &= 16\beta^4/D \\
 \langle X_7 \rangle &= 8\beta[8\Delta\beta^2 + (\Delta+b)(1+\Delta^2)]/D \\
 \langle X_8 \rangle &= 32\Delta\beta^3/D \\
 \langle X_9 \rangle &= -8\beta^2[\Delta(1+a) + (\Delta+b)]/D
 \end{aligned} \tag{61}$$

with

$$D = 64\beta^4 + 16(1 + \Delta^2)\beta^2 + (1 + \Delta^2)[(1 + a)^2 + (\Delta + b)^2] \quad (62)$$

where

$$\begin{aligned} X_1 &= S_1^+ + S_2^+ + S_1^- + S_2^- & X_2 &= S_1^+ S_1^- + S_2^+ S_2^- \\ X_3 &= S_1^+ S_2^- + S_2^+ S_1^- & X_4 &= S_1^+ S_2^+ + S_1^- S_2^- \\ X_5 &= S_1^+ S_1^- S_2^- + S_1^+ S_2^+ S_1^- + S_2^+ S_1^- S_2^- + S_1^+ S_2^+ S_2^- \\ X_6 &= S_1^+ S_2^+ S_1^- S_2^- & X_7 &= -i(S_1^- + S_2^- - S_1^+ - S_2^+) \\ X_8 &= -i(S_1^+ S_1^- S_2^- - S_1^+ S_2^+ S_1^- + S_2^+ S_1^- S_2^- - S_1^+ S_2^+ S_2^-) \\ X_9 &= -i(S_1^- S_2^- - S_1^+ S_2^+) \end{aligned} \quad (63)$$

and

$$\beta = \Omega/4\gamma \quad a = \gamma_{12}/\gamma \quad b = \Omega_{12}/\gamma \quad \Delta = (\omega_0 - \omega)/\gamma \quad (64)$$

The above steady-state solution, which includes the collective damping parameter a , the dipole-dipole interaction parameter b , and the detuning Δ , permits the calculation of photon anticorrelation and squeezing in the fluorescence field emitted by the two-atom system.

Having available the steady-state solution (61), we can calculate the correlation functions (37)–(39) for $\tau = 0$ and $t \rightarrow \infty$. In this limit, we have

$$G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t) = 4\psi^2(\mathbf{R}_1)\psi^2(\mathbf{R}_2)\langle X_6 \rangle \left\{ 1 + \cos[k\mathbf{r}_{12} \cdot (\hat{R}_1 - \hat{R}_2)] \right\} \quad (65)$$

$$G^{(1)}(\mathbf{R}, t) = \psi^2(\mathbf{R}) [\langle X_2 \rangle + \langle X_3 \rangle \cos(k\mathbf{r}_{12} \cdot \hat{R})] \quad (66)$$

$$\begin{aligned} \langle :(\Delta E_\alpha)^2: \rangle &= \frac{1}{2}\psi^2(\mathbf{R}) \left\{ \langle X_4 \rangle \cos(2\alpha) - \langle X_9 \rangle \sin(2\alpha) \right. \\ &\quad \left. + \langle X_2 \rangle + \langle X_3 \rangle \cos(k\hat{R} \cdot \mathbf{r}_{12}) \right. \\ &\quad \left. - \frac{1}{2} [\langle X_1 \rangle \cos \alpha - \langle X_7 \rangle \sin \alpha]^2 \cos^2\left(\frac{1}{2}k\hat{R} \cdot \mathbf{r}_{12}\right) \right\} \end{aligned} \quad (67)$$

Since $\langle X_6 \rangle$ is different from zero, the correlation function $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t)$ is different from zero for $\hat{R}_1 = \hat{R}_2$ and the photon anticorrelation effect is

reduced. However, for $\hat{R}_1 \neq \hat{R}_2$ and

$$\cos \theta' - \cos \theta'' = \lambda/2r_{12} \quad (68)$$

where θ' and θ'' are the angles between \mathbf{r}_{12} and $\hat{R}_1(\hat{R}_2)$, respectively, we have $G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t) = 0$. Thus, we can obtain total anticorrelation between the photons emitted from two atoms, providing that the photons are observed in two different directions. This anticorrelation effect is due to spatial interference [49] causing

$$\left\{ 1 + \cos \left[k \mathbf{r}_{12} \cdot (\hat{R}_1 - \hat{R}_2) \right] \right\} = 0$$

It is interesting to note that for the two-atom resonance fluorescence the phase dependence of the variance (67) of the quadrature component

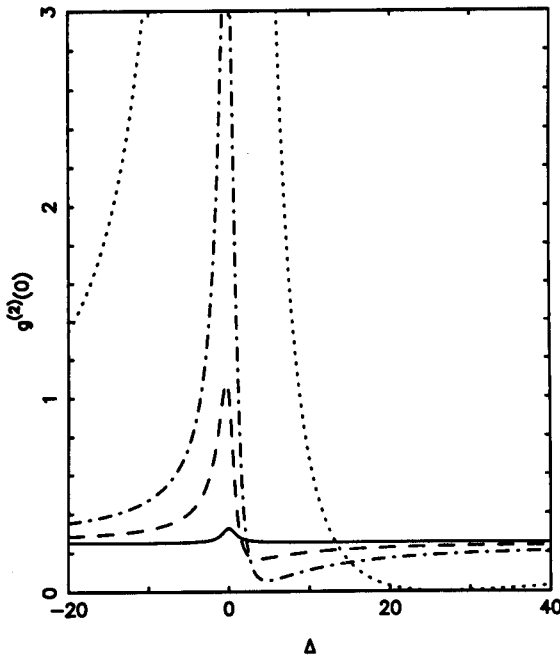


Figure 6. Normalized second-order correlation function $g^{(2)}(0)$ in function of the detuning Δ for $\hat{R}_1 = \hat{R}_2 \perp \mathbf{r}_{12}$, $\beta = 0.2$ and different interatomic separations r_{12} : $r_{12} = 10\lambda$ (solid line), $r_{12} = 0.25\lambda$ (dashed line), $r_{12} = 0.16\lambda$ (dash-dotted line), $r_{12} = 0.08\lambda$ (dotted line).

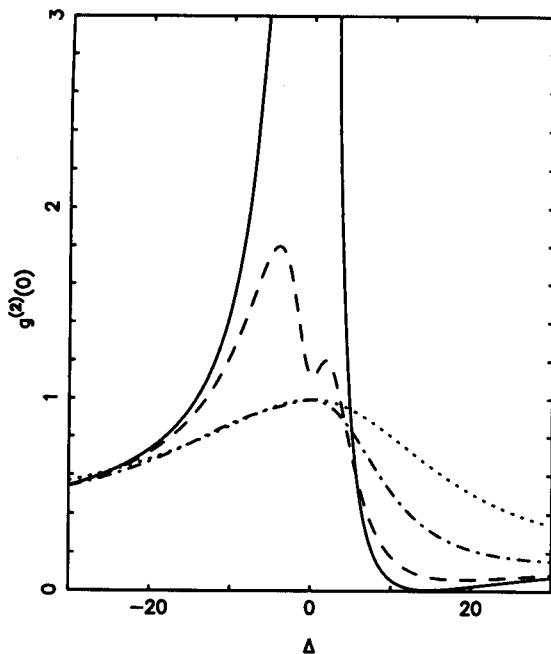


Figure 7. Normalized second-order correlation function $g^{(2)}(0)$ in function of the detuning Δ for $\hat{R}_1 = \hat{R}_2 \perp \mathbf{r}_{12}$, $r_{12} = 0.1\lambda$ for different field strengths β : $\beta = 0.2$ (solid line), $\beta = 2$ (dashed line), $\beta = 5$ (dash-dotted line), $\beta = 10$ (dotted line).

E_α is introduced not only through the dipole moments $\langle X_1 \rangle, \langle X_7 \rangle$ but also through the two-photon correlation functions $\langle X_4 \rangle$ and $\langle X_9 \rangle$.

The normalized second-order correlation function $g^{(2)}(0)$ is illustrated graphically in Figs. 6 and 7 as a function of the detuning Δ for $\mathbf{r}_{12} \perp \hat{R}$ ($\hat{R} = \hat{R}_1 = \hat{R}_2$), and for different values of the interatomic separation r_{12} and of the field strength β . These graphs show that $g^{(2)}(0)$ strongly depends on the detuning Δ , and that the total photon anticorrelation [$g^{(2)}(0) = 0$] can be obtained for certain values of Δ . This happens for $\Delta = -b$, i.e., when the dipole-dipole interaction b and the detuning Δ cancel out mutually. In other words, this means that the laser frequency is tuned to resonance with a particular pair of energy levels of the two-atom system that are shifted by the dipole-dipole interaction. Other levels are far from resonance, and the two-atom system behaves like an individual two-level system.

The variance $F(\theta) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ of the quadrature component E_θ , as given by Eq. (67), is plotted in Figs. 8 and 9 versus the detuning Δ

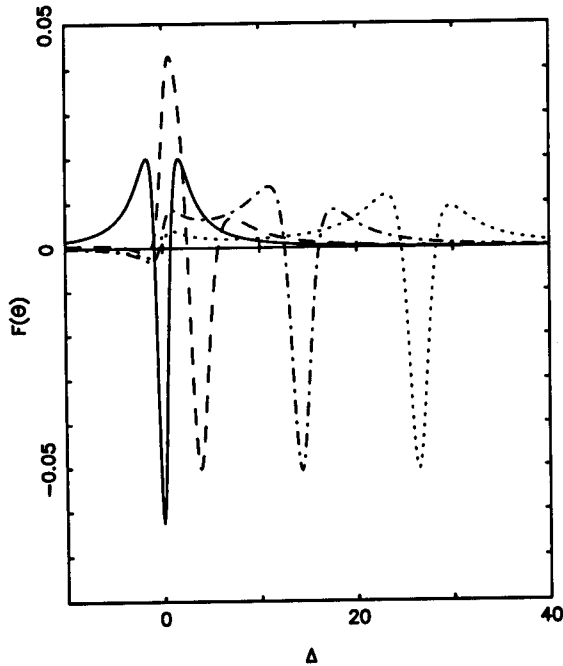


Figure 8. The variance $F(\theta) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ as a function of Δ for $\hat{R} \perp \mathbf{r}_{12}$, $\theta = 0$, $\beta = 0.2$, and for various interatomic separations r_{12} : $r_{12} = 10\lambda$ (solid line), $r_{12} = 0.16\lambda$ (dashed line), $r_{12} = 0.1\lambda$ (dash-dotted line), $r_{12} = 0.08\lambda$ (dotted line).

for $\mathbf{r}_{12} \perp \hat{R}$ and for various interatomic separations r_{12} at fixed θ as well as at different values of the phase θ and fixed r_{12} . It is evident from Fig. 8 that, as the interatomic distance r_{12} becomes sufficiently small and the dipole-dipole interaction between the atoms becomes considerable, the squeezing in $F(\theta)$, which for independent atoms has its maximum for $\Delta = 0$, shifts to region of finite Δ . In fact, as in the normalized second-order correlation function $g^{(2)}(0)$, the minimum in $F(\theta)$ appears for $\Delta = -b$ and can again be attributed to the change in energy-level structure of the two-atom system due to dipole-dipole interaction. Figure 9 shows that unlike in a single-atom resonance fluorescence, large squeezing can appear for a strong driving field and $\theta = \pi/2$. At $\Delta = -b$, values of squeezing in $F(\theta)$ can be obtained that are comparable to the value obtained in the transient regime of $F(\theta)$ for a single atom. Thus, stationary two-atom resonance fluorescence shows squeezing for a strong driving

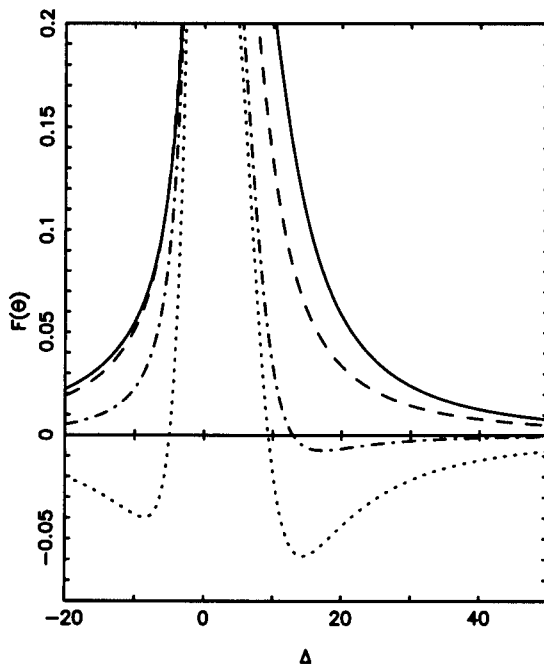


Figure 9. The variance $F(\theta)$ as a function of Δ for $\hat{R} \perp \mathbf{r}_{12}$, $r_{12} = \lambda/6$, $\beta = 2$ and for different phases θ : $\theta = 0$ (solid line), $\theta = \pi/8$ (dashed line), $\theta = \pi/4$ (dash-dotted line), $\theta = \pi/2$ (dotted line).

field contrary to the single-atom resonance fluorescence which for strong driving fields is squeezed only in the transient regime.

VI. SQUEEZING IN TWO-ATOM SPONTANEOUS EMISSION

In Section IV we have shown that only the initially different from zero dipole moment can produce squeezing in the spontaneous emission from a single atom. Here, we examine conditions for squeezing in two-atom spontaneous emission. Equation (67) shows that the variance $\langle:(\Delta E_\alpha)^2: depends on the phase α not only through the nonvanishing dipole moments $\langle S_i^\pm \rangle$, but also through the two-photon coherences $\langle S_1^+ S_2^+ \rangle$ and $\langle S_1^- S_2^- \rangle$. This dependence suggests that there are two different processes that can lead to squeezing in two-atom spontaneous emission. To show this, we start from Eq. (39), which for two atoms and $\hat{R} \perp \mathbf{r}_{12}$ can be$

written in the form

$$\begin{aligned} \langle :(\Delta E_\theta)^2: \rangle = \frac{1}{2} \psi^2(\mathbf{R}) \{ & \langle S_1^+ S_2^+ \rangle e^{-2i\theta} + \langle S_1^- S_2^- \rangle e^{2i\theta} \\ & + \langle S_1^+ S_1^- + S_2^+ S_2^- \rangle + \langle S_1^+ S_2^- + S_2^+ S_1^- \rangle \quad (69) \\ & - \frac{1}{2} [\langle S_1^+ + S_2^+ \rangle e^{-i\theta} + \langle S_1^- + S_2^- \rangle e^{i\theta}]^2 \} \end{aligned}$$

From the master equation (23) it is easy to show that for $\Omega = 0$ the equations of motion for the atomic correlation functions, which appear in Eq. (69), are

$$\begin{aligned} \frac{d}{dt} Y_1 &= -(\gamma + \gamma_{12} + i\Omega_{12}) Y_1 + 2(\gamma_{12} + i\Omega_{12}) Y_3 \\ \frac{d}{dt} Y_2 &= -(\gamma + \gamma_{12} - i\Omega_{12}) Y_2 + 2(\gamma_{12} - i\Omega_{12}) Y_4 \\ \frac{d}{dt} Y_3 &= -(3\gamma + \gamma_{12} - i\Omega_{12}) Y_3 \\ \frac{d}{dt} Y_4 &= -(3\gamma + \gamma_{12} + i\Omega_{12}) Y_4 \\ \frac{d}{dt} Y_5 &= -2\gamma Y_5 - 2\gamma_{12} Y_6 + 8\gamma_{12} Y_7 \\ \frac{d}{dt} Y_6 &= -2\gamma Y_6 - 2\gamma_{12} Y_5 \\ \frac{d}{dt} Y_7 &= -4\gamma Y_7 \\ \frac{d}{dt} Y_8 &= -2\gamma Y_8 \\ \frac{d}{dt} Y_9 &= -2\gamma Y_9 \end{aligned} \quad (70)$$

where

$$\begin{aligned} Y_1 &= \langle S_1^- + S_2^- \rangle & Y_2 &= \langle S_1^+ + S_2^+ \rangle & Y_3 &= \langle S_1^+ S_1^- S_2^- + S_2^+ S_1^- S_2^- \rangle \\ Y_4 &= \langle S_1^+ S_2^+ S_1^- + S_1^+ S_2^+ S_2^- \rangle & Y_5 &= \langle S_1^+ S_2^- + S_2^+ S_1^- \rangle \\ Y_6 &= \langle S_1^+ S_1^- + S_2^+ S_2^- \rangle & Y_7 &= \langle S_1^+ S_2^+ S_1^- S_2^- \rangle & Y_8 &= \langle S_1^+ S_2^+ \rangle \\ Y_9 &= \langle S_1^- S_2^- \rangle \end{aligned} \quad (71)$$

and γ_{12} and Ω_{12} are given by Eqs. (19) and (20), respectively.

Equations (70) are simple in form and can be solved exactly. The solutions have the following form:

$$\begin{aligned}
 Y_1(t) &= Y_1(0)\exp[-(\gamma + \gamma_{12} + i\Omega_{12})t] \\
 &\quad + \frac{Y_3(0)(\gamma_{12} + i\Omega_{12})}{(\gamma - i\Omega_{12})} \{1 - \exp[-2(\gamma - i\Omega_{12})t]\} \\
 &\quad \times \exp[-(\gamma + \gamma_{12} + i\Omega_{12})t] \\
 Y_2(t) &= Y_1^*(t) \\
 Y_3(t) &= Y_4^*(t) = Y_3(0)\exp[-(3\gamma + \gamma_{12} - i\Omega_{12})t] \\
 Y_5(t) &= -\frac{4\gamma\gamma_{12}Y_7(0)}{(\gamma^2 - \gamma_{12}^2)}\exp(-4\gamma t) \\
 &\quad + \left\{ \frac{1}{2}[Y_5(0) - Y_6(0)] + \frac{2\gamma_{12}Y_7(0)}{(\gamma + \gamma_{12})} \right\} \\
 &\quad \times \exp[-(\gamma - \gamma_{12})t] \\
 &\quad + \left\{ \frac{1}{2}[Y_5(0) + Y_6(0)] + \frac{2\gamma_{12}Y_7(0)}{(\gamma - \gamma_{12})} \right\} \\
 &\quad \times \exp[-(\gamma + \gamma_{12})t] \\
 Y_6(t) &= -\frac{4\gamma_{12}^2Y_7(0)}{(\gamma^2 - \gamma_{12}^2)}\exp(-4\gamma t) \\
 &\quad + \left\{ \frac{1}{2}[Y_6(0) - Y_5(0)] - \frac{2\gamma_{12}Y_7(0)}{(\gamma + \gamma_{12})} \right\} \exp[-(\gamma - \gamma_{12})t] \\
 &\quad + \left\{ \frac{1}{2}[Y_6(0) + Y_5(0)] + \frac{2\gamma_{12}Y_7(0)}{(\gamma - \gamma_{12})} \right\} \exp[-(\gamma + \gamma_{12})t] \\
 Y_7(t) &= Y_7(0)\exp(-4\gamma t) \\
 Y_8(t) &= Y_8(0)\exp(-2\gamma t) \\
 Y_9(t) &= Y_8^*(t)
 \end{aligned} \tag{72}$$

where $Y_i(0)$ ($i = 1, \dots, 9$) describe the initial expectation values of the atomic correlation functions. They are dependent on the initial population of the atomic states. Consider the initial ($t = 0$) superposition state [50]

$$|\psi_0\rangle = [\cos\frac{1}{2}\theta_1|e_1\rangle + \sin\frac{1}{2}\theta_1 e^{i\varphi_1}|g_1\rangle][\cos\frac{1}{2}\theta_2|e_2\rangle + \sin\frac{1}{2}\theta_2 e^{i\varphi_2}|g_2\rangle] \tag{73}$$

in which the atoms are in an arbitrary linear combination of their states. If the two-atom system is initially in the states $|\psi_0\rangle$, then

$$\begin{aligned}
 Y_1(0) &= \frac{1}{2}(\sin \theta_1 e^{-i\varphi_1} + \sin \theta_2 e^{-i\varphi_2}) \\
 Y_2(0) &= Y_1^*(0) \\
 Y_3(0) &= \frac{1}{2}(\sin \theta_1 \cos^2(\frac{1}{2}\theta_2) e^{-i\varphi_1} + \sin \theta_2 \cos^2(\frac{1}{2}\theta_1) e^{-i\varphi_2}) \\
 Y_4(0) &= Y_3^*(0) \\
 Y_5(0) &= \frac{1}{2}\sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2) \\
 Y_6(0) &= \cos^2(\frac{1}{2}\theta_1) + \cos^2(\frac{1}{2}\theta_2) \\
 Y_7(0) &= \cos^2(\frac{1}{2}\theta_1)\cos^2(\frac{1}{2}\theta_2) \\
 Y_8(0) &= \frac{1}{4}\sin \theta_1 \sin \theta_2 \exp[i(\varphi_1 + \varphi_2)] \\
 Y_9(0) &= \frac{1}{4}\sin \theta_1 \sin \theta_2 \exp[-i(\varphi_1 + \varphi_2)]
 \end{aligned} \tag{74}$$

If we choose our initial conditions so that $\theta_1 = \theta_2 = \theta_0$, $\varphi_1 = 0$, and $\varphi_2 = \pi$, then $Y_1(0) = Y_2(0) = 0$ and $Y_8(0) = Y_9(0) = -\frac{1}{4}\sin^2 \theta_0$. In such a state the expectation values of the dipole moments are zero. Nevertheless, squeezing is still possible because the two-photon coherences $Y_8(0)$ and $Y_9(0)$ lead to phase sensitivity in the variance (69). At the initial time $t = 0$ the variance (69) is

$$\langle :(\Delta E_\theta)^2: \rangle = \frac{1}{2}\psi^2(\mathbf{R})(1 + \cos \theta_0 - \sin^2 \theta_0 \cos^2 \theta) \tag{75}$$

The minimum value of $\langle :(\Delta E_\theta)^2: \rangle$ corresponding to optimum squeezing occurs for $\theta = 0$ and $\theta_0 = 2\pi/3$ when

$$\langle :(\Delta E_\theta)^2: \rangle = -\frac{1}{8}\psi^2(\mathbf{R}) \tag{76}$$

With the parameters $\theta_0 = 2\pi/3$, $\varphi_1 = 0$ and $\varphi_2 = \pi$ the initial state $|\varphi_0\rangle$ has the form

$$\begin{aligned}
 |\varphi_0\rangle &= \frac{1}{4}[3|g_1\rangle|g_2\rangle - |e_1\rangle|e_2\rangle] \\
 &+ \frac{\sqrt{3}}{4}[|e_1\rangle|g_2\rangle - |e_2\rangle|g_1\rangle] = |\psi_1\rangle + |\psi_2\rangle
 \end{aligned} \tag{77}$$

where $|\psi_1\rangle$ is a linear combination of the atomic states in which both atoms are in their ground or excited states, whereas $|\psi_2\rangle$ is an antisymmetric combination of the excited and ground states of the atoms. In the

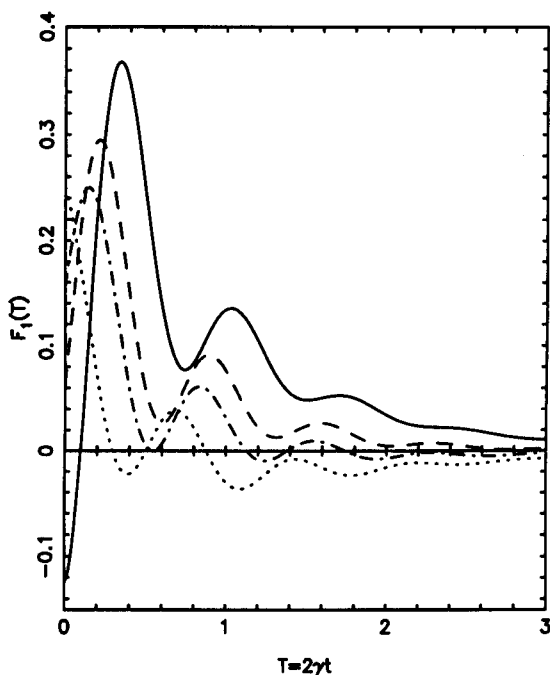


Figure 10. Time dependence of $F(T) = \langle :(\Delta E_\theta)^2: \rangle / \psi^2(\mathbf{R})$ for $r_{12} = 0.12\lambda$, $\theta_1 = \theta_2 = 2\pi/3$, $\varphi_1 = \varphi_2 = 0$, and for different phases θ : $\theta = 0$ (solid line), $\theta = \pi/4$ (dashed line), $\theta = \pi/3$ (dash-dotted line), $\theta = \pi/2$ (dotted line).

notation of the Dicke states [39] the state $|\psi_2\rangle$ corresponds to the antisymmetric state $|0,0\rangle$, whereas the state $|\psi_1\rangle$ corresponds to a linear combination of the Dicke states $|1,1\rangle$ and $|1,-1\rangle$. The state $|\psi_1\rangle$, which is the linear combination of the Dicke states $|1,1\rangle$ and $|1,-1\rangle$ and gives squeezing in the quadrature component $E_{\theta=0}$, is known in the literature as a two-atom squeezed state [51], or pairwise atomic state [52].

In Fig. 10 we plot the variance (69) as a function of time t for $r_{12} = 0.12\lambda$, $\theta_1 = \theta_2 = 2\pi/3$, $\varphi_1 = \varphi_2 = 0$ and for different values of the phase θ . These graphs show that the fluctuations in the quadrature component E_θ strongly depend on phase θ and can be squeezed at different times t . If the quadrature component E_θ were squeezed initially then it is not squeezed at later times, whereas an initially unsqueezed quadrature component can be squeezed at later times. This effect does not appear in a single atom spontaneous emission, and is due to interatomic interactions which create linear superpositions of the atomic states.

The idea of squeezing in spontaneous emission can be extended to higher order quadrature components of the electromagnetic field. Hong and Mandel [53] defined N th-order squeezing and showed that resonance fluorescence predicts this type of squeezing. Another type of higher-order squeezing, called amplitude-squared squeezing, has been defined by Hillery [54]. It has been shown that the amplitude-squared squeezing occurs in multiatom resonance fluorescence, and does not appear in spontaneous emission and resonance fluorescence from a single two-level atom [55–57]. This is another example that shows that the interatomic interactions can produce squeezing in spontaneous emission.

VII. SUMMARY

In this chapter we have considered two nonclassical effects: photon antibunching and squeezing in the spontaneous emission and resonance fluorescence from two-level atoms. Spontaneous emission from a single two-level atom shows squeezing if at the initial time the atom were in a suitable prepared linear combination of its excited and ground states. Resonance fluorescence from a single atom shows photon antibunching and squeezing independent of the initial preparation of the atom. However, squeezing strongly depends on the intensity of the driving laser field, and in the steady-state appears only for a weak exciting field. A strong laser field can produce squeezing only in the transient regime of resonance fluorescence. Interatomic interactions have a destructive effect on these two nonclassical effects. However, a considerable amount of photon antibunching and squeezing can be obtained in a two-atom resonance fluorescence when the detuning of the laser frequency from atomic resonance and the dipole–dipole interaction between the atoms cancel out mutually. Moreover, squeezing can appear in the steady-state resonance fluorescence even for a strong exciting laser field.

We have also discussed the possibility of obtaining squeezed states in spontaneous emission from two interacting atoms. We have shown that the two-atom system can produce squeezing even for the vanishing atomic dipole moments. The system produces squeezing through having two-photon coherences different from zero [50, 51].

In conclusion, we have demonstrated nonclassical effects exhibited by independent as well as correlated atoms. These effects are not evident when the electromagnetic field is treated classically and easily manifest the quantum nature of resonance fluorescence.

As a result of recent successful experiments, which have generated squeezed light, interest is now turning to possible applications. Gardiner [58] first pointed out that squeezed light incident upon a single two-level

atom can in principle inhibit the phase decay of that atom, giving rise to line narrowing in the spectrum of resonance fluorescence [58]. Since that first paper, analyses have been extended to the treatment of atomic absorption spectra [60, 61], atomic level shifts in a squeezed vacuum [33, 62, 63], and squeezed pump lasers [64]. Multiatom and multilevel systems in a squeezed vacuum are now extensively studied [65–74], and show novel effects not observed in an ordinary vacuum.

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