

Phase properties of the field interacting with a three-level atom: II. Two-mode case

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Abstract. In this paper we consider phase properties of the field in the two-mode three-level problem. Phase properties of each individual mode as well as their joint probability distribution and correlation function are studied. We find that phase properties of the field reflect the collapses and revivals of the level occupation probabilities in most situations. However, there exist exceptions, for example, in the case of Raman scattering, which allows us to conclude that the collapses and revivals of the atomic population can be completely decorrelated from the phase of the field. This takes place when the field mode is fed by the photons emitted spontaneously from a real atomic level, which affects the field occupation numbers but not its phase.

1. Introduction

In the previous paper of this series [1] (hereafter referred to as I) we have studied phase properties of a one-mode coherent field interacting with a three-level atom under two-photon resonance. The three levels have been denoted by 1, 2 and 3 with level 1 coupled to level 2 and level 2 coupled to level 3 through corresponding electronic dipole transitions, while the transition $1 \leftrightarrow 3$ is forbidden. We have found that when the electron is initially in level 2, level 2 and the combination of levels 1 and 3 behave like two levels concerning phase properties of the field. Interesting features such as the partial trapping of the field phase have been observed when level 3 is initially occupied. In both cases the collapse and revival phenomenon has been connected to the time behaviour of the phase probability distribution and the phase variance.

In this paper we treat, instead of one mode, two modes. Mode 1 interacts with the atomic dipole moment between levels 1 and 2, and mode 2 with the one between levels 2 and 3 (see I, figure 1). As in I, we assume two-photon resonance and use the formalism worked out by Yoo and Eberly [2] to obtain general expressions for the evolution of the system, and consequently for phase probability distribution, the expectation value and the variance of the phase in terms of matrix elements of the evolution operator. We show that one-mode results hold for the two-mode case with minor modifications if the photon number distributions of the field in the modes are well localized around the mean photon numbers \bar{n}_1 and \bar{n}_2 , respectively. For coherent fields this assumption means $\bar{n}_1, \bar{n}_2 \gg 1$.

More attention is drawn to specific problems of the two-mode system such as different effects of the initial atomic state on the evolution of the field phase in the two

modes, or phase properties of scattered light in Raman scattering. Finally, we use the Hermitian phase formalism of Pegg and Barnett [3–5] to investigate the joint probability distribution and the correlation function of the phase operators for the two modes and discuss phase properties associated with these quantities.

2. Basic equations

In a similar fashion as for one mode, we write the Hamiltonian for two-mode coupling as follows

$$H = H_A + H_F + H' \quad (1)$$

where ($\hbar = 1$)

$$H_A = \sum_{j=1}^3 \Omega_j b_j^\dagger b_j \quad H_F = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 \quad (2)$$

and the interaction Hamiltonian H' given for each atom type in the dipole and rotating wave approximations by

$$\begin{aligned} H'_\Xi &= \xi a_1 b_2^\dagger b_1 + \eta a_2 b_3^\dagger b_2 + \text{HC} \\ H'_\Lambda &= \xi a_1 b_2^\dagger b_1 + \eta a_2^\dagger b_3^\dagger b_2 + \text{HC} \\ H'_\text{V} &= \xi a_1^\dagger b_2^\dagger b_1 + \eta a_2 b_3^\dagger b_2 + \text{HC}. \end{aligned} \quad (3)$$

In the two-mode case, there exist two excitation numbers N_1 and N_2 . For a given pair of (N_1, N_2) the basis vectors in the (N_1, N_2) subspace are

$$\begin{aligned} |j\rangle^{(N_1, N_2)} &= b_j^\dagger |0, 0, 0; N_1 - \mu_j, N_2 - \nu_j\rangle \\ &\equiv |j; N_1 - \mu_j, N_2 - \nu_j\rangle \quad j = 1, 2, 3 \end{aligned} \quad (4)$$

where the values of (μ_1, ν_1) , (μ_2, ν_2) , (μ_3, ν_3) are $(0, 0)$, $(1, 0)$, $(1, 1)$ for Ξ type, $(0, 1)$, $(1, 1)$, $(1, 0)$ for Λ type and $(1, 0)$, $(0, 0)$, $(0, 1)$ for V type. In this subspace the matrix representation of the time evolution operator in the interaction picture is [2]

$$\begin{aligned} U^{(N_1, N_2)}(t) &= e^{i\Delta t/2} \\ &\times \begin{pmatrix} \tilde{\eta}^2 e^{i\Delta t/2} + \tilde{\xi}^2 x(t) & \tilde{\xi} y(t) & \tilde{\xi} \tilde{\eta} [-e^{i\Delta t/2} + x(t)] \\ \tilde{\xi} y(t) & x^*(t) & \tilde{\eta} y(t) \\ \tilde{\xi} \tilde{\eta} [-e^{i\Delta t/2} + x(t)] & \tilde{\eta} y(t) & \tilde{\xi}^2 e^{i\Delta t/2} + \tilde{\eta}^2 x(t) \end{pmatrix} \end{aligned} \quad (5)$$

where Δ is the detuning parameter and we have used the notation

$$\begin{aligned} \xi_{N_1} &= \xi \sqrt{N_1} & \eta_{N_2} &= \eta \sqrt{N_2} \\ g &\equiv g_{N_1, N_2} = (\xi_{N_1}^2 + \eta_{N_2}^2)^{1/2} & f &\equiv f_{N_1, N_2} = (g_{N_1, N_2}^2 + \frac{1}{4} \Delta^2)^{1/2} \\ \tilde{\xi} &\equiv \tilde{\xi}_{N_1, N_2} = \frac{\xi_{N_1}}{g_{N_1, N_2}} & \tilde{\eta} &\equiv \tilde{\eta}_{N_1, N_2} = \frac{\eta_{N_2}}{g_{N_1, N_2}} \end{aligned} \quad (6)$$

$$x(t) \equiv x_{N_1, N_2}(t) = \cos(f_{N_1, N_2} t) + i \frac{\Delta}{2f_{N_1, N_2}} \sin(f_{N_1, N_2} t)$$

$$y(t) \equiv y_{N_1, N_2}(t) = -i \frac{g_{N_1, N_2}}{f_{N_1, N_2}} \sin(f_{N_1, N_2} t).$$

For an electron initially in level i and the field initially in a coherent state, the state vector of the atom-field system at $t=0$ will be

$$|\psi(0)\rangle = \sum_{n_1, n_2} b_{n_1} b_{n_2} \exp[i(n_1\beta_1 + n_2\beta_2)] |i; n_1, n_2\rangle \quad (7)$$

where

$$b_{n_1} = \exp(-\bar{n}_1/2) \left(\frac{\bar{n}_1^{n_1}}{n_1!} \right)^{1/2} \quad b_{n_2} = \exp(-\bar{n}_2/2) \left(\frac{\bar{n}_2^{n_2}}{n_2!} \right)^{1/2} \quad (8)$$

and β_1, β_2 are the phases of the field in the two modes. At a later time t , the state vector equation (7) in the interaction picture becomes

$$|\psi(t)\rangle = \sum_{n_1, n_2} b_{n_1} b_{n_2} \exp[i(n_1\beta_1 + n_2\beta_2)] \sum_{j=1}^3 U_{ij}^{(n_1+\mu_i, n_2+\nu_i)}(t) |j; n_1 + \mu_{ij}, n_2 + \nu_{ij}\rangle \quad (9)$$

where $\mu_{ij} = \mu_i - \mu_j$ and $\nu_{ij} = \nu_i - \nu_j$. For convenience, we introduce the following sums

$$\begin{aligned} U &\equiv U(n_1, k_1, n_2, k_2, i; t) \\ &= \sum_{j=1}^3 U_{ij}^{(N_1, N_2)}(t) U_{ij}^{(K_1, K_2)*}(t) \end{aligned} \quad (10)$$

where

$$\begin{aligned} N_1 &= n_1 + \mu_i & K_1 &= k_1 + \nu_i \\ N_2 &= n_2 + \mu_i & K_2 &= k_2 + \nu_i \end{aligned} \quad (11)$$

and

$$\begin{aligned} U_1 &= \sum_{n_2} b_{n_2}^2 U(n_1, k, n_2 = k_2, i; t) \\ U_2 &= \sum_{n_1} b_{n_1}^2 U(n_1, k, n_2 = k_2, i; t) \end{aligned} \quad (12)$$

which are relevant to phase variables. By means of equations (5), (6) and (10) we have

(i) if $i=2$

$$U = (\xi_{N_1 N_2} \xi_{K_1 K_2} + \bar{\eta}_{N_1 N_2} \bar{\eta}_{K_1 K_2}) y_{N_1 N_2} y_{K_1 K_2}^* + x_{N_1 N_2}^* x_{K_1 K_2} \quad (13)$$

(ii) if $i=1$

$$\begin{aligned} U &= \bar{\eta}_{N_1 N_2}^2 \bar{\eta}_{K_1 K_2}^2 + \xi_{N_1 N_2} \xi_{K_1 K_2} \bar{\eta}_{N_1 N_2}^2 \bar{\eta}_{K_1 K_2}^2 + \xi_{N_1 N_2} \xi_{K_1 K_2} y_{N_1 N_2} y_{K_1 K_2}^* \\ &\quad + (\xi_{N_1 N_2} \xi_{N_1 N_2} \xi_{K_1 K_2} \bar{\eta}_{N_1 N_2}^2 \bar{\eta}_{K_1 K_2}^2) x_{N_1 N_2} x_{K_1 K_2}^* \\ &\quad + (\bar{\eta}_{N_1 N_2}^2 \xi_{K_1 K_2}^2 - \xi_{N_1 N_2} \xi_{K_1 K_2} \bar{\eta}_{N_1 N_2} \bar{\eta}_{K_1 K_2}) \exp(\frac{1}{2}i\Delta t) x_{K_1 K_2}^* \\ &\quad + \xi_{N_1 N_2}^2 \bar{\eta}_{K_1 K_2}^2 - \xi_{N_1 N_2} \xi_{K_1 K_2} \bar{\eta}_{N_1 N_2}^2 \bar{\eta}_{K_1 K_2}^2 \exp(\frac{1}{2}i\Delta t) x_{N_1 N_2}^* \end{aligned} \quad (14)$$

The last two terms in (14) will give insignificant contributions when we average U over the photon number distributions in the strong coherent excitation limit. Therefore, we neglect them from now on whenever this limit is obeyed.

Using the standard procedures [1, 6, 7], the phase probability distribution, the expectation value and the variance of the Hermitian phase operator may be obtained for the field in each mode. For example, for mode 1, one gets

$$P(\theta_1, t) = \frac{1}{2\pi} \left(1 + 2 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \cos[(n_1 - k_1)\theta_1] \operatorname{Re} U_1 + 2 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \sin[(n_1 - k_1)\theta_1] \operatorname{Im} U_1 \right) \quad (15)$$

which is normalized so that

$$\int_{-\pi}^{\pi} P(\theta_1, t) d\theta_1 = 1. \quad (16)$$

and

$$\langle \hat{\Phi}_{\theta_1} \rangle = \beta_1 - 2 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \frac{(-1)^{n_1 - k_1}}{n_1 - k_1} \operatorname{Im} U_1 \quad (17)$$

$$\langle \Delta \Phi_{\theta_1}^2 \rangle = \frac{\pi^2}{3} + 4 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \frac{(-1)^{n_1 - k_1}}{(n_1 - k_1)^2} \operatorname{Re} U_1 - 4 \left(\sum_{n_1 > k_1} b_{n_1} b_{k_1} \frac{(-1)^{n_1 - k_1}}{n_1 - k_1} \operatorname{Im} U_1 \right)^2. \quad (18)$$

Similar results hold for the field phase in mode 2. The joint probability distribution and the correlation function between the two phases will be examined in section 4.

3. Phase properties of the field in individual modes

For definiteness, we consider phase properties of the field in mode 1. Phase properties of the field in mode 2 may be treated in a similar fashion.

Case 1. The electron is initially in level 2

By putting $i = 2$ and using equations (6), (12) and (13) we get

$$\operatorname{Re} U_1 = \sum_{n_2} b_{n_2}^2 \left(\cos(f_{N_1 N_2} t) \cos(f_{K_1 N_2} t) + \frac{1}{f_{N_1 N_2} f_{K_1 N_2}} (\xi_{N_1} \xi_{K_1} + \eta_{N_2}^2 + \frac{1}{4} \Delta^2) \sin(f_{N_1 N_2} t) \sin(f_{K_1 N_2} t) \right) \quad (19)$$

$$\operatorname{Im} U_1 = \frac{\Delta}{2} \sum_{n_2} b_{n_2}^2 \left(\frac{1}{f_{K_1 N_2}} \sin(f_{K_1 N_2} t) \cos(f_{N_1 N_2} t) - \frac{1}{f_{N_1 N_2}} \sin(f_{N_1 N_2} t) \cos(f_{K_1 N_2} t) \right). \quad (20)$$

These expressions have the same time-dependence form as in equation (I.27), (I.28). This implies that, concerning the field phase, the three-level system in this case also behaves like a two-level one consisting of level 2 and the combination of level 1 and level 3, but the ratio of the contribution from level 3 to that from level 1 is now

$\xi^2 \bar{n}_1 / \eta^2 \bar{n}_2$ and the frequency relation is $(\xi^2 N_1 + \eta^2 N_2)^{1/2} = \kappa \sqrt{N}$, where κ is the coupling coefficient and N is the photon number in the two-level system.

On resonance, we now cannot separate $P(\theta, t)$ exactly into two parts $P_+(\theta, t)$ and $P_-(\theta, t)$ to show the two-peak structure as clearly as in (I.31) and (I.32), though it may be done approximately. However, in the large detuning limit

$$\varepsilon_\Delta^2 \equiv \frac{4g_{\bar{n}_1, \bar{n}_2}^2}{\Delta^2} = 4 \frac{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2}{\Delta^2} \ll 1 \quad (21)$$

the phase probability distributions are again simplified considerably, and are given by

$$P(\theta_1, t) = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \cos \left[(n_1 - k_1) \left(\theta_1 + \frac{\xi^2 \tau}{\Delta} \right) \right] \right\} \quad (22)$$

$$P(\theta_2, t) = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n_2 > k_2} b_{n_2} b_{k_2} \cos \left[(n_2 - k_2) \left(\theta_2 + \frac{\eta^2 \tau}{\Delta} \right) \right] \right\} \quad (23)$$

where we have discarded all terms of order ε_Δ^2 and smaller. These results can also be compared with those obtained by Yoo and Eberly in [2], where they have shown that in this case the two-mode three-level system can be described by the effective field Hamiltonian

$$H_{\text{eff}}^F = \frac{\xi^2}{\Delta} a_1^\dagger a_1 + \frac{\eta^2}{\Delta} a_2^\dagger a_2. \quad (24)$$

The phase probability distributions (22) and (23), clearly, may be derived directly from the Hamiltonian (24).

Case 2. The electron is initially in level 1 ($i = 1$).

In the strong coherent excitation limit we get the following approximate expressions for $\text{Re } U_1$ and $\text{Im } U_1$

$$\begin{aligned} \text{Re } U_1 \approx & \frac{\eta^2 \bar{n}_2}{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2} + \frac{\xi^2 \bar{n}_1}{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2} \sum_{n_2} b_{n_2}^2 \left(\cos(f_{N_1 N_2} t) \cos(f_{K_1 N_2} t) \right. \\ & \left. + \frac{1}{f_{N_1 N_2} f_{K_1 N_2}} (g_{N_1 N_2} g_{K_1 N_2} + \frac{1}{2} \Delta^2) \sin(f_{N_1 N_2} t) \sin(f_{K_1 N_2} t) \right) \end{aligned} \quad (25)$$

$$\begin{aligned} \text{Im } U_1 \approx & \frac{\xi^2 \bar{n}_1}{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2} \frac{\Delta}{2} \sum_{n_2} b_{n_2}^2 \left(\frac{1}{f_{N_1 N_2}} \sin(f_{N_1 N_2} t) \cos(f_{K_1 N_2} t) \right. \\ & \left. - \frac{1}{f_{K_1 N_2}} \sin(K_1 N_2 t) \cos(f_{N_1 N_2} t) \right) \end{aligned} \quad (26)$$

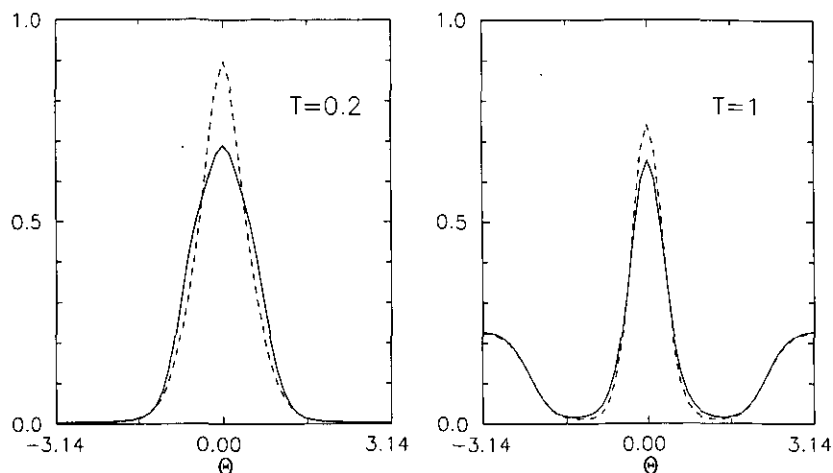


Figure 1. Phase probability distribution $P(\theta_1, t)$ (full curves) and $P(\theta_2, t)$ (broken curves) plotted against θ for Λ type and for $i=1$, $\Delta=0$, $\bar{n}_1=\bar{n}_2=\bar{n}=3$, $\xi=\eta=\gamma$. The scaled time $T=\gamma t/(2\pi\sqrt{2\bar{n}})$.

The phase probability distribution can be written as a sum of time-independent and time-dependent terms where

$$P(\theta_1, t) = \frac{\eta^2 \bar{n}_2}{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2} P_0(\theta_1) + \frac{\xi^2 \bar{n}_1}{\xi^2 \bar{n}_1 + \eta^2 \bar{n}_2} P_1(\theta_1, t) \quad (27)$$

where

$$P_0(\theta_1) = \frac{1}{2\pi} \left(1 + 2 \sum_{n_1 > k_1} b_{n_1} b_{k_1} \cos[(n_1 - k_1)\theta_1] \right) \quad (28)$$

represents an unmoved phase state, and $P_1(\theta, t)$ represents two counterrotating phase states in the polar diagram. We thus observe the partial trapping of the field phase in the initial state as in the one-mode case.

From the similarities between (25), (26) and (I.38), (I.39) it is not difficult to predict that the phase probability distributions, the expectation values and the variances of the phase will show identical behaviour, and the discussion for the two modes considered here can be done in a similar way to that for the one-mode case [1]. Clearly, the connection of the collapse-revival phenomenon with the dynamical behaviour of the field phases is also seen in the two-mode case.

Up to now, we have treated only the phase properties which resemble those for the one-mode case. As a first example of discrepancies between the two cases, one can notice that when the electron is initially in levels 1 or 3, it must affect the evolution of the field phases in mode 1 and 2 in different ways, which is not the case when the electron is initially in level 2. Indeed, in figure 1 we have plotted $P(\theta_1, t)$ (full curves) and $P(\theta_2, t)$ (broken curves) against θ in the Cartesian coordinate system for various times. The mean photon numbers and the coupling coefficients have especially been chosen equal to isolate the effects of the initial atomic state. Figure 1 shows that as the interaction is switched on, the field phase in mode 2 evolves somewhat differently with respect to the field phase in mode 1. This can be interpreted as a result of the fact that

the atom must first interact with the field in mode 1 to transfer the electron to level 2 before the transition $2 \leftrightarrow 3$ may come into operation.

Another situation, which occurs for two modes only, is when the field in mode 1 is initially excited and mode 2 is initially empty. Clearly, for Ξ and V types the field in mode 2 will not show any dynamics in the course of time. For Λ type, however, the field evolves and the occupation probability of level 3 are not only non-vanishing due to spontaneous emission from level 2, but also exhibit collapses and revivals [2]. It is interesting to know then: whether the field phase will show corresponding changes. From the equation similar to (15) for the field phase in mode 2, one gets

$$P(\theta_2, t) = 1/2\pi \quad (29)$$

for all times. Since only spontaneously emitted photons feed mode 2, they do change its occupation numbers but do not affect the phase of the field, which at all times remains uniformly distributed. The time behaviour of the phase probability distribution can not be connected with the collapse-revival phenomenon any more. Below we will also show that in this case the correlation function between the two phases vanishes.

4. The joint probability distribution and the correlation function

In the two-mode, three-level system, we have two phases evolving simultaneously. These two are connected through interaction with the common atomic level 2 and it is desirable to investigate such quantities as the joint probability distribution and the correlation function between them.

Generalization of the Hermitian phase formalism into the two-mode case is straightforward and has been used by Gantsog and Tanaš for the propagation of light in a Kerr medium [8] as well as in pair coherent states [9]. In our case, for the state vector (9) we obtain

$$\begin{aligned} |\langle \theta_{m_1} | \langle \theta_{m_2} | \psi(t) \rangle|^2 &= \frac{1}{(s+1)^2} \sum_{n_1, k_1, n_2, k_2} b_{n_1} b_{k_1} b_{n_2} b_{k_2} \\ &\times \{ \cos[(n_1 - k_1)(\theta_1 - \beta_1) + (n_2 - k_2)(\theta_1 - \beta_2)] \operatorname{Re} U \\ &+ \sin[(n_1 - k_1)(\theta_1 - \beta_1) + (n_2 - k_2)(\theta_2 - \beta_2)] \operatorname{Im} U \} \end{aligned} \quad (30)$$

where $|\theta_{m_1}\rangle$ and $|\theta_{m_2}\rangle$ are phase states of mode 1 and mode 2, respectively, and U is given by equation (10). Since the coherent field at $t=0$ belongs to a class of partial phase states [5], we choose the reference phases θ_{0_1} and θ_{0_2} as

$$\theta_{0_i} = \beta_i - \frac{\pi s_i}{s_i + 1} \quad i = 1, 2 \quad (31)$$

and introduce the new phase labels

$$\mu_i = m_i - \frac{1}{2}s_i \quad m_i = 0, 1, 2, \dots, s_i \quad (32)$$

which go in integer steps from $-\frac{1}{2}s_i$ to $\frac{1}{2}s_i$. In the limit when s_i tend to infinity, the continuous phase variables can be introduced replacing $\mu_i 2\pi/(s_i + 1)$ by θ_i and $2\pi/(s_i + 1)$ by $d\theta_i$ and one obtains the continuous joint probability distribution in the form

$$P(\theta_1, \theta_2, t) = \frac{1}{(2\pi)^2} \sum_{n_1, k_1, n_2, k_2} b_{n_1} b_{k_1} b_{n_2} b_{k_2} \{ \cos[(n_1 - k_1)\theta_1 + (n_2 - k_2)\theta_2] \operatorname{Re} U \\ + \sin[(n_1 - k_1)\theta_1 + (n_2 - k_2)\theta_2] \operatorname{Im} U \} \quad (33)$$

which is normalized according to

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} P(\theta_1, \theta_2, t) d\theta_1 d\theta_2. \quad (34)$$

It is easy to verify that integrating $P(\theta_1, \theta_2, t)$ over one of the phases gives the marginal phase distributions $P(\theta_1, t)$ or $P(\theta_2, t)$ for phases θ_1 or θ_2

$$P(\theta_1, t) = \int_{-\pi}^{\pi} P(\theta_1, \theta_2, t) d\theta_2 \quad (35)$$

$$P(\theta_2, t) = \int_{-\pi}^{\pi} P(\theta_1, \theta_2, t) d\theta_1 \quad (36)$$

as expected.

Some examples of the time behaviour of $P(\theta_1, \theta_2, t)$ are presented in figures 2 and 3 for the resonance case, and for various moments of the scaled times. The advantage of figures 2 and 3 is that they allow observation of the evolution of the field phases in mode 1 and mode 2 simultaneously. In figure 2, where the level 2 is assumed to be initially occupied, we clearly see the splitting of the phase states of both modes into two as the time goes on. When the level 1 is initially occupied (figure 3) the phase state of each mode splits into three satellite phase states. The central unmoved peak in figure 3 is associated with the first two time-dependent terms in (14) and indicates the partial trapping of the field phases in the modes. Figures 2 and 3 also reveal that at $T=1$, i.e. when the level occupation probabilities show their first revivals, the sideband peaks of $P(\theta_1, t)$ and $P(\theta_2, t)$ approach the borders of the phase windows which means the overlapping of these peaks in the polar diagram. Note that in figures 2 and 3, $P(\theta_1, \theta_2, t)$ exhibits some additional symmetry

$$P(\theta_1, \theta_2, t) = P(-\theta_1, -\theta_2, t) \quad (37)$$

which is due to the exact resonance condition.

The correlation function between the two phases can be calculated according to

$$\begin{aligned}
 C_{\theta_1\theta_2} &= \langle \hat{\Phi}_{\theta_1} \hat{\Phi}_{\theta_2} \rangle - \langle \hat{\Phi}_{\theta_1} \rangle \langle \hat{\Phi}_{\theta_2} \rangle \\
 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \theta_1 \theta_2 P(\theta_1, \theta_2, t) d\theta_1 d\theta_2 \\
 &\quad - \left(\int_{-\pi}^{\pi} \theta_1 P(\theta_1, t) d\theta_1 \right) \left(\int_{-\pi}^{\pi} \theta_2 P(\theta_2, t) d\theta_2 \right). \quad (38)
 \end{aligned}$$

For $P(\theta_1, \theta_2, t)$, $P(\theta_1, t)$ and $P(\theta_2, t)$ given by equations (33), (15) we have

$$\begin{aligned}
 C_{\theta_1\theta_2} &= - \sum_{n_1 \neq k_1, n_2 \neq k_2} b_{n_1} b_{k_1} b_{n_2} b_{k_2} \frac{(-1)^{n_1-k_1+n_2-k_2}}{(n_1-k_1)(n_2-k_2)} \operatorname{Re} U \\
 &\quad - \left(\sum_{n_1 \neq k_1} b_{n_1} b_{k_1} \frac{(-1)^{n_1-k_1}}{n_1-k_1} \operatorname{Im} U_1 \right) \left(\sum_{n_2 \neq k_2} b_{n_2} b_{k_2} \frac{(-1)^{n_2-k_2}}{n_2-k_2} \operatorname{Im} U_2 \right). \quad (39)
 \end{aligned}$$

This correlation coefficient in the case of exact resonance is plotted against the scaled time T in figures 4 and 5. As the evolution proceeds, the correlation coefficient, which is equal to zero at $t=0$, goes up initially. After a while, it begins to oscillate near the value depending on the initial atomic state and intensities of the field in the modes. In

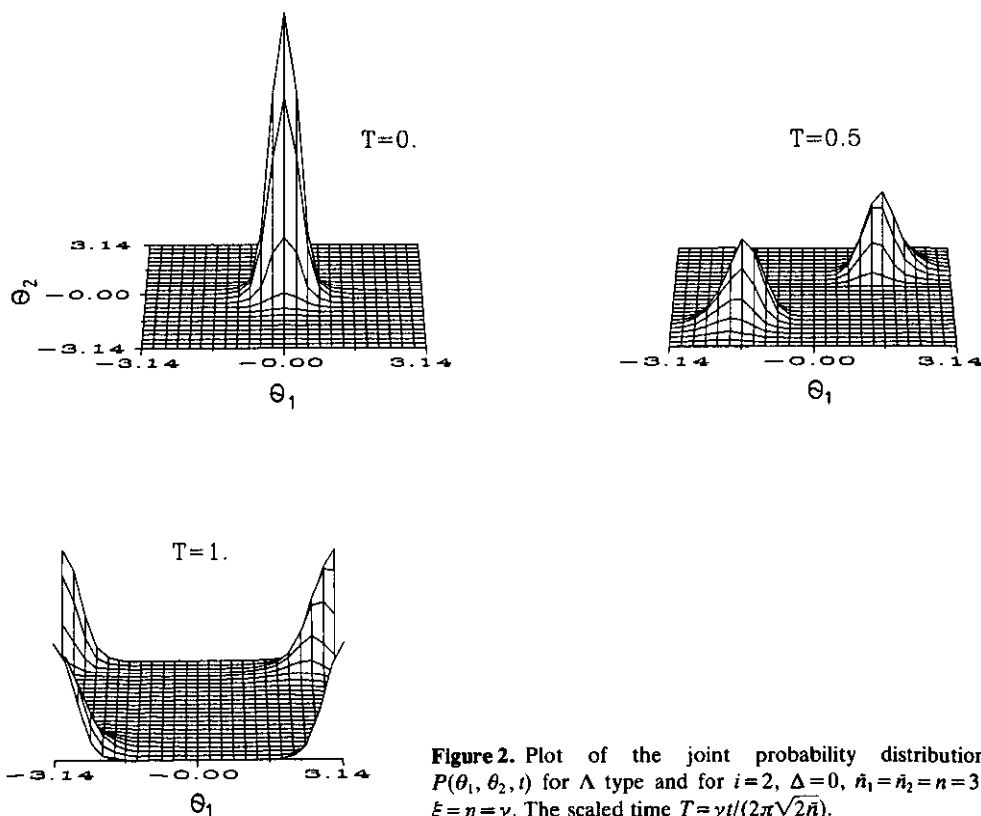


Figure 2. Plot of the joint probability distribution $P(\theta_1, \theta_2, t)$ for Λ type and for $i=2$, $\Delta=0$, $\hat{n}_1=\hat{n}_2=n=3$, $\xi=\eta=\gamma$. The scaled time $T=\gamma t/(2\pi\sqrt{2\bar{n}})$.

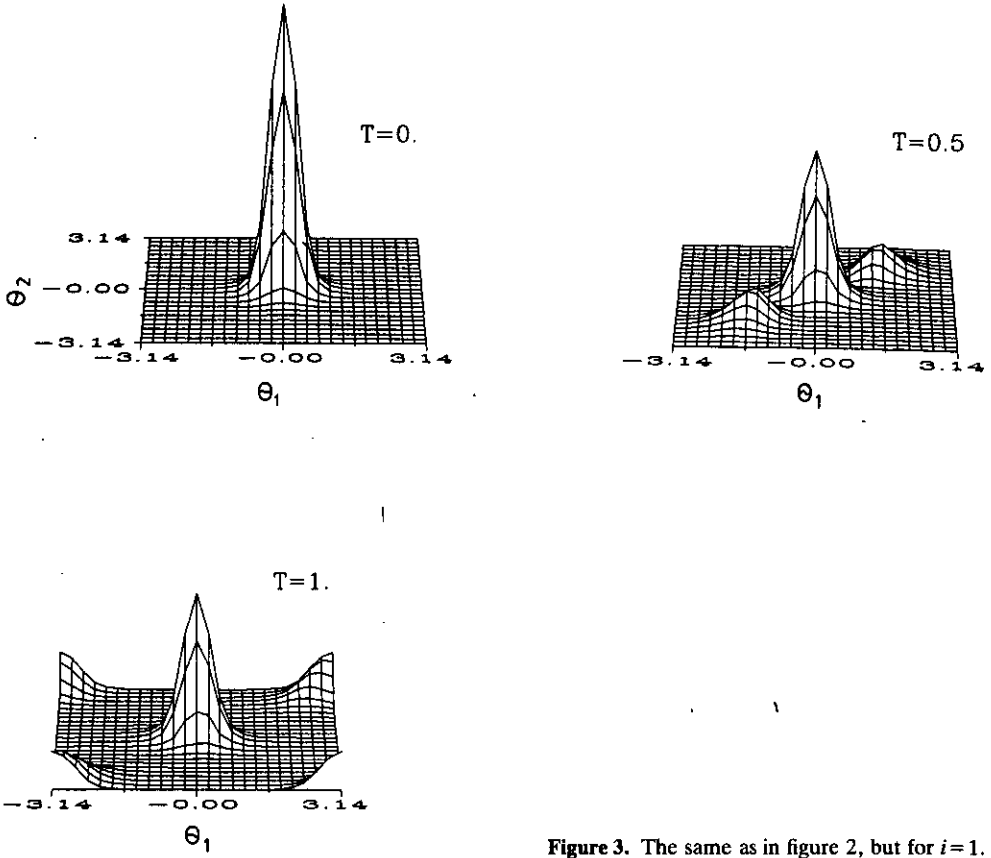


Figure 3. The same as in figure 2, but for $i=1$.

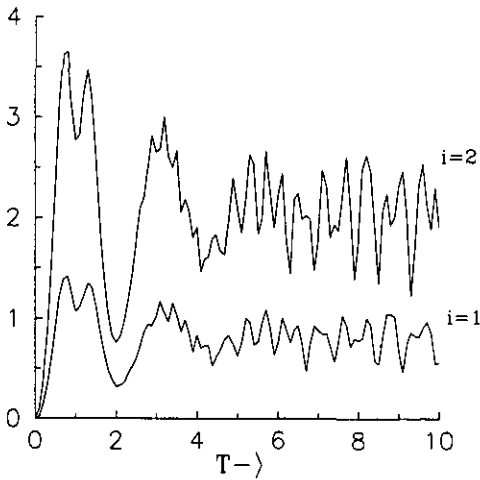


Figure 4. Phase correlation coefficient $C_{\theta_1 \theta_2}$ plotted as a function of the scaled time T for various initial atomic states and for Λ type, $\Delta=0$, $\bar{n}_1=\bar{n}_2=\bar{n}=3$, $\xi=\eta=\gamma$.

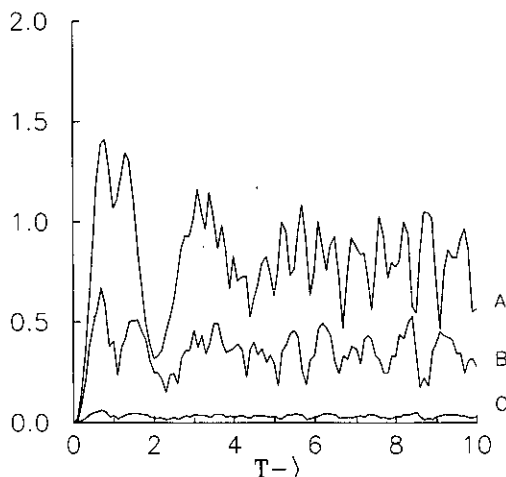


Figure 5. Phase correlation coefficient C_{θ_1, θ_2} plotted as a function of the scaled time T for Λ type and for $i=1$, $\Delta=0$, $\bar{n}_1=3$, $\xi=\eta=\gamma$. The curves A, B, C are for $\bar{n}_2=3, 0.1$ and 0.001 , respectively. The scaled time $T=\gamma t/[2\pi(\bar{n}_1+\bar{n}_2)^{1/2}]$.

figure 4, where the correlation coefficients C_{θ_1, θ_2} for $i=1$ and $i=2$ are compared, we see that when the electron is initially in the common level 2, the two phases are more strongly correlated than when the electron is initially in level 1.

In figure 5 we have fixed the mean photon number \bar{n}_1 of the field in mode 1 and have varied the mean photon number \bar{n}_2 of the field in mode 2. It can be seen that the value of C_{θ_1, θ_2} decreases when \bar{n}_2 decreases (the same holds true when both \bar{n}_1 and \bar{n}_2 decrease). For $\bar{n}_2=0$, C_{θ_1, θ_2} vanishes. The vanishing correlation coefficient for the case when one of the two modes is initially vacuum can be found directly from (39). This decorrelation between the two phases additionally explains the fact mentioned above that the phase of the scattered light always remains randomly distributed although the average photon number exhibits collapses and revivals.

5. Conclusion

We have used the Hermitian phase formalism of Pegg and Barnett to study the phase properties of a two mode coherent field interacting with a three-level atom in an ideal cavity. Similarly to the case of a one-mode three-level system, the collapses and revivals of the level occupation probabilities have been shown to be accompanied by subsequent splittings and overlappings of the phase states, except for the case of the scattered light in Raman scattering, which may be explained in part by the decorrelation between the phases of the two modes. We have investigated the joint probability distribution and the correlation coefficient between the two phases. We have shown that this correlation coefficient decreases with decreasing in the mean photon numbers of the field modes.

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