Full length article

Phase properties of pair coherent states

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Phase properties of pair coherent states are re-examined from the point of view of the Pegg–Barnett hermitian phase formalism. The joint probability distribution for the phases of the two modes is calculated and illustrated graphically. Strong correlation between the two phases is shown to exist, and the correlation coefficient calculated. The variance of the cosine of the phase-sum-operator is calculated. The results are compared to the earlier results of Agarwal.

1. Introduction

Pair coherent states introduced recently by Agarwal [1,2] are quantum states of the two-mode electromagnetic field which are simultaneous eigenstates of the pair-annihilation operator and of the difference in the number operators of the two modes of the field. Agarwal [2] has discussed the nonclassical properties of such states showing that they have remarkable quantum features such as sub-poissonian statistics, correlations in the number fluctuations, squeezing, and violations of Cauchy–Schwarz inequalities. He has also presented results for the fluctuations in the phase operators using the Susskind–Glogower [3] definition of the phase operator. It is known that the Susskind–Glogower definition of the phase exponential operator leads to the nonunitary operators. Recently, Pegg and Barnett [4–6] have introduced a new hermitian phase formalism in which the hermitian phase operator is constructed and the problem of nonunitarity is avoided.

In this paper we re-examine the problem of quantum phase fluctuations in the pair coherent states from the point of view of the new Pegg–Barnett phase formalism. The joint probability distribution for the phases of the two modes is obtained, and it is shown that this distribution depends only on the sum of the phases of the two modes. The strong correlation between the phases of the two modes in pair coherent states is shown to exist, and the correlation function of the hermitian phase operators for the two modes is calculated. The variance of the cosine function of the sum of the phase operators of the two modes is also calculated and compared to the results obtained by Agarwal [2].

Pair coherent states have very interesting quantum features that are worth of experimental effort to reveal them. Agarwal [1,2] has shown that such states can be produced by the competition of processes corresponding to nonlinear gain and nonlinear absorption in a two-photon medium. The competition between nonlinear-absorption and four-wave-mixing processes studied by Malcuit et al. [7] makes hopes for experimental success more realistic. Recently, Lee [8] has generalized pair coherent states by replacing the Fock state for one mode by a coherent state claiming that it should be easier to produce such states. He has discussed many-photon antibunching in generalized pair coherent states.

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2. Phase properties of pair coherent states

Pair coherent states introduced by Agarwal [1,2] are defined for a two-mode field. If a and b are the annihilation operators associated with the two modes, the operator ab, acting on a Fock state, simultaneously annihilates photons of modes a and b, and it can be referred to as the pair-annihilation operator. The pair coherent states are defined as eigenstates of the pair-annihilation operator [2]

$$ab|\zeta, q\rangle = \zeta|\zeta, q\rangle \,, \tag{1}$$

where ζ is a complex eigenvalue and q is the degeneracy parameter, which can be fixed by the requirement that $|\zeta, q\rangle$ is an eigenstate of the difference in the number operators for the two modes

$$(a^\dagger a - b^\dagger b)|\zeta, q\rangle = q|\zeta, q\rangle \,. \tag{2}$$

When photons are created and destroyed in pairs, the difference in the number of photons remains constant, and if the pair creation starts from vacuum, the parameter q will be zero.

The solution to the above eigenvalue problem, assuming q to be positive, is [2]

$$|\zeta, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\zeta^n}{[n!\,(n+q)!]^{1/2}} |n+q, n\rangle \,, \tag{3}$$

where N_q is the normalization constant,

$$N_q = \left(\sum_{n=0}^{\infty} \frac{|\zeta|^{2n}}{n!\,(n+q)!} \right)^{-1/2} = [(+\mathrm{i}|\zeta|)^{-q} J_q(2\mathrm{i}|\zeta|)]^{-1/2}. \tag{4}$$

The state $|n+q, n\rangle$ is the Fock state with $n+q$ photons in mode a and n photons in mode b. Eqs. (1)–(4) define the pair coherent states, phase properties of which we are going to study in this paper.

We shall use the hermitian phase formalism of Pegg and Barnett [4–6], which is based on introducing a finite $(s+1)$-dimensional subspace Ψ spanned by the number states $|0\rangle$, $|1\rangle$, ..., $|s\rangle$. The hermitian phase operator operates on this finite subspace, and after all necessary expectation values have been calculated in Ψ, the value of s is allowed to tend to infinity. A complete orthonormal basis of $(s+1)$ states is defined on Ψ as

$$|\theta_m\rangle = (s+1)^{-1/2} \sum_{n=0}^{s} \exp(\mathrm{i}n\theta_m)|n\rangle \,, \tag{5}$$

where

$$\theta_m \equiv \theta_0 + 2\pi m/(s+1) \,, \quad (m=0, 1, ..., s) \,. \tag{6}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The hermitian phase operator is defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m| \,. \tag{7}$$

Of course, the phase states (5) are eigenstates of the phase operator (7) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0+2\pi$. The unitary phase operator $\exp(\mathrm{i}\hat{\phi}_\theta)$ can be defined as the exponential function of the hermitian operator $\hat{\phi}_\theta$. This operator acting on the eigenstate $|\theta_m\rangle$ gives the eigenvalue $\exp(\mathrm{i}\theta_m)$, and can be written as [4–6]

$$\exp(\mathrm{i}\hat{\phi}_\theta) \equiv |0\rangle\langle 1| + |1\rangle\langle 2| + ... + |s-1\rangle\langle s| + \exp(\mathrm{i}(s+1)\theta_0]\,|s\rangle\langle 0| \,, \tag{8}$$

and its hermitian adjoint is

$$[\exp(\mathrm{i}\hat{\phi}_\theta)]^\dagger = \exp(-\mathrm{i}\hat{\phi}_\theta) \tag{9}$$

with the same set of eigenstates $|\theta_m\rangle$ but with eigenvalues $\exp(-\mathrm{i}\theta_m)$.

To make the further comparisons easier, it is useful to relate this new operator to the Susskind–Glogower phase operator. This gives following relation [9]

$$\langle\exp(\mathrm{i}m\hat{\phi}_\theta)\rangle=\langle[\exp(\mathrm{i}\hat{\phi}_\theta)]^m\rangle=\lim_{s\to\infty}\left\langle\left(\sum_{n=0}^{s-m}|n\rangle\langle n+m|+\exp[\mathrm{i}(s+1)\theta_0]\sum_{n=0}^{m-1}|s-n\rangle\langle m-1-n|\right)\right\rangle$$

$$=\langle\widehat{\exp}\,(\mathrm{i}m\phi_{\mathrm{SG}})\rangle+\lim_{s\to\infty}\left\langle\left(\exp[\mathrm{i}(s+1)\theta_0]\sum_{n=0}^{m-1}|s-n\rangle\langle m-1-n|\right)\right\rangle, \tag{10}$$

where the Susskind–Glogower phase operator is given by

$$\widehat{\exp}\,(\mathrm{i}m\theta_{\mathrm{SG}})\equiv\sum_{n=0}^{\infty}|n\rangle\langle n+m|\,. \tag{11}$$

The Susskind–Glogower phase operator defined by (11) is not unitary. From (11) and the definition

$$\widehat{\exp}\,(-\mathrm{i}m\phi_{\mathrm{SG}})\equiv[\widehat{\exp}\,(\mathrm{i}m\phi_{\mathrm{SG}})]^\dagger\,, \tag{12}$$

one gets, for $m=1$,

$$\widehat{\exp}\,(\mathrm{i}\phi_{\mathrm{SG}})\,\widehat{\exp}\,(-\mathrm{i}\phi_{\mathrm{SG}})=1\,,\quad\widehat{\exp}\,(-\mathrm{i}\phi_{\mathrm{SG}})\,\widehat{\exp}\,(\mathrm{i}\phi_{\mathrm{SG}})=1-|0\rangle\langle 0|\,, \tag{13}$$

This is in a sharp contrast to the unitary exponential phase operator in the Pegg-Barnett formalism.

When the expectation values are calculated in "physical states", according to their definition by Pegg and Barnett [5,6], the last term in eq. (10) becomes zero and there is no difference between the expectation values of the exponential phase operators in the two formalisms. The differences do appear, however, when the variances of the cosine and sine functions of the phase are considered [9].

It is worth noting that in the Pegg-Barnett formalism the hermitian phase operator exist, and the exponential phase operator is simply the exponential function of this hermitian phase operator. The existence of the hermitian phase operator makes it possible to discuss quantum fluctuations in the phase itself, and to derive the phase probability distribution function.

Generalization of the Pegg-Barnett formalism into the two-mode case is straightforward, and we can directly apply this formalism to the pair coherent states. Before doing this we rewrite eq. (3) defining the pair coherent states in a slightly different form. If the complex number ζ is written in the form

$$\zeta=|\zeta|\exp(\mathrm{i}\varphi)\,, \tag{14}$$

the state (3) can be written as

$$|\zeta,q\rangle=\sum_{n=0}^{\infty}b_n\exp(\mathrm{i}n\varphi)\,|n+q,n\rangle\,, \tag{15}$$

where

$$b_n=N_q\frac{|\zeta|^n}{[n!\,(n+q)!]^{1/2}}\geqslant 0\,. \tag{16}$$

When the state (15) is projected onto the phase states (5) of modes a and b, one gets the joint probability amplitude

$$\langle\theta_{m_a}|\langle\theta_{m_b}|\zeta,q\rangle=(s+q+1)^{-1/2}(s+1)^{-1/2}\exp(-\mathrm{i}q\theta_{m_a})\sum_{n=0}^{s}b_n\exp[\mathrm{i}n(\varphi-\theta_{m_a}-\theta_{m_b})]\,, \tag{17}$$

where we have taken into account the fact that the dimension of the Fock space for mode a is larger by q from that for mode b. For the joint probability of the phase of modes a and b having the values θ_{m_a} and θ_{m_b} we obtain the following expression

$$|\langle\theta_{m_a}|\langle\theta_{m_b}|\zeta, q\rangle|^2=(s+q+1)^{-1}(s+1)^{-1}\sum_{n=0}^{s}\sum_{k=0}^{s} b_n b_k \exp\{i(n-k)[\varphi-(\theta_{m_a}+\theta_{m_b})]\} . \tag{18}$$

Since the phase φ of the complex number ζ makes the pair coherent state a partial phase state, one can choose θ_{0_a} and θ_{0_b} as to symmetrize the two phases with respect to $\varphi/2$. This means the choice of θ_{0_a} and θ_{0_b} the values [4,5]

$$\theta_{0_i}=\varphi/2-\pi s_i/(s_i+1)\,, \quad (i=\text{a or b}) \tag{19}$$

and the new phase labels

$$\mu_i=m_i-s_i/2$$

that run in unit steps from $-s_i/2$ to $s_i/2$. After such symmetrization and performing the continuum limit transition one arrives at the continuous joint probability distribution for the continuous phase variables θ_a and θ_b, which is given by

$$P(\theta_a, \theta_b)=(2\pi)^{-2}\left(1+2\sum_{n>k} b_n b_k \cos[(n-k)(\theta_a+\theta_b)]\right), \tag{20}$$

where b_n is given by eq. (16). The distribution (20) is normalized such that

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} P(\theta_a, \theta_b)\, d\theta_a\, d\theta_b=1 . \tag{21}$$

One important phase property of pair coherent states is seen directly from the form of formula (20). It is clear that the joint probability distribution (20) depends on the sum of the two phases only

$$P(\theta_a, \theta_b)=P(\theta=\theta_a+\theta_b) . \tag{22}$$

This means the strong correlation of the two modes.

Integrating $P(\theta_a, \theta_b)$ over one of the phases gives the marginal phase distribution $P(\theta_a)$ or $P(\theta_b)$ for the phase θ_a or θ_b, which are uniformly distributed

$$P(\theta_a)=\int_{-\pi}^{\pi} P(\theta_a, \theta_b)\, d\theta_b=\frac{1}{2\pi}, \quad P(\theta_b)=P(\theta_a)=\frac{1}{2\pi} . \tag{23,24}$$

Formulas (23) and (24) allow for direct calculations of the expectation values of the phase operators, defined by eq. (7), for modes a and b. We have

$$\langle\hat{\phi}_{\theta_a}\rangle=\sum_{m_a=0}^{s_a}\theta_{m_a}|\langle\theta_{m_a}|\zeta, q\rangle|^2=\frac{\varphi}{2}+\sum_{\mu=-s_a/2}^{s_a/2}\theta_\mu|\langle\theta_\mu|\zeta, q\rangle|^2=\frac{\varphi}{2}+\int_{-\pi}^{\pi}\theta_a P(\theta_a)\, d\theta_a=\frac{\varphi}{2}, \tag{25}$$

and similarly

$$\langle\hat{\phi}_{\theta_b}\rangle=\langle\hat{\phi}_{\theta_a}\rangle=\varphi/2 . \tag{26}$$

Consequently,

$$\langle\hat{\phi}_{\theta_a}-\hat{\phi}_{\theta_b}\rangle=0\,, \quad \langle\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b}\rangle=\varphi . \tag{27}$$

For the variances of the individual phase we obtain the results for uniformly distributed phases

$$\langle (\Delta\hat{\phi}_{\theta_a})^2 \rangle = \langle (\Delta\hat{\phi}_{\theta_b})^2 \rangle = \pi^2/3 \,. \tag{28}$$

Thus pair coherent states have very interesting phase properties: the individual phases θ_a and θ_b as well as the phase difference $\theta_a - \theta_b$ are uniformly distributed, and the only nonuniformly distributed phase quantity is the phase sum $\theta_a + \theta_b$. The mean value of the phase sum is given by (27), and is equal to the phase φ of the complex number ζ. Example of the joint probability distribution $P(\theta_a, \theta_b)$ is shown in fig. 1. The symmetry with respect to the diagonals of the square of the phase windows for θ_a and θ_b is clearly visible, and it reflects the dependence of $P(\theta_a, \theta_b)$ on the sum $\theta_a + \theta_b$ only. Thus the phase properties of pair coherent states can be illustrated by plotting $P(\theta = \theta_a + \theta_b)$, i.e., looking at the section in a plane perpendicular to the symmetry axis. The dependence of $P(\theta)$ on $|\zeta|$ is shown in fig. 2, and the dependence on q for given $|\zeta|$ in fig. 3. It is seen that as $|\zeta|$ increases the distribution $P(\theta)$ becomes narrower. Quite opposite, increase of q broadens the distribution $P(\theta)$, which is seen from fig. 3.

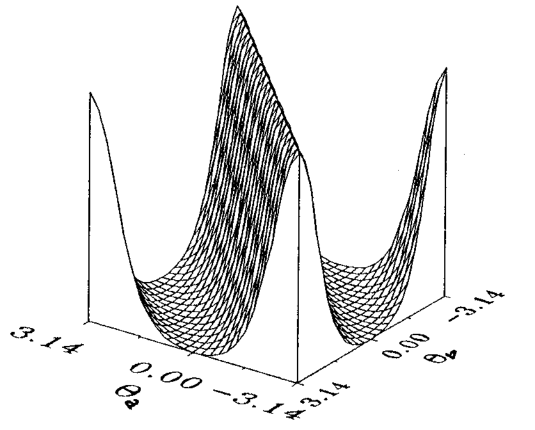

Fig. 1. Plot of the joint probability distribution $P(\theta_a, \theta_b)$, for $|\zeta| = 1$ and $q = 1$.

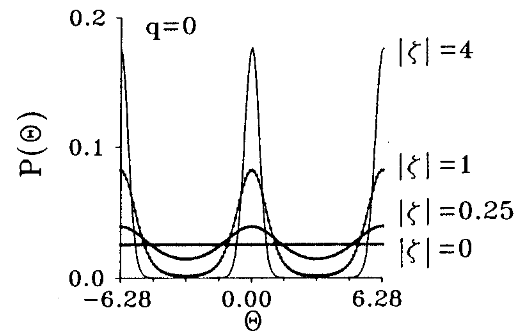

Fig. 2. The section of $P(\theta_a, \theta_b) = P(\theta = \theta_a + \theta_b)$ in a plane perpendicular to the symmetry axis plotted against $\theta = \theta_a + \theta_b$, for $q = 0$ and various $|\zeta|$.

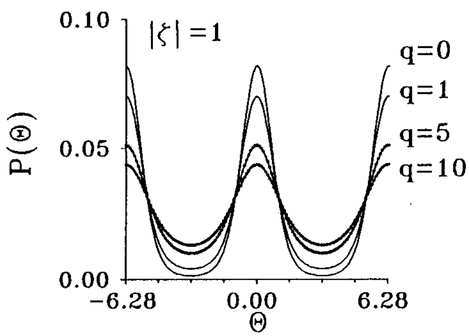

Fig. 3. Same as fig. 2, but for $|\zeta| = 1$ and various q.

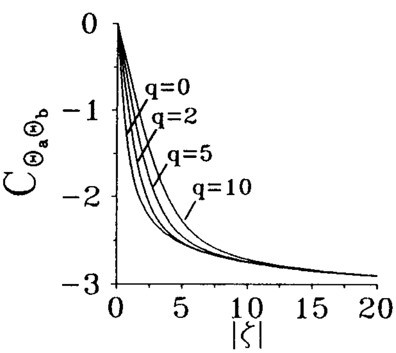

Fig. 4. The phase correlation coefficient $C_{\theta_a\theta_b}$ plotted against $|\zeta|$ for various values of q.

The variance of the sum of the phase operators is given by

$$\langle [\Delta(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})]^2\rangle = \langle (\Delta\hat{\phi}_{\theta_a})^2\rangle + \langle (\Delta\hat{\phi}_{\theta_b})^2\rangle + 2(\langle \hat{\phi}_{\theta_a}\hat{\phi}_{\theta_b}\rangle - \langle \hat{\phi}_{\theta_a}\rangle\langle \hat{\phi}_{\theta_b}\rangle)\,, \tag{29}$$

where the variances for the individual phases are given by eq. (28), and the correlation between the two phases is given by

$$C_{\theta_a\theta_b} = \langle \hat{\phi}_{\theta_a}\hat{\phi}_{\theta_b}\rangle - \langle \hat{\phi}_{\theta_a}\rangle\langle \hat{\phi}_{\theta_b}\rangle = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \theta_a\theta_b P(\theta_a,\theta_b)\,\mathrm{d}\theta_a\,\mathrm{d}\theta_b = -2\sum_{n>k}\frac{b_n b_k}{(n-k)^2}\,. \tag{30}$$

This correlation coefficient which is negative lowers the variance (29) of the phase-sum-operator. The dependence of this correlation coefficient on $|\zeta|$ is plotted in fig. 4. It is seen that as $|\zeta|$ increases the correlation coefficient (30) decreases (its absolute value increases) meaning that the phase-sum-operator variance also decreases, i.e., the sum of the two phases becomes less uncertain. Asymptotically, for $|\zeta|\to\infty$, this coefficient approaches $-\pi^2/3$, and the variance (29) approaches zero, and we have the classical situation of perfectly defined phase sum. This phase correlation coefficient can be contrasted with the photon number correlation coefficient between modes a and b considered by Agarwal [2]. The photon number correlation coefficient increases as $|\zeta|$ increases.

Agarwal [2] has considered phase properties of pair coherent states introducing, following Carruthers and Nieto [10], the exponential phase operator of the two-mode field being the product of the exponential phase operators of the Susskind–Glogower type for the two modes. He has calculated and discussed the variance of the cosine phase operator of the two-mode field. This would correspond to the cosine function of the phase-sum-operator in the Pegg–Barnett formalism. Introducing the two-mode version of eq. (10) and taking into account that the pair coherent state is a physical state, one obtains for the phase-sum-operator exponential

$$\langle \exp[im(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})]\rangle_p = \langle \exp(im\hat{\phi}_{\theta_a})\exp(im\hat{\phi}_{\theta_b})\rangle_p$$

$$= \left\langle \sum_{n=0}^{\infty}(|n\rangle\langle n+m|)_a \sum_{k=0}^{\infty}(|k\rangle\langle k+m|)_b\right\rangle_p = \langle \widehat{\exp}_a(im\phi_{SG})\,\widehat{\exp}_b(im\phi_{SG})\rangle_p\,. \tag{31}$$

Taking the expectation value (31) in the pair coherent states leads to the expectation value of the exponential phase operator obtained by Agarwal [2]

$$\langle \zeta,q|\exp[i(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})]|\zeta,q\rangle = e^{i\varphi}\sum_{n=0}^{\infty} b_n b_{n+1} = |\zeta|e^{i\varphi}N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{n!\,(n+q)!\,[(n+1)(n+q+1)]^{1/2}}\,, \tag{32}$$

and for the cosine function of the phase-sum operator we have

$$\langle \zeta,q|\cos(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})|\zeta,q\rangle = \cos\varphi\sum_{n=0}^{\infty} b_n b_{n+1} = \cos\varphi|\zeta|N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{[n!\,(n+q)!\,(n+1)!\,(n+q+1)!]^{1/2}} \tag{33}$$

which agrees with the corresponding expression obtained by Agarwal [2].

For the square of the cosine of the phase-sum operator we obtain

$$\langle \zeta,q|\cos^2(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})|\zeta,q\rangle = \tfrac{1}{4}\langle \exp[2i(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})] + \exp[-2i(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})]+2\rangle$$

$$= \tfrac{1}{2}+\tfrac{1}{2}\cos 2\varphi\sum_{n=0}^{\infty} b_n b_{n+2} = \tfrac{1}{2}+\tfrac{1}{2}\cos 2\varphi|\zeta|^2N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{[n!\,(n+2)!\,(n+q)!\,(n+q+2)!]^{1/2}}\,, \tag{34}$$

which is different from the expression obtained by Agarwal [2] who used the Susskind–Glogower type phase operators.

The variance of the cosine of the phase-sum operator is equal to

$$\langle\zeta,q|\cos^2(\hat\phi_{\theta_a}+\hat\phi_{\theta_b})|\zeta,q\rangle-\langle\zeta,q|\cos(\hat\phi_{\theta_a}+\hat\phi_{\theta_b})|\zeta,q\rangle^2$$
$$=\tfrac12\left[1-\left(\sum_{n=0}^{\infty}b_nb_{n+1}\right)^2\right]+\tfrac12\cos2\varphi\left[\sum_{n=0}^{\infty}b_nb_{n+2}-\left(\sum_{n=0}^{\infty}b_nb_{n+1}\right)^2\right]$$
$$=\tfrac12\left[1-\left(|\zeta|N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{[n!\,(n+q)!\,(n+1)!\,(n+q+1)!]^{1/2}}\right)^2\right]$$
$$+\tfrac12\cos2\varphi\left[|\zeta|^2N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{[n!\,(n+2)!\,(n+q)!\,(n+q+2)!]^{1/2}}\right.$$
$$\left.-\left(|\zeta|N_q^2\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{[n!\,(n+q)!\,(n+1)!\,(n+q+1)!]^{1/2}}\right)^2\right].\qquad(35)$$

As one could expect, our formula (35) differs from the Agarwal formula, and the difference is most important for $|\zeta|\ll1$. For the two-mode vacuum ($|\zeta|=0$) formula (35) gives the value 1/2 in contrast to 1/4 obtained from the Agarwal formula. This once more proves the advantage of the Pegg–Barnett [4–6] approach over that of Susskind–Glogower [3] in interpreting the vacuum as the state with randomly distributed phase.

For $q=0$, formula (35) can be expressed by the modified Bessel functions $I_n(2|\zeta|)$, which gives

$$\langle\cos^2(\hat\phi_{\theta_a}+\hat\phi_{\theta_b})\rangle-\langle\cos(\hat\phi_{\theta_a}+\hat\phi_{\theta_b})\rangle^2=\tfrac12\left[1-\left(\frac{I_1(2|\zeta|)}{I_0(2|\zeta|)}\right)^2\right]+\tfrac12\cos2\varphi\left[\frac{I_2(2|\zeta|)}{I_0(2|\zeta|)}-\left(\frac{I_1(2|\zeta|)}{I_0(2|\zeta|)}\right)^2\right].\qquad(36)$$

Asymptotically, for large $|\zeta|$, the variance (36) behaves as $|\zeta|^{-1}$, i.e., the asymptotic behaviour is the same as in the case of Agarwal's formula. In the limit $|\zeta|\to\infty$, the variance (36) tends to zero, which means the classical behaviour of the field with perfectly defined sum of phases. This is in agreement with our results (29) and (30) for the fluctuations in the phase-sum-operator itself.

3. Conclusions

We have re-examined phase properties of pair coherent states from the point of view of the Pegg–Barnett hermitian phase formalism. The joint probability distribution for the phases of the two modes has been obtained and shown to depend only on the sum of the two phases. It has been shown that in the pair coherent states the only nonrandom phase distribution is that for the sum of phases. The strong correlation between the phases of the two modes has been shown, and the correlation function of the hermitian phase operators for the two modes has been calculated. This correlation lowers the variance of the phase-sum-operator, and in the limit of large $|\zeta|$ the sum of the two phases becomes well defined, although the individual phases are randomly distributed. The variance of the cosine of the phase-sum-operator has also been calculated. It has been shown that the result obtained within the framework of the Pegg–Barnett formalism differs essentially from the result of Agarwal [2] obtained within the Susskind–Glogower phase formalism when $|\zeta|\ll1$. For large $|\zeta|$, the asymptotic behaviour of the two results is identical, as one would expect.

We should also emphasize that the results for the phase expectation values and the variances depend on the choice of the phase windows for θ_a and θ_b, or in other words they depend on the choice of the square over which the integrations are performed. It is seen from fig. 1 that there is the second ridge of the distribution which is split into two pieces that are located in the corners of the integration square when the choice of the phase windows is taken as in eq. (19). As $|\zeta|$ increases, parts of this additional ridge of the distribution are pressed more and more into the corners and do not contribute essentially to the integrals. Were the windows chosen differently, the two-ridge structure of the distribution would affect essentially all the expectation values and obscure their physical interpretation. For example, the phase-sum-operator variance would not tend to zero for $|\zeta| \to \infty$, and the clear picture of asymptotically classical behaviour would be disturbed.

Generally, pair coherent states have very specific phase features, and the new Pegg–Barnett phase formalism, as we have shown, allows to reveal fine details of these features.

Acknowledgements

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