J. Phys. France 49 (1988) 787-794 MAI 1988, PAGE 787

Classification
Physics Abstracts
42.50F

Collective resonance fluorescence in an intense laser field with phase and amplitude fluctuations

A. S. Shumovsky, R. Tanas (*) and Tran Quang

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Head Post Office P.O. Box 79, Moscow 101000 U.S.S.R.

(*Reçu le 28 juillet 1987, révisé le 7 décembre 1987, accepté le 13 janvier 1988*)

Résumé. — On considère la fluorescence de résonance collective d'un système d'atomes à deux niveaux interagissant de manière résonnante avec un champ laser fort présentant des fluctuations de phase et d'amplitude. On discute en détail, dans le cas non résonnant, de l'effet des fluctuations du champ laser sur le spectre et aussi sur le degré de cohérence du second ordre.

Abstract. — The collective resonance fluorescence from a system of two-level atoms resonantly driven by a strong laser field with the phase and amplitude fluctuations is considered. The effect of the laser field fluctuations on the spectrum as well as the degrees of the second-order coherence of the fluorescent field is discussed in detail for the off-resonance case.

1. Introduction.

During the last years a large number of works has been concentrated on the problems of collective interactions of atoms with a laser field and with the vacuum of radiation such as superfluorescence (see [1] and Refs. therein), collective resonance fluorescence [2-8], collective double optical resonance [9], collective Raman scattering [10-14], etc.

In the one-atom case the effects due to the fluctuating driving fields have been discussed for resonance fluorescence [12-19], double optical resonance [29] and Raman scattering [20-23]. Puri and Hassan [8] have studied the effects due to a fluctuating driving laser field in collective spectral and statistical properties of resonance fluorescence. Contrarily to the paper [8] where only the exact resonance has been considered, in this paper we discuss the off-resonance case for the collective resonance fluorescence in an intense fluctuating laser driving field and investigate the spectral and statistical properties due to the phase and amplitude fluctuations of the driving field.

(*) *Permanent address:* Institute of Physics, A. Mickiewicz university 60-780, Poznan, Poland.

2. The master equation.

We consider a system of N two-level atoms concentrated in a region small compared to the wavelength of all the relevant radiation modes (Dicke model) interacting with a mode of monochromatic driving field of frequency ω_L and with the vacuum of other modes (Fig. 1). In treating the external field classically and using the Markov and rotating wave approximation when describing the coupling of the system with the vacuum field, one can find a master equation for the reduced density matrix ρ for the atomic system alone in the form [6, 8, 26]

$$\begin{aligned}\frac{\partial \rho}{\partial t} = & -id[E^*(t) J_{21} + E(t) J_{12}, \rho] \\ & - \frac{i}{2}\delta [J_{22} - J_{11}, \rho] \\ & - \gamma_{21}(J_{21} J_{12} \rho - 2 J_{12} \rho J_{21} + \rho J_{21} J_{12}) ,\end{aligned} \tag{1}$$

with $2\gamma_{21}$ being the transition rate from the level $|2\rangle$ to $|1\rangle$ due to the atomic interaction with the reservoir; $\delta = \omega_{21} - \omega_L$ is the detuning of laser frequency from the atomic resonance frequency ω_{21}; d is the dipole matrix element; $E(t)$ is the driving laser field; $J_{k\ell}$ $(k, \ell = 1, 2)$ are the collective operators (angular momenta) describing the atomic

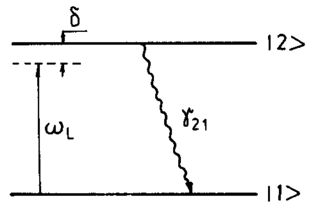

Fig. 1. — Schematic diagram of the two-level atoms, interacting with the laser driving field and with the vacuum of radiation.

system and having in the Schwinger representation [27, 7] the following form :

$$J_{k\ell} = C_k^+ C_\ell \quad (k, \ell = 1, 2),$$

where the operators C_k and C_k^+ obey the boson commutation relations

$$[C_k, C_\ell^+] = \delta_{k\ell}$$

and can be treated as annihilation and creation operators for atoms populating the level $|k\rangle$.

In the following, the laser field is assumed to have the form

$$E(t) = (E_0 + E_1(t))\, e^{-i\phi(t)}, \; \phi_0 = \phi(0), \quad (2)$$

where E_0 and ϕ_0 are the nonstochastic parts of the field amplitude and phase while $E_1(t)$, $\phi(t)$ are the stochastic variables.

Following Puri *et al.* [6, 8] we discuss the case in which the phase fluctuations are described by the *phase diffusion model* [24] and the amplitude fluctuations are described by the nonwhite Gaussian process, i.e. the following relations are satisfied :

$$\frac{d\phi(t)}{dt} = \mu(t), \quad (3)$$

where $\mu(t)$ is Gaussian white noise with

$$\overline{\mu(t)} = 0, \quad \overline{\mu(t)\,\mu(t')} = 2\,\gamma_c\,\delta(t - t'), \quad (4)$$

and

$$\overline{\varepsilon_1(t)} = 0, \; \overline{\varepsilon_1(t)\,\varepsilon_1(t')} = (\Delta\varepsilon_1)^2\, e^{-\gamma_a|t-t'|}, \quad (5)$$

$$\overline{\varepsilon_1(t)\,\mu(t')} = 0, \quad (6)$$

where $\varepsilon_1(t) = dE_1(t)$; the quantities γ_c and γ_a describe the band-widths due to the frequency and amplitude fluctuations of the laser field, and $(\Delta\varepsilon_1)^2$ is a measure of the fluctuations of the Rabi frequency.

After Puri *et al.* [6, 8], for the later use, we introduce the transformation

$$W_m(t) = e^{-im\phi(t)}\, e^{-i\frac{\phi(t)}{2}J_3}\, \rho\, e^{i\frac{\phi(t)}{2}\cdot J_3}, \quad (7)$$

where $J_3 = J_{22} - J_{11}$.

It is easy to show that an equation for $W_m(t)$ has the form

$$\frac{dW_m(t)}{dt} = [L_0 - i(m + L_1)\,\dot\phi(t) - \\ - i\,\varepsilon_1(t)\,L_2] W_m(t) \quad (8)$$

here

$$\begin{aligned} L_0 W_m = & -i\,\varepsilon_0[J_{12} + J_{21}, W_m] - i\,\frac{\delta}{2}\,[J_3, W_m] \\ & - \gamma_{21}(J_{21}\,J_{12}\,W_m - 2\,J_{12}\,W_m\,J_{21} \\ & + W_m\,J_{21}\,J_{12}), \\ L_1\,W_m = & \frac{1}{2}\,[J_3, W_m], \\ L_2\,W_m = & [J_{21} + J_{12}, W_m], \end{aligned}$$

with $\varepsilon_0 = dE_0$ being the resonance Rabi frequency. Our next step is to obtain the master equation for $\bar W_m(t)$, the transformed density operator averaged over the ensemble with respect to the distribution of the phases. After Van Kampen [25], using the theory of multiplicative stochastic processes, one can find

$$\begin{aligned} \frac{d}{dt}\,\bar W_m(t) = & [L_0 - \gamma_c(m + L_1)^2 - \\ & - i\,\varepsilon_1(t)\,L_2]\,\bar W_m(t) \quad (9) \\ = & L\bar W_m(t). \end{aligned}$$

Equation (9) has been investigated in the work by Puri and Lawande [6] where the exact steady-state operator $\bar W_m^{(s)}$ is given for the case without the amplitude fluctuations, i.e. for the case with $\varepsilon_1(t) = 0$.

Since the operator L_2 which is multiplied by the time-dependent coefficient $\varepsilon_1(t)$ does not commute with all other operators in (9), it is impossible to use the theory of multiplicative stochastic processes to obtain the master equation for the density matrix averaged over the amplitude fluctuations in the same fashion as for the phase fluctuations. We thus restrict our consideration to strong laser field or to large detuning δ so that the Rabi frequency Ω satisfied the following relation

$$\Omega = \left(\frac{1}{4}\,\delta^2 + \varepsilon_0^2\right)^{1/2} \gg N\gamma_{21}. \quad (10)$$

After performing the canonical (dressing) transformation

$$\begin{aligned} C_1 &= Q_1 \cos\varphi + Q_2 \sin\varphi, \\ C_2 &= -Q_1 \sin\varphi + Q_2 \cos\varphi, \end{aligned} \quad (11)$$

where $\operatorname{tg} 2\varphi = 2\,\varepsilon_0/\delta$,

one can split the Liouville operator appearing in equation (9) into the slowly varying part and the terms oscillating at frequencies $2\,\Omega$ and $4\,\Omega$. Since we assume that the Rabi frequency Ω is sufficiently large according to the relation (10) the secular approximation [3, 8, 9] is justified and we retain only the slowly varying part of the Liouville operator. We have then

$$\frac{d}{dt}\bar{W}'_m(t) = [\mathfrak{L}_0 + \mathfrak{L}_1 - z_3\,\mathfrak{L}_2 - 2\,i\,\varepsilon_1(t)\sin\varphi\cos\varphi\,.\,\mathfrak{L}_3]\,.\,\bar{W}'_m(t)\,, \tag{12}$$

where $\bar{W}'_m = U\bar{W}_m\,U^+$ with U being the unitary operator representing the canonical transformation (11)

$$\begin{aligned}\mathfrak{L}_0\,\bar{W}'_m(t) = & -\gamma_c\,m^2\,\bar{W}'_m(t) - i\Omega\,[D_3, \bar{W}'_m(t)] - \\ & -\gamma_c\,m(\cos^2\varphi - \sin^2\varphi)[D_3, \bar{W}'_m(t)]\,,\end{aligned} \tag{13}$$

$$\begin{aligned}\mathfrak{L}_1\,\bar{W}'_m(t) = & -Z_1(R_{21}\,R_{12}\,\bar{W}'_m(t) - 2\,R_{12}\,\bar{W}'_m(t)\,R_{21} + \bar{W}'_m(t)\,R_{21}\,R_{12}) \\ & -Z_2(R_{12}\,R_{21}\,\bar{W}'_m(t) - 2\,R_{21}\,\bar{W}'_m(t)\,R_{12} + \bar{W}'_m(t)\,R_{12}\,R_{21})\,,\end{aligned} \tag{14}$$

$$\begin{aligned}&\mathfrak{L}_2\,\bar{W}'_m(t) = D_3^2\,\bar{W}'_m(t) - 2\,D_3\,\bar{W}'_m(t)\,D_3 + \bar{W}'_m(t)\,D_3^2\,,\\ &\mathfrak{L}_3\,\bar{W}'_m(t) = [D_3, \bar{W}'_m(t)]\,,\end{aligned} \tag{15}$$

where $D_3 = R_{22} - R_{11}$ with $R_{ij} = Q_i^+\,Q_j$ $(i, j = 1, 2)$ are the collective angular momenta of *dressed* atoms ;

$$\begin{aligned} Z_1 &= \gamma_{21}\cos^4\varphi + \gamma_c\sin^2\varphi\,.\,\cos^2\varphi\,,\\ Z_2 &= \gamma_{21}\sin^4\varphi + \gamma_c\sin^2\varphi\,.\,\cos^2\varphi\,,\\ Z_3 &= \gamma_{21}\sin^2\varphi\cos^2\varphi + \\ &\quad + \frac{\gamma_c}{4}(\cos^2\varphi - \sin^2\varphi)^2\,. \end{aligned} \tag{16}$$

Now we derive an equation for $\bar{\bar{W}}'_m(t)$ (i.e. $\bar{\bar{W}}'_m(t)$ averaged over the distribution of the amplitude fluctuations). Again, we use the theory of multiplicative stochastic processes [25] and arrive at the equation for $\bar{\bar{W}}'_m(t)$ in the form

$$\frac{d}{dt}\bar{\bar{W}}'_m(t) = [\mathfrak{L}_0 + \mathfrak{L}_1 - (X_0 + X_1(t))\,\mathfrak{L}_2]\,\bar{\bar{W}}'_m(t)\,, \tag{17}$$

where

$$\begin{aligned} X_0 = & \ \gamma_{21}\sin^2\varphi\,.\,\cos^2\varphi + \\ & + \frac{\gamma_c}{4}(\cos^2\varphi - \sin^2\varphi)^2 \\ & + 4\left(\frac{\Delta\varepsilon_1}{\gamma_a}\right)^2\cdot\sin^2\varphi\cdot\cos^2\varphi\,,\\ X_1(t) = & -4\,\frac{(\Delta\varepsilon_1)}{\gamma_a}^2\sin^2\varphi\,.\,\cos^2\varphi\,.\,\ell^{-\gamma_a t}\,, \end{aligned}$$

and the Liouville operators $\mathfrak{L}_0$, $\mathfrak{L}_1$, $\mathfrak{L}_2$ have the same forms as given in (13), (15). For the case of exact resonance, *i.e.* $\mathrm{ctg}^2\,\varphi$ equation (17) reduces to that derived by Puri and Hassan [8].

The stationary solution of equation (17) takes the form

$$\bar{\bar{W}}'^{(S)}_m = \begin{cases} 0 & \text{for } m \neq 0\,,\\ A^{-1}\sum\limits_{M=0}^{N} Z^M\,|M\rangle\,\langle M| & \text{for } m = 0\,,\end{cases} \tag{18}$$

where

$$\begin{aligned} Z &= \frac{Z_1}{Z_2} = \frac{\gamma_{21}\,\mathrm{ctg}^4\,\varphi + \gamma_c\,\mathrm{ctg}^2\,\varphi}{\gamma_{21} + \gamma_c\,\mathrm{ctg}^2\,\varphi}\,,\\ A &= \frac{Z^{N+1} - 1}{Z - 1}\,,\end{aligned}$$

$|M\rangle$ is an eigenstate of the operators R_{11} and $\hat{N} = R_{11} + R_{22}$ where $R_{ij} = Q_i^+\,Q_j$ $(i, j = 1, 2)$. The operators Q_i and Q_i^+ satisfy the boson commutation relations

$$[Q_i, Q_j^+] = \delta_{ij}\,, \tag{19}$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij'}\,\delta_{i'j} - R_{i'j}\,\delta_{ij'}\,. \tag{20}$$

For the later use, we introduce the characteristic function

$$\begin{aligned}\chi_{R_{11}}(\eta) &= \mathrm{Tr}\,\{e^{i\eta R_{11}}\,.\,\bar{\bar{W}}'^{(S)}_0\} = \langle e^{i\eta R_{11}}\rangle_S \\ &= A^{-1}\frac{Y^{N+1} - 1}{Y - 1}\,,\end{aligned}$$

where $\quad Y = Z\,.\,e^{i\eta}\,.$

All statistical moments of *dressed* atoms of the form $\langle R_{11}^q \rangle_S$ can be found from the characteristic function $\chi_{R_{11}}(\eta)$ and have the following form :

$$\langle R_{11}^q \rangle_S = \frac{\partial^q}{\partial (i\eta)^q} \chi_{R_{11}}(\eta) \bigg|_{i\eta = 0} . \qquad (21)$$

The expectation values of the atomic observables averaged over the distributions of the phase and amplitude are given by

$$\begin{aligned} \langle \overline{\overline{J_{21}^{n+m} J_3^p J_{12}^n}} \rangle &= \mathrm{Tr}\ \overline{\overline{\{J_{21}^{n+m} J_3^p J_{12}^n\, \varphi(t)\}}} \\ &= \mathrm{Tr}\ \left\{ (J_{21}^{n+m} J_3^p J_{12}^n)'\, \bar{\bar{W}}_m'(t) \right\} , \quad (22) \end{aligned}$$

where as denoted above $A' = UAU^+$ with U being the unitary operator representing the canonical transformation (11).

It is easy to show from the stationary solution (18) that

$$\langle \overline{\overline{J_{21}^{n+m} J_3^p J_{12}^n}} \rangle_S = \\ = \begin{cases} \mathrm{Tr}\ \left\{ (J_{21}^n J_3^p J_{12}^n)'\, \bar{\bar{W}}_0'^{(S)} \right\} & \text{if } m = 0 , \\ 0 & \text{if } m \neq 0 . \end{cases} \quad (23)$$

By using the canonical transformation (18) one can write the expectation value (23) *via* the statistical moments $\langle R_{11}^q \rangle_S$ given in the relation (21).

As it is seen from (18) and (23), in the secular approximation, only the phase fluctuations (but not the amplitude fluctuations) of the laser driving field affect the stationary atomic density matrix and consequently atomic observables and their correlation functions (23). In the case of exact resonance ($\mathrm{ctg}^2\, \varphi = 1$) we have $Z = 1$ for all values γ_c and the phase fluctuations also do not influence the atomic observables and their correlation functions of the form (23). The influence of the phase fluctuations on the steady-state intensities and photon statistics of the spectral components will be considered in the following sections.

3. Steady-state fluorescent spectrum.

In this section we consider the effects that may arise in the collective steady-state fluorescence spectrum due to the phase and amplitude fluctuations.

Since the operator $\mathcal{L}_2$ in master equation (17) commutes with the operators $\mathcal{L}_0$ and $\mathcal{L}_1$ this equation describes a Markov process despite the time-dependent coefficient $X_1(t)$. Thus, the two-time averages may derived from the one-time averages by taking the advantage of the quantum regression theorem [28]. The steady-state spectrum of the fluorescent light is proportional to the Fourier transforms of the following atomic correlation functions

$$\langle \overline{\overline{J_{21}(\tau) J_{12}}} \rangle_S = \lim_{t \to \infty} \langle \overline{\overline{J_{21}(t+\tau) J_{12}(t)}} \rangle . \quad (24)$$

By using the transformation (7) and canonical transformation (11) one finds

$$\begin{aligned} \langle \overline{\overline{J_{21}}} \rangle &= \mathrm{Tr}\ \{\overline{\overline{J_{21}\, \rho(t)}}\} \\ &= \sin\varphi \cos\varphi \langle D_3 \rangle_1 + \\ &\quad + \cos^2\varphi \langle R_{21} \rangle_1 - \sin^2\varphi \langle R_{12} \rangle_1 , \quad (25) \end{aligned}$$

where

$$\langle R_{ij} \rangle_m = \mathrm{Tr}\ \left\{ R_{ij}\, \bar{\bar{W}}_m'(t) \right\} . \quad (26)$$

Equations of motion for $\langle R_{ij} \rangle_m$ can be derived by using master equation (17) and have the following forms

$$\begin{aligned} \frac{d}{dt} \langle D_3 \rangle_m = &- (\gamma_c m^2 + 2 Z_1 + 2 Z_2) \langle D_3 \rangle_m - \gamma_{21} (\sin^2\varphi - \cos^2\varphi) \langle D_3^2 \rangle_m \\ &+ (\sin^2\varphi - \cos^2\varphi) \langle \hat{N}^2 + 2 \hat{N} \rangle_m , \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{d}{dt} \langle R_{21} \rangle_m = &\ 2 i\Omega \langle R_{21} \rangle_m - [\gamma_c m^2 + 4 m \gamma_c (\sin^2\varphi - \cos^2\varphi) + 4 X_0 + 4 X_1(t) + 2 Z_1] \langle R_{21} \rangle_m \\ &- \gamma_{21} (\sin^2\varphi - \cos^2\varphi) \langle D_3 R_{21} \rangle_m , \end{aligned} \quad (28)$$

In the case of exact resonance, $\mathrm{ctg}^2\,\varphi = 1$, equations (27), (28) reduce to the linear differential equations derived by Puri and Hassan [8].

For the one-atom case, one can use the well-known operator relation

$$R_{ij} R_{i'j'} = R_{ij'} \delta_{ji'} \quad (i, j, i', j' = 1, 2)$$

and equations (27), (28) also reduce to the linear differential equations of the paper [12, 18].

For a general case, we use the factorization

$$\langle D_3 R_{ij} \rangle_m = \langle D_3 \rangle_S \cdot \langle R_{ij} \rangle_m . \quad (29)$$

By using the steady-state density matrix (18) one can show that the factorization (29) yields an error of an order of $N^{-1/2}$ in the calculation of the steady-state fluorescent spectrum which can be neglected when N is large [2, 11].

It is easy to see that with factorization (29) the equations (27), (28) have simple exponential solutions. Using the relation (25), the solutions of equations (27), (28), and applying the quantum regression theorem one obtains the following expressions for the correlation function (24)

$$\langle \overline{\overline{J_{21}(\tau)\, J_{12}}} \rangle_S = \sin^2\varphi \,.\, \cos^2\varphi \left(\langle D_3^2 \rangle_S - \frac{\Gamma_a}{\Gamma_0} \right) e^{-\Gamma_0 \tau} + \sin^2\varphi \cdot \cos^2\varphi \cdot \frac{\Gamma_a}{\Gamma_0}$$
$$+ \cos^4\varphi \,\langle R_{21}\, R_{12} \rangle_S\, e^{2 i \Omega \tau - \Gamma_1 \tau - \eta (e^{-\gamma_a \tau} - 1)} + \sin^4\varphi \,\langle R_{12}\, R_{21} \rangle_S\, e^{-2 i \Omega \tau - \Gamma_1 \tau - \eta (e^{-\gamma_a \tau} - 1)} ,$$

where

$$\eta = 16 \sin^2\varphi \cdot \cos^2\varphi \cdot \frac{(\Delta \varepsilon_1)^2}{\gamma_a^2} ,$$

$$\Gamma_0 = 2\,\gamma_{21}(\cos^4\varphi + \sin^4\varphi) + \gamma_c(1 + 4 \sin^2\varphi \,.\, \cos^2\varphi) + \gamma_{21}(\sin^2\varphi - \cos^2\varphi)(N - 2 \langle R_{11} \rangle_S) , \tag{30}$$

$$\Gamma_1 = 2\,\gamma_{21} \cos^2\varphi\,(1 + \sin^2\varphi) + \gamma_c(1 + 5 \sin^4\varphi - 3 \cos^4\varphi)$$
$$+ 16\, \frac{(\Delta \varepsilon_1)^2}{\gamma_a} \sin^2\varphi \,.\, \cos^2\varphi + \gamma_{21}(\sin^2\varphi - \cos^2\varphi)(N \langle R_{11} \rangle_S) \tag{31}$$

$$\Gamma_a = \gamma_{21}(\sin^2\varphi - \cos^2\varphi)(N^2 + 2\,N) \langle D_3 \rangle_S \,. \tag{32}$$

By using the commutation relations (19), (20) one can write

$$\langle D_3^2 \rangle_S = 4 \langle R_{11}^2 \rangle_S - 4\,N \langle R_{11} \rangle_S + N^2 , \tag{33}$$

$$\langle R_{12}\, R_{21} \rangle_S = - \langle R_{11}^2 \rangle_S + (N + 1) \langle R_{11} \rangle_S , \tag{34}$$

$$\langle R_{21}\, R_{12} \rangle_S = - \langle R_{11}^2 \rangle_S + (N - 1) \langle R_{11} \rangle_S + N , \tag{35}$$

where the statistical moments $\langle R_{11} \rangle_S$ and $\langle R_{11}^2 \rangle_S$ are found from equation (21).

The steady-state spectrum of the fluorescent light is proportional to the Fourier transform of the correlation function $\langle \overline{\overline{J_{21}(\tau)\, J_{12}}} \rangle_S$ and has the following form

$$S(\nu) \sim \frac{1}{2} \operatorname{Re} \left[\int_0^\infty e^{-i(\nu - \omega_L)\tau} \langle \overline{\overline{J_{21}(\tau)\, J_{12}}} \rangle_{SS}\, d\tau \right]$$
$$= \sin^2\varphi \cdot \cos^2\varphi \cdot \left(\langle D_3^2 \rangle_S - \frac{\Gamma_a}{\Gamma_0} \right) \cdot \frac{\Gamma_0}{(\nu - \omega_L)^2 + \Gamma_0^2}$$
$$+ \cos^4\varphi \,\langle R_{21}\, R_{12} \rangle_S \cdot e^{\eta} \sum_{n=0}^{\infty} \frac{(-1)^n\, \eta^n}{n\,!} \cdot \frac{\Gamma_1 + n\gamma_a}{(\nu - \omega_L - 2\,\Omega)^2 + (\Gamma_1 + n\gamma_a)^2}$$
$$+ \sin^4\varphi \,\langle R_{12}\, R_{21} \rangle_S\, e^{\eta} \sum_{n=0}^{\infty} \frac{(-1)^n\, \eta^n}{n\,!} \cdot \frac{\Gamma_1 + n\gamma_a}{(\nu - \omega_L + 2\,\Omega)^2 + (\Gamma_1 + n\gamma_a)^2}$$
$$+ \frac{\mu \Gamma_a}{2\,\Gamma_0} \cdot \sin^2\varphi \cdot \cos^2\varphi \cdot \delta(\nu - \omega_L) \,. \tag{36}$$

The steady-state fluorescence spectrum (36) contains three spectral lines centered at $\nu = \omega_L$, $\omega_L \pm 2\,\Omega$. In the off-resonance case, i.e. when $\operatorname{ctg}^2\varphi \neq 1$ the central line at $\nu = \omega_L$ contains the elastic component with the intensity proportional to N^2 and the Lorentzian shaped component with the width Γ_0 and intensity $\sin^2\varphi \,.\, \cos^2\varphi \left(\langle D_3^2 \rangle_S - \frac{\Gamma_a}{\Gamma_0} \right)$. The two sidebands are sums of the Lorentzians of widths $\Gamma_1 + n\gamma_a$, $n = 0, 1, \dots$ centered at frequencies $\nu = \omega_L - 2\,\Omega$ and $\nu = \omega_L + 2\,\Omega$, and having the intensities which are proportional to $\sin^4\varphi \,\langle R_{12}\, R_{21} \rangle_S$

and $\cos^4 \varphi \langle R_{21} R_{12} \rangle_S$ respectively. For off-resonance case and a large number of atoms, as for the non fluctuating laser driving field [2], the widths and intensities of the three inelastic lines are proportional to N (i.e. their peak intensities are independent of N).

In the case of exact resonance ($\mathrm{ctg}^2 \varphi = 1$) one can see that $\Gamma_a = 0$ and the elastic component vanishes. In this case the spectrum (36) is in agreement with the work [8] and is the same as that for $N = 1$, except for the fact that the intensities of all inelastic components are proportional to N^2.

In contrast with the exact resonance case [8] the spectrum (36) is not symmetric whenever $\mathrm{ctg}^2 \varphi \neq 1$. In figure 2 the relative intensities of the two sidebands, i.e. the values $I_{-1}/N = \sin^4 \varphi \langle R_{12} R_{21} \rangle_S / N$ (solid curves) and $I_{+1}/N = \cos^4 \varphi \langle R_{21} R_{12} \rangle_S / N$ (dashed curves) are plotted as functions of the parameter $\mathrm{ctg}^2 \varphi$. As it is clear from figure 2, the intensities of the two sidebands are

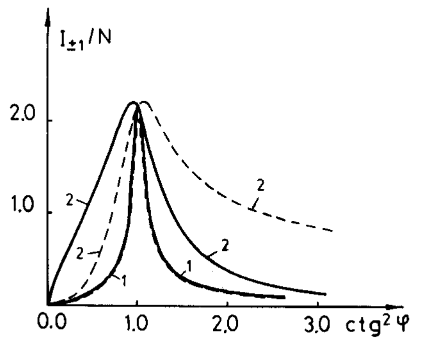

Fig. 2. — Relative intensities I_{+1}/N (dashed curves) and I_{-1}/N (solid curves) as functions of the parameter $\mathrm{ctg}^2 \varphi$ for fixed $N = 50$. Curves (1)-(2) correspond to $\gamma_c/\gamma_{21} = 0$ and 4, respectively.

equal only in the case of exact resonance ($\mathrm{ctg}^2 \varphi = 1$) or in the case of $\gamma_c = 0$ which means that the asymmetry of the spectrum is caused by the phase fluctuations of the laser driving field. We note that the values $I_{\pm 1}$ are the integrated intensities of the two sidebands and the value $I_0 = \sin^2 \varphi \,.\, \cos^2 \varphi \langle D_3^2 \rangle_S$ is the integrated intensity (the sum of elastic and inelastic components) of the central peak which is plotted in figure 3. As it has been noted above, in the off-resonance case (see Figs. 2, 3) the phase fluctuations strongly affect the integrated intensities of the spectral lines while the amplitude fluctuations have no influence whatsoever on these quantities. In the case of exact resonance, the integrated intensities of the spectral lines are the same as these for the case when the phase and amplitude fluctuations of the laser driving field are absent.

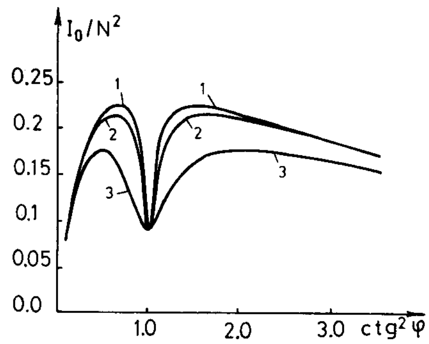

Fig. 3. — Relative intensity I_0/N^2 as a function of the parameter $\mathrm{ctg}^2 \varphi$ for fixed $N = 50$. Curves (1)-(3) correspond to $\gamma_c/\gamma_{21} = 0$; 0.5 ; 4 respectively.

4. Photon statistics of the spectral components.

In this section we discuss the influence of the phase and amplitude fluctuations on the photon statistics of the spectral components. By using the canonical transformation (11) one can write the collective angular momentum of atoms J_{21} in the following form :

$$J_{21} = \sin \varphi \cos \varphi D_3 + \cos^2 \varphi R_{21} - \sin^2 \varphi R_{12} \,. \quad (37)$$

As it is clear from the previous section, the operators $\cos^2 \varphi R_{21}$, $\sin \varphi \cos \varphi \,.\, D_3$ and $-\sin^2 \varphi R_{12}$ can be considered as operator sources of the spectral lines centered at $\nu = \omega_L + 2\,\Omega$, ω_L and $\omega_L - 2\,\Omega$ and for later use these operators will be denoted by S_{+1}^+, S_0^+ and S_{-1}^+, respectively.

We introduce the degree of second-order coherence for the spectral line S_ℓ ($\ell = 0, \pm 1$) in the following form :

$$G_{\ell,\ell}^{(2)} = \frac{\langle S_\ell^+ S_\ell^+ S_\ell S_\ell \rangle_S}{(\langle S_\ell^+ S_\ell \rangle_S)^2} \,. \quad (38)$$

By using the stationary atomic density matrix (18) and commutation relations (19), (20) one can find

$$G_{0,0}^{(2)} = \langle D_3^4 \rangle_S / (\langle D_3^2 \rangle_S)^2 \,, \quad (39)$$

$$G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = \langle R_{12} R_{12} R_{21} R_{21} \rangle_S / (\langle R_{12} R_{21} \rangle_S)^2 \,, \quad (40)$$

where

$$\langle D_3^4 \rangle_S = 16 \langle R_{11}^4 \rangle_S - 32\, N \langle R_{11}^3 \rangle_S + + 24\, N^2 \langle R_{11}^2 \rangle_S - 8\, N^3 \langle R_{11} \rangle_S + N^4 \,, \quad (41)$$

$$\langle R_{12} R_{12} R_{21} R_{21} \rangle_S = \langle R_{11}^4 \rangle_S - 2(N+2) \langle R_{11}^3 \rangle_S + (N^2 + 5\,N + 5) \langle R_{11}^2 \rangle_S - (N^2 + 3\,N + 2) \langle R_{11} \rangle_S \,, \quad (42)$$

the statistical moments $\langle D_3^2 \rangle_S$ and $\langle R_{12} R_{21} \rangle_S$ are found from (33), (34) and $\langle R_{11}^n \rangle_S$ can be found according to equation (21). As it has been mentioned in section 2, the statistical moments $\langle R_{11}^n \rangle_S$ are independent of the amplitude fluctuations and as a consequence the amplitude fluctuations of the laser driving field do not influence the photon statistics of the spectral components.

For the one- atom case, by using the well-known operators relation

$$R_{ij} R_{i'j'} = R_{ij'} \delta_{i'j} \quad (i, j, i', j' = 1, 2),$$

one can obtain

$$G_{0,0}^{(2)} = \langle R_{11} + R_{22} \rangle_S / (\langle R_{11} + R_{22} \rangle_S)^2 = 1,$$
$$G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = 0.$$

Thus, the photon statistics of the central components remains Poissonian and the sidebands have subpoissonian statistics as for the case when the phase fluctuations are absent. In other words, for the one-atom case the phase fluctuations do not affect the photon statistics of the spectral components.

Contrary to the one-atom case, the phase fluctuations strongly affect the photon statistics of the collective spectral components. The degrees of second-order coherence $G_{0,0}^{(2)}$ and $G_{\pm 1,\pm 1}^{(2)}$ as functions of the parameter $\mathrm{ctg}^2\,\varphi$ are plotted in figures 4 and 5, respectively. As it is seen from figures 4, 5, except for the point of exact resonance, i.e. $\mathrm{ctg}^2\,\varphi = 1$, the phase fluctuations of the laser field play an important role in determining photon statistics of particular spectral components in the collective resonance fluorescence. The character of the photon statistics (Poissonian, superpoissonian) can even be changed due to phase fluctuations as it is evident from figure 4.

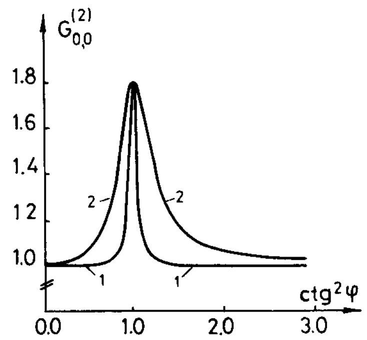

Fig. 4. — Degree of second-order coherence $G_{0,0}^{(2)}$ as a function of the parameter $\mathrm{ctg}^2\,\varphi$ for fixed $N = 50$. Curves (1)-(2) correspond to $\gamma_c/\gamma_{21} = 0$; 4 respectively.

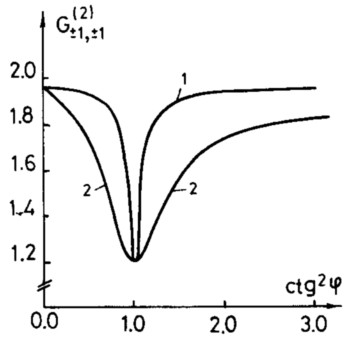

Fig. 5. — Degree of second-order coherence $G_{\pm 1,\pm 1}^{(2)}$ as a function of the parameter $\mathrm{ctg}^2\,\varphi$ for fixed $N = 50$. The curves (1)-(2) correspond to $\gamma_c/\gamma_{21} = 0$; 4, respectively.

5. Conclusions.

We have considered the problem of collective resonance fluorescence in the strong laser field with phase and amplitude fluctuations. The steady-state solution for the atomic density matrix averaged over the phase and amplitude fluctuations has been obtained for the general off-resonance case within the secular approximation. Analytical formulas for the spectrum of resonance fluorescence have been derived. It has been shown that in the off-resonance case the spectrum is asymmetric with an asymmetry which is determined by the phase fluctuations solely. The explicit analytical formulas for the degrees of the second order coherence for particular spectral components have also been obtained. It has been shown that, unlike the one-atom case, in the collective resonance fluorescence the photon statistics of individual spectral components depends strongly on the phase fluctuations of the laser field. It should be noted here that the approach presented in this paper can be used to investigate the influence of the phase and amplitude fluctuations on the two-time intensity-intensity correlation functions, the cross-correlations between the spectral components and the macroscopic nonclassical effects such as squeezing and violation of the Cauchy-Schwarz inequality. The research in this field in progress.

The authors thank Bogolubov for his help and valuable discussions.

References

[1] GROSS, M. HAROCHE, S., *Phys. Rep.* **93** (1982) 301.

[2] COMPAGNO, G., PERSICO, F., *Phys. Rev. A* **25** (1982) 3138.

[3] AGARWAL, G. S., NARDUCCI, L. M., FENG, P. H., GILMORE, R., *Phys. Rev. Lett.* **42** (1979) 1260.

[4] NARDUCCI, L. M., FENG, P. H., GILMORE, R., AGARWAL, G. S., *Phys. Rev. A* **18** (1978) 1571.

[5] DRUMMOND, P. D., *Phys. Rev. A* **22** (1980) 1179.

[6] PURI, R. R., LAWANDE, S. V., *Physica* **120A** (1983) 43.

[7] BOGOLUBOV, N. N., ALISKENDEROV, Jr., E. I., SHUMOVSKY, A. S., TRAN QUANG, *J. Phys. B* **20** (1987) 1885.

[8] PURI, R. R., HASSAN, S. S., *Physica* **120A** (1983) 55.

[9] BOGOLUBOV, N. N., SHUMOVSKY, Jr., A. S., TRAN QUANG, *Phys. Lett.* **112A** (1985) 323 and **122A** (1987) 25.

[10] RAYMER, M. G., WALMSLEY, I. A., MOSTOWSKI, J., SOBOLEVSKA, B., *Phys. Rev. A* **32** (1985) 332.

[11] BOGOLUBOV, N. N., SHUMOVSKY, Jr., A. S., TRAN QUANG, *J. Phys.* **B 20** (1987) 629, *Physica* **144A** (1987) 503.

[12] TANAS, R., KIELICH, S., *Opt. Commun.* **32** (1980) 399.

[13] AGARWAL, G. S., *Phys. Rev. A* **181** (1978) 1490.

[14] EBERLY, J. H., *Phys. Rev. Lett.* **37** (1976) 1387.

[15] AVAN, P., COHEN-TANNOUDJI, C., *J. Phys. B* **10** (1977) 155.

[16] ZOLLER, P., *J. Phys. B* **10** (1977) L 321, **B 11** (1978) 805 and *Phys. Rev. A* **19** (1979) 1151, *A* **20** (1979) 2420.

[17] ZOLLER, P., EHLOTAZLEY, F., *J. Phys. B* **10** (1977) 3023.

[18] GEORGER, A. T., DIXIT, S. N., *Phys. Rev. A* **23** (1981) 2580.

[19] CHATURVEDI, S., GARDINER, C. W., *J. Phys. B* **14** (1981) 1119.

[20] TRIPPENBACH, M., RZAZEWSKI, K., RAYMER, M. G., *J. Opt. Soc. Am. B* **1** (1984) 671.

[21] RAYMER, M. G., MOSTOWSKI, J., CARLSTEN, J. L., *Phys. Rev. A* **19** (1979) 2304.

[22] AGARWAL, G. S., *Opt. Commun.* **35** (1980) 267.

[23] TRIPPENBACH, M., CAO LONG VAN, *JO SA* **B 3** (1986) 879.

[24] HAKEN, H., *Handb. Phys.* **25/2c** (Springer, Berlin, 1970).

[25] VAN KAMPEN, N. G., *Physica* **74** (1974) 215 and 239.

[26] AGARWAL, G. S., *Springer Tracts Mod. Phys.* (Springer, Berlin, 1974).

[27] SCHWINGER, J. V., Quantum Theory of Angular Momentum, Eds. L. C. Biedenharn and H. Van Dam (Academic Press, New York, 1965).

[28] LAX, M., *Phys. Rev.* **172** (1968) 350.

[29] LAWANDE, S. V., PURI, R. R., SOUZA, R. D., *Phys. Rev. A* **33** (1986) 2504.