COLLECTIVE JUMPS IN A SYSTEM OF THREE-LEVEL ATOMS

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The collective jumps in the steady-state intensity of resonance fluorescence and the collective population trapping in a system of three-level atoms interacting with an intense external field are considered. It is shown that for a proper choice of parameters the atomic populations and the steady-state intensity of resonance fluoresence from such a system display discontinuous behaviour (jumps) in a collective limit $N \to \infty$. Potential applications of collective jumps to measure weak transition linewidths are shortly discussed.

Jumps in the collective limit [2–7] and bistability [1 and refs. cited therein] in an atomic system interacting with an electromagnetic field have attracted considerable interest. There have recently been a number of works concentrated on the novel problem of observing quantum jumps in as a single atomic system and applications of such jumps to measure linewidths of weak transitions in spectroscopy [8–15]. Since the weak transition linewidth may be exceptionally narrow, this scheme has been proposed for an ultimate laser frequency standard [12,13].

In this paper we investigate the collective jumps and collective population trapping in a system of three level atoms interacting with an intense external field and discuss potential applications of the collective jumps to measure narrow linewidths of weak transitions.

We consider N three-level atoms in the Λ configuration shown in fig. 1. The states $|1\rangle$ and $|2\rangle$ are the ground and excited states, respectively. The state $|3\rangle$ may be a low-lying vibrational or rotational excitation accessible from the ground state in Raman scattering [6], or it may be a metastable state [13]. In order to keep the discussion general, we will not specify these states and will return to this question later on.

The three-level atoms (Dicke model) interact with one mode of monochromatic driving field of frequency ω_L and with the vacuum of other modes (fig. 1). The external field is assumed to be intense and can be treated classically. By using the rotating wave approximation and Born and Markov approximations one can obtain a master equation for the reduced density matrix ρ of the atomic system alone in the following form [16]

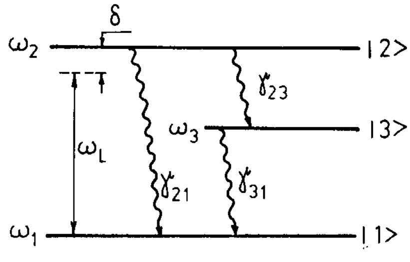

Fig. 1. Level scheme of the atomic system.

$$\begin{aligned}\partial\rho/\partial t = &-\mathrm{i}[(\delta/2)(J_{22}-J_{11})\\ &+G\,(J_{21}+J_{12})-\Omega_3 J_{33},\rho]\\ &-\gamma_{21}(J_{21}J_{12}\rho-J_{12}\rho J_{21}+\mathrm{h.c.})\\ &-\gamma_{23}(J_{23}J_{32}\rho-J_{32}\rho J_{23}+\mathrm{h.c.})\\ &-\gamma_{31}(J_{31}J_{13}\rho-J_{13}\rho J_{31}+\mathrm{h.c.})\equiv L\rho\,, \qquad (1)\end{aligned}$$

where $2\gamma_{kl}(k, l=1, 2, 3)$ are the transition rates from level $|k\rangle$ to $|l\rangle$ due to the atomic interaction with

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the reservoir; $\Omega_3=\omega_{23}-\omega_{21}/2$ (where $\omega_{kl}=\omega_k-\omega_l$, $\hbar\equiv 1$); $\delta=\omega_{21}-\omega_L$ is the detuning of laser frequency from the atomic resonance frequency ω_{21}; $G=-d_{21}E_0$ is the resonance Rabi frequency describing the interaction of the driving field with the atomic system; $J_{kl}(k, l=1, 2, 3)$ are the collective angular momenta of the atomic system having in Schwinger representation [17] the following form

$$J_{kl}=C_k^\dagger C_l ,$$

where the operators C_k and $C_k^\dagger$ obey the boson commutation relation

$$[C_k, C_l^\dagger]=\delta_{kl} ,$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level $|k\rangle$.

Further, we investigate the case of an intense external field or large detuning δ only so that the following relation is fulfilled

$$\Omega=(\tfrac{1}{4}\delta^2+G^2)^{1/2}\gg N\gamma_{kl} . \quad (2)$$

After the canonical transformation

$$C_1=Q_1\cos\zeta+Q_2\sin\zeta ,$$

$$C_2=-Q_1\sin\zeta+Q_2\cos\zeta ,$$

$$C_3=Q_3 , \quad (3)$$

where $\tan 2\zeta=2G/\delta$, and after performing the secular approximation [5–15,18], i.e. ignoring the part of the Liouville operator L containing rapidly oscillating terms with frequencies $nG(n=2, 4)$, one can find a stationary solution of the master equation in the form [19]

$$\tilde{\rho}=U\rho U^\dagger$$

$$=A^{-1}\sum_{P=0}^{N}X^P\sum_{M=0}^{P}Z^M|P, M\rangle\langle M, P| , \quad (4)$$

where U is the unitary operator representing the canonical transformation (3),

$$X=\gamma_{31}/(\gamma_{23}\operatorname{cotan}^2\zeta) , \quad Z=\operatorname{cotan}^4\zeta ,$$

$$A=\frac{Z}{Z-1}\frac{(XZ)^{N+1}-1}{XZ-1}-\frac{1}{Z-1}\frac{X^{N+1}-1}{X-1} . \quad (5)$$

$|P, M\rangle$ is an eigenstate of the operators $R=R_{11}+R_{22}$, R_{11} and the operator of the number of atoms $\hat{N}=R_{11}+R_{22}+R_{33}$, here $R_{kl}=Q_k^\dagger Q_l$ $(k, l=1, 2, 3)$ are the collective angular momenta of "dressed" atoms. The operators Q_k, $Q_k^\dagger$ satisfy the boson commutation relation

$$[Q_k, Q_l^\dagger]=\delta_{kl} , \quad (6)$$

so

$$[R_{kl}, R_{k'l'}]=R_{kl'}\delta_{k'l}-R_{k'l}\delta_{kl'} . \quad (7)$$

As in ref. [19], for later use we introduce the characteristic function

$$\chi_{R_{11},R}(\eta,\zeta)=\langle\exp(\mathrm{i}\eta R_{11}+\mathrm{i}\zeta R)\rangle=A^{-1}$$

$$\times\left[\frac{Y_2}{Y_2-1}\frac{(Y_1-Y_2)^{N+1}-1}{Y_1Y_2-1}-\frac{1}{Y_2-1}\frac{Y_1^{N+1}-1}{Y_1-1}\right],$$

where $Y_1=X\exp(\mathrm{i}\zeta)$, $Y_2=Z\exp(\mathrm{i}\eta)$, and $\langle\ldots\rangle$ denotes the expectation value in the steady state described by the density matrix (4). Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R^n R_{11}^m\rangle$$

$$=\frac{\partial^n}{\partial(\mathrm{i}\zeta)^n}\frac{\partial^n}{\partial(\mathrm{i}\eta)^m}\chi_{R_{11},R}(\eta,\zeta)|_{\mathrm{i}\eta=0,\,\mathrm{i}\zeta=0} .$$

In particular, we have

$$\langle R\rangle=A^{-1}\left(\frac{Z}{Z-1}f_1(XZ)-\frac{1}{Z-1}f_1(X)\right), \quad (8)$$

$$\langle R^2\rangle=A^{-1}\left(\frac{Z}{Z-1}f_2(XZ)-\frac{1}{Z-1}f_2(X)\right), \quad (9)$$

$$\langle R_{11}\rangle=A^{-1}\left[\frac{Z}{Z-1}f_0(XZ)\right.$$

$$\left.-\frac{Z}{(Z-1)^2}\Big(f_0(XZ)-f_0(X)\Big)\right], \quad (10)$$

$$\langle R_{11}^2\rangle=A^{-1}\left[\frac{Z}{Z-1}f_2(XZ)-\frac{2Z}{(Z-1)^2}f_1(XZ)\right.$$

$$\left.+\frac{Z^2+Z}{(Z-1)^3}(f_0(XZ)-f_0(X))\right], \quad (11)$$

$$\langle RR_{11}\rangle = A^{-1}\left[\frac{Z}{Z-1}f_2(XZ) - \frac{Z}{(Z-1)^2}\left(f_1(XZ)-f_1(X)\right)\right], \tag{12}$$

where

$$f_0(\alpha)=(\alpha^{N+1}-1)/(\alpha-1)\,,$$

$$f_1(\alpha)=[N\alpha^{N+2}-(N+1)\alpha^{N+1}+\alpha]/(\alpha-1)^2\,,$$

$$f_2(\alpha)=[N^2\alpha^{N+3}-(2N^2+2N-1)\alpha^{N+2} + (N+1)^2\alpha^{N+1}-\alpha^2]/(\alpha-1)^3\,.$$

Now we discuss the stationary population of the atomic level $|3\rangle$. By using the canonical transformation (3) one can write the number of atoms populating the level $|3\rangle$ in the form

$$N_3=\langle J_{33}\rangle = N-\langle R\rangle\,, \tag{13}$$

where the statistical moment $\langle R\rangle$ can be found according to eq. (8).

First, let us consider the case of $\gamma_{31}/\gamma_{23}<1$. By using the relations (13) and (8) one can show that:

(i) For $\gamma_{31}/\gamma_{23}<1$, $\gamma_{31}/\gamma_{23}<\mathrm{cotan}^2\zeta<\gamma_{23}/\gamma_{31}$ (i.e. when $X<1$, $XZ<1$) and $N\gg 1$ so that N^N, $(XZ)^N<N^{-1}$, almost all of the atoms are populated on the level $|3\rangle$, $N_3\approx N$, thus the atomic level $|3\rangle$ plays a role of a "trap" for the atoms.

(ii) For $\mathrm{cotan}^2\zeta<\gamma_{31}/\gamma_{23}<1$ (i.e. when $X>1$, $XZ<1$) and $N\gg 1$ so that $X^N\gg 1$, $(XZ)^N<N^{-1}$ and in the case of $\mathrm{cotan}^2\zeta>\gamma_{23}/\gamma_{31}>1$ (i.e. when $X<1$, $XZ>1$) and for $N\gg 1$ so that $X^N<N^{-1}$, $(XZ)^N\gg 1$ the population of the level $|3\rangle$ is small in comparison with N.

In the points where $\mathrm{cotan}^2\zeta=\gamma_{31}/\gamma_{23}$ or $\mathrm{cotan}^2\zeta=\gamma_{23}/\gamma_{31}$, nearly half of the atoms ($N_3\approx N/2$) is populated on the level $|3\rangle$.

The jump-like behaviour of the atomic population on the level $|3\rangle$ (per atom) i.e. the quantity N_3/N, is plotted in fig. 2 as a function of the parameter $\mathrm{cotan}^2\zeta$ for $\gamma_{31}/\gamma_{23}=0.5$. In the collective limit $N\to\infty$ (the dotted curve) the function N_3/N has a discontinuous behaviour (jump) at the critical points $\mathrm{cotan}^2\zeta=\gamma_{31}/\gamma_{23}$ and $\mathrm{cotan}^2\zeta=\gamma_{23}/\gamma_{31}$.

In a similar manner one can show that in the case

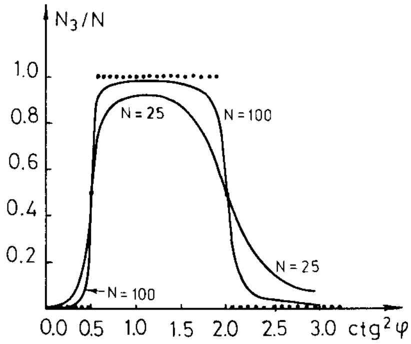

Fig. 2. Population (per atom) of the level $|3\rangle$ as a function of $\mathrm{cotan}^2\zeta$ for $\gamma_{31}/\gamma_{23}=0.5$. The dotted curve illustrates the case $N\to\infty$.

$\gamma_{31}/\gamma_{23}>1$ and for $N\gg 1$ the population of the level $|3\rangle$ is small compared to N for all values of the parameter $\mathrm{cotan}^2\xi$, thus in this case the collective jump in the function N_3/N is absent. In the case when $\gamma_{31}/\gamma_{23}=1$ we have

$$\begin{aligned} N_3/N &\overset{N\to\infty}{=} 0, && \text{if } \mathrm{cotan}^2\zeta>1,\\ &\overset{N\to\infty}{=} 1/3, && \text{if } \mathrm{cotan}^2\zeta=1,\\ &\overset{N\to\infty}{=} 0, && \text{if } \mathrm{cotan}^2\zeta<1, \end{aligned}$$

thus in the collective limit $N\to\infty$ the function N_3/N shows discontinuous behaviour at the critical point $\mathrm{cotan}^2\zeta$.

We note that in the one atom case the level $|3\rangle$ is the "trap" of the atom, i.e. $N_3/N\to 1$, only in the case of $\gamma_{31}/\gamma_{23}\to 0$.

The effects of the collective population trapping and collective jump of the atomic population of the level $|3\rangle$ strongly affect the behaviour of the stationary intensity I of the fluorescent field due to the atomic transition $|2\rangle\to|1\rangle$. The explicit form for the intensity I can be found by applying the canonical transformation (3) and the stationary density matrix solution (4)

$$I\sim\langle J_{21}J_{12}\rangle = \sin^2\zeta\cos^2\zeta\langle(R-2R_{11})^2\rangle + \cos^4\zeta\langle R_{21}R_{12}\rangle + \sin^4\zeta\langle R_{12}R_{21}\rangle\,, \tag{14}$$

where

$$\langle(R-2R_{11})^2\rangle = \langle R^2\rangle + 4\langle R_{11}^2\rangle - 4\langle RR_{11}\rangle\,, \tag{15}$$

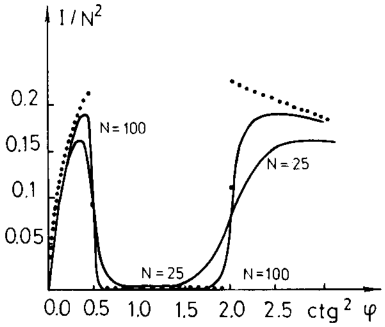

Fig. 3. Scaled intensity of fluorescent light I/N^2 as a function of $\cot\mathrm{an}^2\zeta$ for $\gamma_{31}/\gamma_{23}=0.5$. The dotted curve illustrates the case $N\to\infty$.

$$\langle R_{21}R_{12}\rangle=\langle R\rangle-\langle R_{11}\rangle+\langle RR_{11}\rangle-\langle R_{11}^2\rangle, \quad (16)$$

$$\langle R_{12}R_{21}\rangle=\langle R_{11}\rangle+\langle RR_{11}\rangle-\langle R_{11}^2\rangle . \quad (17)$$

In the equations (15)–(17), the statistical moments $\langle R\rangle$, $\langle R^2\rangle$, $\langle R_{11}\rangle$, $\langle R_{11}^2\rangle$ and $\langle RR_{11}\rangle$ can be found according to relations (8)–(12). By using the equations (14)–(17) one can show that in the case of $\gamma_{31}/\gamma_{23}<1$, $\gamma_{31}/\gamma_{23}<\mathrm{cotan}^2\zeta<\gamma_{23}/\gamma_{31}$ and $N\gg 1$ (the condition (i)), i.e., when atoms are "trapped" on the level $|3\rangle$, the intensity I is independent of the numbers of atoms N. For all other values of the parameters γ_{31}/γ_{23} and $\mathrm{cotan}^2\zeta$ the intensity I is proportional to N^2. The jump-like behaviour of the quantity I/N^2 as a function of parameter $\mathrm{cotan}^2\zeta$ for $\gamma_{31}/\gamma_{21}=0.5$ is plotted in fig. 3, where the dotted curve indicates the collective limit $N\to\infty$. As it is seen from fig. 3, in the case of $\gamma_{31}/\gamma_{23}<1$ and $n\to\infty$ the function I/N^2 shows discontinuous behaviour at the critical points $\mathrm{cotan}^2\zeta=\gamma_{31}/\gamma_{23}$ and $\mathrm{cotan}^2\zeta=\gamma_{23}/\gamma_{31}$.

The collective jump in the atomic population of the level $|3\rangle$ and the intensity of the fluorescence field is caused only by the collective interaction between atoms and the driving field and it could be used to measure the narrow linewidth of the weak transition.

Let the level $|3\rangle$ be a metastable state, the transition $|3\rangle\to|1\rangle$ is forbidden and other transitions $|2\rangle\to|1\rangle$ and $|2\rangle\to|3\rangle$ are allowed transitions [13]. It has been argued that the weak transition $|3\rangle\to|1\rangle$, which is difficult to detect, could be monitored by the scattered light of the strong transition $|2\rangle\to|1\rangle$. Changing the parameter $\mathrm{cotan}^2\zeta$, i.e. changing the detuning δ or intensity of the external field one can observe the jump (see fig. 3) in the intensity of the fluorescence corresponding to the strong transition $|2\rangle\to|1\rangle$ at the critical points $\mathrm{cotan}^2\zeta=\gamma_{31}/\gamma_{23}$, $\mathrm{cotan}^2\zeta=\gamma_{23}/\gamma_{31}$, and this allows us in principle to measure the quantity γ_{31}.

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