Squeezed states in the transient regime of resonance fluorescence

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Resonance fluorescence from a two-level atom is first shown to permit squeezing for an arbitrary intensity of the exciting field provided that the resonance is time dependent. When the exciting field increases in intensity, the squeezing shifts toward shorter times and its maximum value becomes double, for strong exciting fields, its maximum value in the steady state. Moreover, it is shown that intervals of time exist in which the normally ordered variance of the fluorescence field is positive, despite the presence of squeezing in the atomic variables. Also, the signal-to-noise ratio in the transient regime attains high values, predictably making the observation of squeezed states experimentally feasible.

1. INTRODUCTION

The fluorescent radiation field from a two-level atom interacting with a coherent-laser field is known to display two distinct quantum features. First, the fluorescent photons exhibit antibunching and, second, the fluorescent radiation occurs in a squeezed quantum state under appropriate conditions.

Photon antibunching in resonance fluorescence has been theoretically predicted by Carmichael and Walls¹ and Kimble and Mandel² and observed in experiment by Kimble et al.^{3,4} The antibunching effect reflects the existence of discrete objects, i.e., the photons, and may be directly understood as resulting from the fact that, immediately after emission of a photon, the atom is found in its ground state and a subsequent photon emission can occur only after a finite recovery time.

Squeezing, on the other hand, is a phase-sensitive effect that describes quite another feature of the quantized electromagnetic field. This effect consists in the reduction of fluctuations in one of the two quadrature components (the inphase and the out-of-phase components) of the fluorescent field at the expense of increased fluctuations in the other component. Squeezing in resonance fluorescence was recently discussed by Walls and Zoller⁵ and Mandel.⁶ Squeezed states have a negative normally ordered variance of one of the two field components, which means that these states have no classical analog since their diagonal coherent-state representation cannot be nonnegative. 7,8 Walls and Zoller and Walls9 have shown that either of the fluorescent-field components can become squeezed under certain conditions in the steady state if the Rabi frequency Ω of the exciting field is sufficiently low. To satisfy this condition, when the exciting field is perfectly tuned to the atomic transitions, the Rabi frequency Ω has to be less than $\sqrt{2}\gamma$, with $2\gamma = A$ (the Einstein coefficient for spontaneous emission). Off-resonance excitation further lowers this critical value of Ω . Walls and Zoller⁵ have also pointed out, without giving the details, that squeezing can double in the transient regime of resonance fluorescence. Arnoldus and Nienhuis¹⁰ have considered the conditions for squeezing in steady-state resonance fluorescence showing that the condition for the presence of squeezed states corresponds to the requirement that the intensity of the coherent Rayleigh line be more than half of the total intensity of the fluorescence radiation.

Mandel⁶ has shown that the use of a one-port homodyne detection scheme to detect squeezing in steady-state resonance fluorescence always leads to sub-Poissonian photon statistics if the detected component is squeezed. Loudon¹¹ recently considered the two-time photon-number correlation function resulting in this homodyne technique. Yuen and Chan¹² have proposed a new homodyne detection scheme that allows elimination of the local oscillator noise, thus increasing the chances for the detection of squeezed states.

In this paper, we consider the possibility of obtaining and recording squeezed states in the transient regime of resonance fluorescence from a coherently driven two-level atom. We show that, in the transient regime, contrary to the steady-state regime, there is no restriction on the required values of Ω . Therefore squeezing can occur for any value of the Rabi frequency of the exciting field, provided that the interval of time is chosen appropriately. As the intensity of the exciting field increases, the squeezing shifts toward shorter times and its maximum value increases to double, for strong exciting fields $(\Omega = 200\gamma)$, its steady-state value. However, this takes place only if the atom started in its ground state and if the initial phase of the exciting field was zero. The dependence of squeezing on the initial phase of the laser field is discussed for both the steady-state and the transient regimes. It is found that certain intervals of time exist in which the normally ordered variance of the fluorescent field is positive, despite the fact that squeezing is exhibited during the same intervals of time by the relevant atomic variables. The signal-to-noise ratio (SNR) is also considered for the fluorescent field. It is shown that this ratio attains high values in the transient regime, in contrast to low ratio values in the steady-state regime of resonance fluorescence.

2. SQUEEZED STATES OF ATOMIC OPERATORS

To define squeezed states in resonance fluorescence of a two-level atom driven by a coherent-laser field, we use pseudospin operators that satisfy the well-known commutation relations

$$[S^{+}(t), S^{-}(t)] = 2S_{3}(t),$$

$$[S_{3}(t), S^{\pm}(t)] = \pm S^{\pm}(t),$$
(1)

where $S^{\pm}(t)$ are the raising (lowering) operators of the energy of the atom, whereas $S_3(t)$ describes its energy.

Defining the Hermitian operators

$$S_1(t) = \frac{1}{2}[S^+(t) + S^-(t)],$$

$$S_2(t) = -\frac{i}{2}[S^+(t) - S^-(t)],$$
(2)

we have

$$[S_1(t), S_2(t)] = iS_3(t).$$
 (3)

A squeezed state is then characterized by the condition ${\rm that}^5$

$$\langle \left[\Delta S_1(t) \right]^2 \rangle < \frac{1}{2} |\langle S_3(t) \rangle|$$

 \mathbf{or}

$$\langle [\Delta S_2(t)]^2 \rangle < \frac{1}{2} |\langle S_3(t) \rangle|. \tag{4}$$

The time dependence of the expectation values of the slowly varying atomic operators $S^{\pm}(t)$ and $S_3(t)$ is well documented^{2,13} and has the form

$$\langle S^{-}(t) \rangle = \frac{-\frac{1}{2}\beta \exp(-i\varphi)(1 - i\Delta)}{(\frac{1}{4} + \frac{1}{4}\Delta^{2} + 2\beta^{2})} + 2\beta \exp(-i\varphi) u_{0} \sum_{\substack{i=1\\i\neq j\neq k}}^{3} \frac{[z_{i} + \frac{1}{2}(1 - i\Delta)]\exp(z_{i}t)}{(z_{i} - z_{j})(z_{i} - z_{k})} - \beta \exp(-i\varphi) \sum_{\substack{i=1\\i\neq j\neq k}}^{3} \frac{(z_{i} + 1)[z_{i} + \frac{1}{2}(1 - i\Delta)]\exp(z_{i}t)}{z_{i}(z_{i} - z_{j})(z_{i} - z_{k})},$$

$$\langle S_{3}(t) \rangle = -\frac{\frac{1}{8}(1 + \Delta^{2})}{(\frac{1}{4} + \frac{1}{4}\Delta^{2} + 2\beta^{2})}$$
(5a)

$$\langle S_3(t) \rangle = -\frac{\frac{\frac{1}{2}(1+\Delta^2)}{(\frac{1}{4}+\frac{1}{4}\Delta^2+2\beta^2)}}{+2\beta^2 \sum_{\substack{i=1\\i\neq j\neq k}}^3 \frac{(z_i+\frac{1}{2})\exp(z_it)}{z_i(z_i-z_j)(z_i-z_k)}} + u_0 \sum_{\substack{i=1\\i\neq j\neq k}}^3 \frac{[(z_i+\frac{1}{2})^2+\frac{1}{4}\Delta^2]\exp(z_it)}{(z_i-z_j)(z_i-z_k)},$$
 (5b)

where z_i (i = 1, 2, 3) are the three roots (assumed unequal) of the cubic equation

$$(z+1)[(z+\frac{1}{2})^2 + \frac{1}{4}\Delta^2] + 4\beta^2(z+\frac{1}{2}) = 0.$$
 (6)

In Eqs. (5) we have used the notation

$$\beta = \Omega/4\gamma, \qquad \Delta = \frac{\omega_L - \omega_0}{\gamma}, \qquad u_0 = \langle S^+(0)S^-(0) \rangle, \quad (7)$$

where Ω is the Rabi frequency of the driving field. Ω is real since the factor $\exp(-i\varphi)$, which includes the initial phase of the laser field, is excluded. Δ is the detuning of the laser frequency ω_L from the atomic resonance ω_0 , whereas u_0 is the initial population of the excited state of the atom. If the atom is initially in its ground state, then $u_0=0$, whereas $u_0=1$ if the atom starts in its excited state.

By definition (4) and Eqs. (5), we find that

$$F_{1}(t) = \langle [\Delta S_{1}(t)]^{2} \rangle - \frac{1}{2} | \langle S_{3}(t) \rangle |$$

$$= \frac{1}{4} - \frac{1}{4} \left(\frac{-\beta(\cos \varphi - \Delta \sin \varphi)}{(\frac{1}{4} + \frac{1}{4}\Delta^{2} + 2\beta^{2})} \right)$$

$$+ \left\{ \beta \exp(-i\varphi) \sum_{\substack{i=1\\i\neq j\neq k}}^{3} \frac{[2u_{0}z_{i} - (z_{i} + 1)][z_{i} + \frac{1}{2}(1 - i\Delta)]}{z_{i}(z_{i} - z_{j})(z_{i} - z_{k})} \right\}$$

$$\times \exp(z_{i}t) + \text{c.c.}$$

$$= \frac{1}{2} \left(\frac{1}{4} + \frac{1}{4}\Delta^{2} + 2\beta^{2} + \sum_{\substack{i=1\\i\neq j\neq k}}^{3} \sum_{\substack{i=1\\i\neq j\neq k}}^{4} \frac{1}{4} \right)$$

$$\times \frac{u_{0}z_{i}[(z_{i} + \frac{1}{2})^{2} + \frac{1}{4}\Delta^{2}] + 2\beta^{2}(z_{i} + \frac{1}{2})}{z_{i}(z_{i} - z_{j})(z_{i} - z_{k})} \exp(z_{i}t) \right\},$$

$$= \frac{1}{4} \left(\frac{1}{4} + \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2}}{2} \right) + \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2}}{2} + \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2}} + \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2}}{2} + \frac{1}{4}\Delta^{2} + 2\beta^{2} - \frac{1}{4}\Delta^{2}}{2} + \frac{1}{4}\Delta^{2} + 2\beta^{2}(z_{i} + \frac{1}{4}z_{j})} \exp(z_{i}t) \right\}.$$

$$\times \exp(z_{i}t) - \text{c.c.}$$

The time dependence of $F_1(t)$ is illustrated graphically in Figs. 1–3 for $\Delta=0$ and various values of the parameters β , φ , and u_0 . It is seen from Fig. 1 that, for a weakly exciting field $(\beta=0.2)$ and $\varphi=0$, $F_1(t)$ is always negative if the atom was in its ground state at t=0. If the atom was in its excited state at t=0, $F_1(t)$ is positive for short times and, after reaching a peak, decreases to become negative for very long times. A change in phase by $\pi/2$ causes an interchange of $F_1(t)$ and $F_2(t)$, as is obvious from Eqs. (8) and (9).

As the intensity of the laser field increases, squeezing in $F_1(t)$ shifts to the region of shorter times, and $F_1(t)$ itself shows an oscillatory behavior reflecting the Rabi oscillations. This is shown in Figs. 2 and 3.

The maximum value of squeezing that can be obtained in the transient regime becomes greater than the steady-state maximum, marked by the dashed lines in Figs. 2 and 3. The amount of squeezing increases as the laser field becomes in-

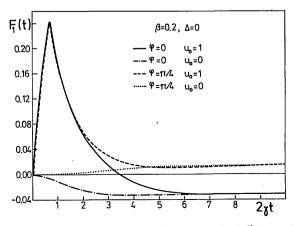


Fig. 1. Plot of $F_1(t)$ versus time t [in units of $(2\gamma)^{-1}$] for various values of the parameters on which $F_1(t)$ is dependent.

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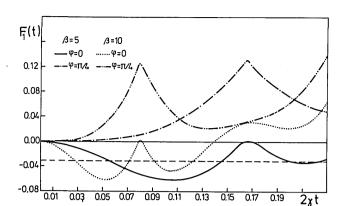


Fig. 2. Plot of $F_1(t)$ versus time t for strong exciting field. Dashed line marks the maximum value of squeezing in steady state. Atom starts in its ground state: $u_0 = 0$.

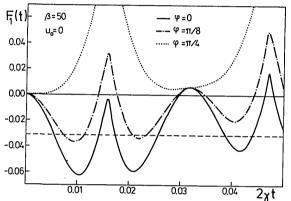


Fig. 3. The same as Fig. 2, except for an extremely strong exciting field $(\beta = 50)$.

creasingly stronger and reaches its limiting value $F_1(t) = -\frac{1}{16}$ at the time t=0.0105 for $\beta=50$ and $u_0=\varphi=0$. This is the absolute maximum of squeezing [the absolute minimum of $F_1(t)$] that can be obtained in one-atom resonance fluorescence. This limit is related to the fact that $\langle S^{\pm}(t) \rangle$ and $\langle S_3(t) \rangle$ in the case of a two-level atom can vary within a strictly bounded range of values only.

For steady-state resonance fluorescence, we have by Eqs. (8) and (9)

$$F_{1,2}(\infty) = \frac{2\beta^2}{(1+\Delta^2+8\beta^2)} \left[8\beta^2 \mp (1-\Delta^2) \cos 2\varphi \pm 2\Delta \sin 2\varphi \right].$$

(10)

For $\varphi=0$, this formula goes over into that derived by Walls and Zoller.⁵ It is clear from Eq. (10) that a change by $\pi/2$ in the initial phase of the exciting field interchanges F_1 and F_2 . At this point, we should also note that, because of our different choice of phase, our F_1 corresponds to what would be F_2 for Walls and Zoller.⁵ Maximum squeezing in steady-state resonance fluorescence occurs for $\Delta=\varphi=0$ and $\beta^2=\frac{1}{24}$, when $F_1=-\frac{1}{32}$. This value, compared with the $-\frac{1}{16}$ derived above, is one half of that obtained by us for transient resonance fluorescence, as already predicted by Walls and Zoller.⁵ From the experimental point of view, however, it is interesting to note that, in steady-state resonance fluorescence, squeezing occurs in the F_1 component for $8\beta^2+\Delta^2<1$ and in the F_2 component for $\Delta^2>8\beta^2+1$, i.e., in the weak-field limit.⁹

Similar conditions for squeezing hold for the case of two interacting atoms. 14

For time-dependent resonance fluorescence, as is shown in Figs. 1-3, squeezing occurs for all values of the incident field intensity.

3. SQUEEZING IN THE FLUORESCENT FIELD

By defining the in-phase component E_1 and the out-of-phase component E_2 of the fluorescent-field amplitude as the positive- (negative-) frequency parts $E^{(+)}$ ($E^{(-)}$),

$$E_1 = E^{(+)} + E^{(-)},$$

 $E_2 = -i(E^{(+)} - E^{(-)}),$ (11)

we obtain the communication rule

$$[E_1, E_2] = 2iC, (12)$$

where C is a positive C number if the field is described by boson operators. Squeezed states of the fluorescent field are defined by the requirement that the variance of one of two noncommuting observables $(E_1 \text{ or } E_2)$ shall be less than C, i.e.,

$$\langle (\Delta E_1)^2 \rangle < C$$
 or $\langle (\Delta E_2)^2 \rangle < C$. (13)

Since the following relations hold for the field operators:

$$\langle (\Delta E_1)^2 \rangle = C + \langle : (\Delta E_1)^2 : \rangle,$$

$$\langle (\Delta E_2)^2 = C + \langle : (\Delta E_2)^2 : \rangle, \tag{14}$$

where the colon stands for normal ordering of the operators, the squeezing conditions [relations (13)] imply negative values of the normally ordered variance for the squeezed-field component. This condition may be written as

$$\langle : (\Delta E_1)^2 : \rangle = \int (\Delta \operatorname{Re} \mathscr{E})^2 P(\mathscr{E}) d^2 \mathscr{E}, \tag{15}$$

where $P(\mathcal{E})$ is the Glauber P representation for the fluorescent field with the complex amplitude \mathcal{E} . Hence we see that squeezed states have no classical analog in the sense that their diagonal coherent-state representation cannot be nonnegative.

The normally ordered variance of the fluorescent field may be derived by using the following relation between the radiation field and the atomic operators in the far-field limit^{2,15}:

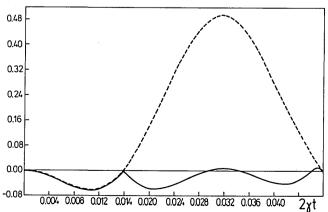


Fig. 4. The normally ordered variance $\langle :(\Delta E_1(t))^2 : \rangle/\psi^2(R)$ (dashed curve) and $F_1(t)$ (solid curve) for $\beta = 50$, $\Delta = u_0 = 0$ and $\varphi = 0$. For $0 \le 2\gamma t \le 0.015$ the two curves merge.

$$E^{(+)}(\mathbf{R},t) = E_0^{(+)}(\mathbf{R},t) + \frac{1}{2}\psi(\mathbf{R})S^{-}\left(t - \frac{R}{C}\right), \quad (16)$$

with $\psi^2(R) = (3\hbar k \gamma/2R^2) \sin^2 \theta$, where θ is the angle between the observation direction **R** and the atomic transition dipole moment μ .

According to Eq. (16), the normally ordered variance of the fluorescent field in any direction other than that of the laser beam is given by

$$\langle : (\Delta E_1(t))^2 : \rangle = \psi^2(R) \{ \langle [\Delta S_1(t)]^2 \rangle + \frac{1}{2} \langle S_3(t) \rangle \},$$

$$\langle : (\Delta E_2(t))^2 : \rangle = \psi^2(R) \{ \langle [\Delta S_2(t)]^2 \rangle + \frac{1}{2} \langle S_3(t) \rangle \}.$$
(17)

In the steady state $\langle S_3(\infty) \rangle < 0$, i.e., $\langle S_3(\infty) \rangle = -|\langle S_3(\infty) \rangle|$, and squeezing in the atomic variables given by condition (4) implies a negative value of the normally ordered variance of the corresponding component of the fluorescent field. In the transient regime, however, $\langle S_3(t) \rangle$ evolves in time and can take positive as well as negative values. It is obvious that intervals of time exist in which the atomic squeezing does not necessarily lead to a nonclassical nature of the emitted field (Fig. 4). The reduction of the atomic Hilbert space to two states determines, by way of the relation [Eq. (16)], the operator nature of the fluorescent field that causes, in the transient regime, squeezing in the atomic variables, which does not necessarily signify negative values of the normally ordered variance.

4. SIGNAL-TO-NOISE RATIO

Recently, a great deal of research has been directed toward the design of practical schemes that would make the detection of squeezed states experimentally feasible. Since squeezing is phase dependent, homodyne or heterodyne schemes are preferred. Mandel⁶ has shown that the one-port homodyne detection scheme always yields sub-Poissonian photon statistics if the detected component of the field is squeezed. This scheme, however, has one important drawback: It introduces the local oscillator noise. In this context, the two-port homodyne scheme proposed by Yuen and Chan,12 which is insensitive to the local-oscillator noise, is especially promising. The quality of a particular detection scheme is often characterized by its SNR. In order to circumvent various sources of noise that can intrude in the detection process itself, one can introduce the intrinsic SNR characterizing the relative fluctuations of the field

$$SNR = |\langle E_1 \rangle| / \sqrt{\langle (\Delta E_1)^2 \rangle}. \tag{18}$$

For time-dependent resonance fluorescence, this parameter is given by the corresponding atomic variables

$$SNR = |\langle S_1(t)\rangle|/\sqrt{\langle (\Delta S_1(t))^2\rangle}.$$
 (19)

According to definition (4), a squeezed state must satisfy the inequality

$$SNR = \frac{\left| \langle S_1(t) \rangle \right|}{\sqrt{\langle \left[\Delta S_1(t) \right]^2 \rangle}} > \frac{\left| \langle S_1(t) \rangle \right|}{\left[\frac{1}{2} \right] \langle S_3(t) \rangle \left| \right]^{1/2}}.$$
 (20)

This means that even a high value of the SNR does not necessarily mean squeezing if the right-hand side of inequality (20) is high. In fact, for times t for which $\langle S_3(t) \rangle = 0$, we have infinity on the right-hand side; however, in this case, the

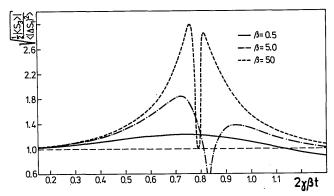


Fig. 5. The excess SNR versus $2\gamma\beta t$ for various strengths of the exciting field.

quantities $S_1(t)$ and $S_2(t)$ can be measured simultaneously with arbitrary accuracy, and the problem of squeezing does not arise at all. So a measure of squeezing in resonance fluorescence can be provided by the excess SNR:

$$\frac{\text{SNR}}{\frac{|S_1(t)\rangle|}{[\frac{1}{2}|\langle S_3(t)\rangle|]^{1/2}}} = \left[\frac{\frac{1}{2}|\langle S_3(t)\rangle|}{\langle (\Delta S_1(t))^2\rangle}\right]^{1/2}.$$
 (21)

Whenever the ratio [Eq. (21)] is greater than unity, the variable $S_1(t)$ is squeezed.

For steady-state resonance fluorescence, expressions (4), (5), and (21) at $\varphi = 0$ lead to

$$\left[\frac{\frac{1/2|\langle S_3(\infty)\rangle|}{\langle (\Delta S_1(\infty))^2 \rangle} \right]^{1/2} = \frac{\frac{1/2[(1+\Delta^2)(\frac{1}{4}+\frac{1}{4}\Delta^2+2\beta^2)]^{1/2}}{[\frac{1}{8}(\frac{1}{2}+\Delta^2)+(\frac{1}{4}\Delta^2+2\beta^2)^2]^{1/2}} \cdot$$
(22)

The ratio [Eq. (22)] is greater than unity for $8\beta^2 + \Delta^2 < 1$ and takes its maximum value 1.105 for $\Delta = 0$, $\beta^2 = \frac{1}{8}(\sqrt{2} - 1)$.

In the transient regime, the ratio [inequality (21)] can attain much greater values, as is shown in Fig. 5, where this ratio is plotted as a function of time for $\Delta=0=\varphi$ and various values of the field strength β (note the different time scales for different β). For strong fields, the time dependence of this ratio displays a pronounced peak with values considerably greater than unity and then a narrow dip. When designing an experiment to measure this ratio directly, it would be advantageous to use strong fields for the detection of squeezed states. This, however, would require sufficiently short detection times (of the order of Ω^{-1}). It should also be noted that the maximum of this ratio does not coincide with the maximum of squeezing defined by Eq. (8). This is because $\langle S_3(t) \rangle$ also changes in time.

5. CONCLUSION

We have studied the problem of the occurrence of squeezed states in time-dependent resonance fluorescence from a two-level atom interacting with a resonant laser field. Our results show that in the transient regime of resonance fluorescence there is no restriction on the values of the Rabi frequency Ω for which squeezing can occur. As the Rabi frequency of the driving field increases, the region of squeezing shifts toward shorter times and the maximum of squeezing that can be obtained increases. For strong driving fields it can attain a value two times greater than the steady-state

value, provided that the atom started in its ground state and that the initial phase of the driving field was zero.

We have also shown that, in the transient regime, certain time intervals exist in which squeezing in the atomic variables does not proceed in step with the negative values of the normally ordered variance of the fluorescent field.

Our excess SNR, which can be a useful parameter to describe squeezing, is also considered. This parameter is slightly greater than unity in the steady state and can be considerably greater than unity in the transient regime, as is evident from Fig. 5.

We conclude that some new properties of the fluorescent field that are not met in the steady state appear in the transient regime of resonance fluorescence. We infer that it should be easier to detect squeezed states in the transient regime. This, however, would require a fast detection technique, such as the picosecond technique.

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