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SQUEEZED STATES OF AN ANHARMONIC OSCILLATOR

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In the present paper the very simple model of an anharmonic oscillator accessible to strict solution and, nonetheless, offering the opportunity of obtaining squeezed states, is considered.

The Hamiltonian of the system is taken on discarding the non-energy-conserving terms as:

$$H = \hbar\omega a^{+}a + \hbar k a^{+2}a^{2} \quad , \tag{1}$$

where k is the anharmonicity parameter assumed to be real and the creation and annihilation operators are taken in normal order. This Hamiltonian describes, for example, a single mode of the electromagnetic field interacting with a non-absorbing nonlinear medium. In this case the anharmonicity parameter k is related to the third-order susceptibility of the medium (Drummond and Walls, 1980) by the equation:

$$\hbar k = 6(\pi\hbar\omega)^{2}\int d^{3}r\, \chi^{(3)}(r)\,|u(r)|^{4} \quad , \tag{2}$$

where u(r) is the cavity mode function.

According to (1) , the Heisenberg equation of motion for the annihilation operator a reads

$$\dot{a} = -\frac{i}{\hbar}[a, H] = -i(\omega + 2k a^{+}a)\,a \quad . \tag{3}$$

Since a^+a is a constant of motion, equation (3) has the solution

$$a(t) = \exp\left[-it(\omega + 2k a^+(0) a(0))\right] a(0) \quad . \tag{4}$$

The obvious $e^{-i\omega t}$ dependence (the free evolution) will be dropped in the following.

With the solution (4) available, one can easily answer the question of squeezing in such a system. Introducing the operators $Q = a^+ + a$ and $P = -i(a - a^+)$ which describe the in-phase and out-of-phase components of the field and obey the commutation relation

$$[Q, P] = 2i \ , \tag{5}$$

the squeezing condition can be written (Walls and Zoller, 1981; Mandel, 1982) as

$$\langle :(\Delta Q)^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta P)^2: \rangle < 0 \ , \tag{6}$$

where $\Delta Q = Q - \langle Q \rangle$, $\Delta P = P - \langle P \rangle$, and the colon stands for normal ordering of the operators.

If the oscillator at t=0 starts in a coherent state $|\alpha\rangle$ such that $a(0)|\alpha\rangle = \alpha|\alpha\rangle$, with the mean number of photons $n = |\alpha|^2$, on insertion of (4) into (6) and taking the expectation value in this state we have:

$$\langle :(\Delta Q(\tau))^2: \rangle = 2\,\mathrm{Re}\Big\{\alpha^2 \exp\left[-i\tau + n(e^{-2i\tau} - 1)\right]$$

$$-\alpha^2 \exp\left[2n(e^{-i\tau} - 1)\right]\Big\} + 2n\Big\{1 - \exp\left[2n(\cos\tau - 1)\right]\Big\} , \tag{7}$$

$$\langle :(\Delta P(\tau))^2: \rangle = -2\,\mathrm{Re}\{\cdots\cdots\} + 2n\{\cdots\cdots\} , \tag{8}$$

with $\tau = 2kt$. The expressions in the parentheses of formula (8) are the same as those of formula (7). If the expression (7) or (8) is negative it means squeezing in the in-phase or in the out-of-phase component of the field at time t. Both formulae are illustrated graphically in Fig.1 as a function of the product $n\tau$ for $\tau = 1 \times 10^{-6}$. The initial phase is chosen to have α real.

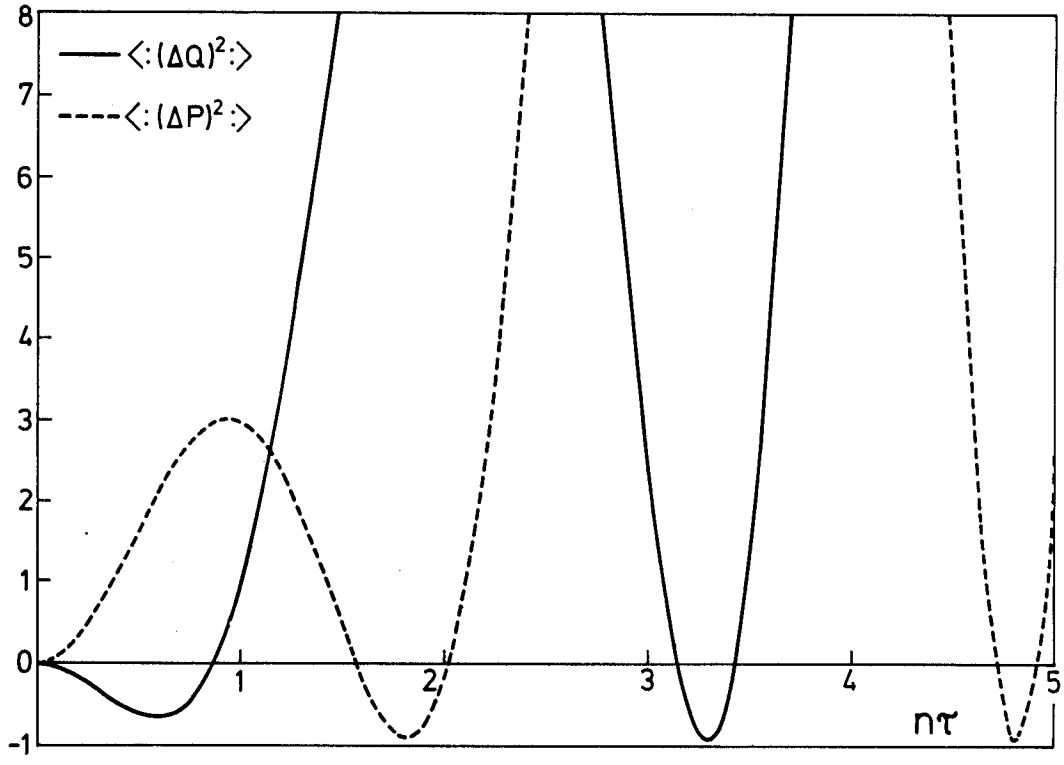

Fig.1. Fluctuations in the in-phase and out-of-phase components of the field against $n\tau$.

It is seen from figure 1 that both curves show an oscillatory behavior with positive as well as negative values. For small $n\tau$ the in-phase component Q shows squeezing while the out-of-phase component P does not. The first minimum of $\langle:(\Delta Q(\tau))^2:\rangle$ occurring for $n\tau \approx 0.6$ has the value -0.66, which is 2/3 of the limit value -1 allowed by quantum mechanics. The first minimum of $\langle:(\Delta P(\tau))^2:\rangle$ has the value -0.92, which occurs for greater values of $n\tau \approx 1.8$. This minimum means over 90 per cent of the allowed squeezing. The next minima of $\langle:(\Delta Q(\tau))^2:\rangle$ and $\langle:(\Delta P(\tau))^2:\rangle$ are even deeper giving as much as 97 per cent of squeezing, but they are much more difficult to tune to. Generally, to obtain these considerable amounts of squeezing a great number of photons is needed if τ is very small (as it usually is in real physical situations), but it is still quite realistic to meet this requirement in practice.

However, one has to keep in mind that the model under consideration is very simple and is far from

including all the effects that can modify the final results. Nevertheless, this model presents the advantage of being strictly solvable and providing a clear interpretation of squeezing. One sees clearly from the form of the solution (4) that it is the intensity dependent change in the phase of the field (the intensity dependent refractive index of the medium) which is responsible for producing squeezed states at time t if there were no squeezed states at t=0. However, this change in phase does not contribute to photon antibunching, which is phase insensitive. On the other hand, as Quattropani et al. (1980) have shown, the same anharmonic oscillator, when interacting with a thermal reservoir at thermal equilibrium, leads to sub-Poissonian photon statistics. In this case, the field is described by the density operator

$$\rho = N \exp\left[-\beta(\hbar\omega a^{+}a + \hbar k a^{+2}a^{2})\right] , \qquad (9)$$

which is diagonal in the number state representation and one can easily check that there is no squeezing in such a field. So, one can say that the nonlinear change in phase of the field contributes to squeezing but not to photon antibunching, whereas the nonlinear change in number of photons contributes to photon antibunching but not squeezing. If the phase and the number of photons are both affected by nonlinear interaction then states can arise that exhibit both squeezing and photon antibunching.

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