

Entanglement induced by spontaneous emission in spatially extended two-atom systems

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(February 18, 2003)

We investigate the role of the collective antisymmetric state in entanglement creation by spontaneous emission in a system of two non-overlapping two-level atoms. We calculate and illustrate graphically populations of the collective atomic states and the Wootters entanglement measure (concurrence) for two sets of initial atomic conditions. Our calculations include the dipole-dipole interaction and a spatial separation between the atoms that the antisymmetric state of the system is included throughout even for small interatomic separations. It is shown that spontaneous emission can lead to a transient entanglement between the atoms even if the atoms were prepared initially in an unentangled state. We find that the ability of spontaneous emission to create the transient entanglement relies on the absence of population in the collective symmetric state of the system. For the initial state of only one atom excited, the entanglement builds up rapidly in time and reaches a maximum for the parameter values corresponding roughly to zero population in the symmetric state. On the other hand, for the initial condition of both atoms excited, the atoms remain unentangled until the symmetric state is depopulated. A simple physical interpretation of these results is given in terms of the diagonal states of the density matrix of the system. We also study entanglement creation in a system of two non-identical atoms of different transition frequencies. It is found that the entanglement between the atoms can be enhanced compared to that for identical atoms, and can decay with two different time scales resulting from the coherent transfer of the population from the symmetric to the antisymmetric state. In addition, we find that a decaying initial entanglement between the atoms can display a revival behaviour.

32.80.-t, 32.80.Qk, 42.50.Gy, 42.50.Ar

I. INTRODUCTION

The subject of generation of entangled states has attracted much interest since it became clear that entanglement lies at the heart of many new applications which come under the general heading of quantum information and quantum computation. Several methods of creating entanglement have been proposed involving trapped and cooled ions or neutral atoms [1–8]. Of particular interest is generation of entangled states in two-atom systems, since it is generally believed that entanglement of only two microscopic quantum systems (two qubits) is essential to implement quantum protocols such as quantum computation. It has been shown that entangled states in a two-atom system can be created by a continuous driving of the atoms with a coherent or chaotic thermal field [5,9–11], or by spontaneous emission from two distant atoms initially prepared in a coherent superposition state or in an entangled state [12–15]. In particular, Cabrillo *et al.* [14] demonstrated that two three-level atoms initially prepared by a weak pulse in an entangled state can evolve under spontaneous emission into an entangled state of the ground states of the atoms.

The effect of spontaneous emission on entanglement creation has also been discussed by others [16–19]. These studies, however, have been limited to the small sample (Dicke) model [20]. The disadvantage of the Dicke model is that it does not include the dipole-dipole interaction among the atoms and does not correspond to realistic experimental situations of atoms located (trapped) at different positions. In fact, the model corresponds to a very specific geometrical configuration of the atoms confined to a volume much smaller compared with the atomic resonant wavelength (the small-sample model). The present atom trapping and cooling techniques can trap two atoms at distances of order of a resonant wavelength [21–23], which makes questionable the applicability of the Dicke model to physical systems.

In this paper we study what may be termed “spontaneously induced transient entanglement” in a system of two interacting atoms. In related works, Kim *et al.* [10] and Zhou *et al.* [11] have shown that a transient entanglement can be created in the Dicke model driven by a thermal (chaotic) field. Sørensen *et al.* [24] have proposed a method to produce a transient entanglement in Bose-Einstein condensate excited by a single pulse. Turchette *et al.* [25] have recently realised experimentally a transient entanglement in two trapped ions. Unlike previous works [10–15,24], we consider entanglement creation by spontaneous emission from *initially uncorrelated* atoms and without the presence of external coherent or incoherent fields. We are particularly interested in three aspects of entanglement creation by

spontaneous emission: (1) The dependence of the entanglement creation on specific arrangements of initial uncorrelated states, (2) the role of the antisymmetric state in the entanglement creation, and (3) sharing and transfer of entanglement between two entangled states. We do not make the small-sample approximation, so that our results are valid for arbitrary interatomic separations. The antisymmetric state and its dynamics are not neglected, even when dealing with small interatomic separations. We show that spontaneous emission from two spatially separated atoms can lead to a transient entanglement of initially unentangled atoms. This result contrasts the Dicke model where spontaneous emission cannot produce entanglement from initially unentangled atoms [10,18]. Moreover, we show that the entanglement creation relies crucially on the population distribution between the entangled symmetric and antisymmetric states and attains maximal values when the population of the symmetric state becomes zero. This is a rather surprising prediction, since the symmetric state is an example of maximally entangled state and one might conclude that its participation in the atomic dynamics would enhance entanglement.

II. COLLECTIVE TWO-ATOM SYSTEMS

We consider a system of two non-overlapping two-level atoms with ground states $|g_i\rangle$ and excited states $|e_i\rangle$ ($i = 1, 2$) connected by dipole transition moments $\vec{\mu}_i$. The atoms are located at fixed positions \vec{r}_1 and \vec{r}_2 and coupled to all modes of the electromagnetic field, which we assume are in the vacuum state. We consider spontaneous emission from identical as well as non-identical atoms prepared in two different initial states. In the case of nonidentical atoms, we assume that atoms have equal dipole moments $\vec{\mu}_1 = \vec{\mu}_2 = \vec{\mu}$, but different transition frequencies ω_1 and ω_2 , such that $\omega_2 - \omega_1 \ll \omega_0 = (\omega_1 + \omega_2)/2$, so that the rotating-wave approximation can be applied to calculate the dynamics of the system.

The time evolution of the system of atoms coupled through the vacuum field is given by the following master equation [26–28]

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} = & -i \sum_{i=1}^2 \omega_i [S_i^z, \hat{\rho}] - i \sum_{i \neq j}^2 \Omega_{ij} [S_i^+ S_j^-, \hat{\rho}] \\ & - \frac{1}{2} \sum_{i,j=1}^2 \Gamma_{ij} (\hat{\rho} S_i^+ S_j^- + S_i^+ S_j^- \hat{\rho} - 2 S_j^- \hat{\rho} S_i^+) , \end{aligned} \quad (1)$$

where S_i^+ (S_i^-) are the dipole raising (lowering) operators and S^z is the energy operator of the i th atom. In Eq. (1), Γ_{ij} ($i = j$) are the spontaneous emission rates of the atoms, equal to the Einstein A coefficient for spontaneous emission, whereas Γ_{ij} and Ω_{ij} ($i \neq j$) describe the interatomic coupling [26–28], and are the collective damping and the dipole-dipole interaction potential defined, respectively, by

$$\begin{aligned} \Gamma_{ij} = \Gamma_{ji} = & \frac{3}{2} \Gamma \left\{ \left[1 - (\vec{\mu} \cdot \vec{r}_{ij})^2 \right] \frac{\sin(k_0 r_{ij})}{k_0 r_{ij}} \right. \\ & \left. + \left[1 - 3(\vec{\mu} \cdot \vec{r}_{ij})^2 \right] \left[\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\} , \end{aligned} \quad (2)$$

and

$$\begin{aligned} \Omega_{ij} = & \frac{3}{4} \Gamma \left\{ - \left[1 - (\vec{\mu} \cdot \vec{r}_{ij})^2 \right] \frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} \right. \\ & \left. + \left[1 - 3(\vec{\mu} \cdot \vec{r}_{ij})^2 \right] \left[\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\} , \end{aligned} \quad (3)$$

where $k_0 = \omega_0/c$, $r_{ij} = |\vec{r}_j - \vec{r}_i|$ is the distance between the atoms, $\vec{\mu}$ is unit vector along the atomic transition dipole moments, that we assume are parallel to each other, and \vec{r}_{ij} is the unit vector along the interatomic axis.

The master equation (1) has been used for many years to study a wide variety of problems involving the interaction of collective atomic systems with the radiation field [29]. Using the master equation (1), we can write down the equations of motion for the components of the density matrix of the two-atom system in the basis of the product states $|e_1\rangle|e_2\rangle$, $|e_1\rangle|g_2\rangle$, $|g_1\rangle|e_2\rangle$ and $|g_1\rangle|g_2\rangle$ of the individual atoms. However, the problem simplifies by working in the basis of the collective states of the system which contains symmetric and antisymmetric combinations of the product states. For identical atoms ($\omega_1 = \omega_2$) the collective states are [20,26]

$$\begin{aligned}
|e\rangle &= |e_1\rangle |e_2\rangle , \\
|s\rangle &= \frac{1}{\sqrt{2}} (|e_1\rangle |g_2\rangle + |g_1\rangle |e_2\rangle) , \\
|a\rangle &= \frac{1}{\sqrt{2}} (|e_1\rangle |g_2\rangle - |g_1\rangle |e_2\rangle) , \\
|g\rangle &= |g_1\rangle |g_2\rangle .
\end{aligned} \tag{4}$$

In the collective state representation, the two-atom system behaves as a single four-level system with the ground state $|g\rangle$, the upper state $|e\rangle$, and two intermediate states: the symmetric $|s\rangle$ and antisymmetric $|a\rangle$ states. The most important property of the collective states is that the symmetric and antisymmetric states are maximally entangled states. The states are linear superpositions of the product states which cannot be separated into product states of the individual atoms.

For non-identical atoms, the collective states of the system contain non-maximally entangled states, which can be written as linear combinations of the maximally entangled states

$$\begin{aligned}
|e\rangle &= |e_1\rangle |e_2\rangle , \\
|s'\rangle &= \frac{1}{\sqrt{2}} [(\alpha + \beta) |s\rangle + (\beta - \alpha) |a\rangle] , \\
|a'\rangle &= \frac{1}{\sqrt{2}} [(\alpha - \beta) |s\rangle + (\alpha + \beta) |a\rangle] , \\
|g\rangle &= |g_1\rangle |g_2\rangle ,
\end{aligned} \tag{5}$$

where $\alpha = d/\sqrt{d^2 + \Omega_{12}^2}$, $\beta = \Omega_{12}/\sqrt{d^2 + \Omega_{12}^2}$, $d = \Delta + \sqrt{\Omega_{12}^2 + \Delta^2}$, and $\Delta = (\omega_2 - \omega_1)/2$.

Thus, in both cases of identical or non-identical atoms, we can limit the considerations to the basis of the collective states (4). In this basis, the density matrix elements satisfy the following set of simple differential equations

$$\begin{aligned}
\dot{\rho}_{ee} &= -2\Gamma \rho_{ee} , \\
\dot{\rho}_{ss} &= -(\Gamma + \Gamma_{12}) (\rho_{ss} - \rho_{ee}) + i\Delta (\rho_{as} - \rho_{sa}) , \\
\dot{\rho}_{aa} &= -(\Gamma - \Gamma_{12}) (\rho_{aa} - \rho_{ee}) - i\Delta (\rho_{as} - \rho_{sa}) , \\
\dot{\rho}_{as} &= -(\Gamma + 2i\Omega_{12}) \rho_{as} + i\Delta (\rho_{ss} - \rho_{aa}) , \\
\dot{\rho}_{se} &= -\left[\frac{1}{2} (3\Gamma + \Gamma_{12}) - i(\omega_0 - \Omega_{12}) \right] \rho_{se} + i\Delta \rho_{ae} , \\
\dot{\rho}_{ae} &= -\left[\frac{1}{2} (3\Gamma - \Gamma_{12}) - i(\omega_0 + \Omega_{12}) \right] \rho_{ae} + i\Delta \rho_{se} , \\
\dot{\rho}_{gs} &= -\left[\frac{1}{2} (\Gamma + \Gamma_{12}) - i(\omega_0 + \Omega_{12}) \right] \rho_{gs} + (\Gamma + \Gamma_{12}) \rho_{se} - i\Delta \rho_{ga} , \\
\dot{\rho}_{ga} &= -\left[\frac{1}{2} (\Gamma - \Gamma_{12}) - i(\omega_0 - \Omega_{12}) \right] \rho_{ga} - (\Gamma - \Gamma_{12}) \rho_{ae} - i\Delta \rho_{gs} , \\
\dot{\rho}_{eg} &= -(\Gamma + 2i\omega_0) \rho_{eg} .
\end{aligned} \tag{6}$$

Equations (6) show that all transitions rates to and from the symmetric state are equal to $(\Gamma + \Gamma_{12})$. On the other hand, all transitions rates to and from the antisymmetric state are equal to $(\Gamma - \Gamma_{12})$. Thus, the symmetric state decays with an enhanced (superradiant) rate, while the antisymmetric state decays with a reduced (subradiant) rate. Hence, the population of the antisymmetric state experiences a variation on a time scale of order $(\Gamma - \Gamma_{12})^{-1}$, which can lead to interesting effects not observed in the Dicke model. These effects result from the fact that the set of equations (6) has two different solutions depending on whether $\Gamma_{12} = \Gamma$ or $\Gamma_{12} \neq \Gamma$. The case of $\Gamma_{12} = \Gamma$ corresponds to the small sample (Dicke) model, whereas the case of $\Gamma_{12} \neq \Gamma$ corresponds to spatially extended atomic systems. The existence of two different solutions of Eq. (6) is connected with conservation of the total spin S^2 , that S^2 is a constant of motion for the Dicke model and S^2 not being a constant of motion for a spatially extended system of atoms [30,31]. We can explain it by expressing the square of the total spin of the two-atom system in terms of the density matrix elements of the collective system as

$$S^2(t) = 2 - 2\rho_{aa}(t) . \tag{7}$$

It is clear from Eq. (7) that S^2 is conserved only in the Dicke model, in which the antisymmetric state is ignored. For a spatially extended system the antisymmetric state participates fully in the dynamics and S^2 is not conserved. The

Dicke model evolves between the triplet states $|e\rangle$, $|s\rangle$, and $|g\rangle$, while the spatially extended two-atom system evolves between the triplet and the antisymmetric states.

III. TRANSIENT ENTANGLEMENT

The entanglement creation by spontaneous emission is illustrated most clearly if one assumes that a system of two atoms decays spontaneously from initially unentangled (uncorrelated) states. Several different measures have been proposed to identify entanglement between two atoms, and we choose the Wootters entanglement measure [32], the concurrence C , defined as

$$C = \max \left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right), \quad (8)$$

where $\lambda_1, \dots, \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ and σ_y is the Pauli matrix. The range of the concurrence is from 0 to 1. For unentangled atoms $C = 0$ whereas $C = 1$ for the maximally entangled atoms. The concurrence involves eigenvalues of the complicated matrix $\tilde{\rho}$ and, in general, is difficult to calculate analytically. Therefore, for the understanding and explanation of the entanglement creation via spontaneous emission, we will use the Peres-Horodecki (negativity) measure for entanglement [33,34]. The negativity criterion is given by the quantity

$$E = \max \left(0, -2 \sum_i \mu_i \right), \quad (9)$$

where the sum is taken over the negative eigenvalues μ_i of the partial transposition of the density matrix $\hat{\rho}$ of the system. The value $E = 1$ corresponds to maximum entanglement between the atoms whilst $E = 0$ describes completely separated atoms.

The two entanglement measures, the concurrence C and negativity E , give the same results for criteria for entanglement, but they can give different results for a degree of entanglement [35].

A. Identical atoms

We begin with spontaneous emission from two identical atoms and consider entanglement creation for two different sets of initial atomic conditions at $t = 0$. In the first, one of the atoms is in its excited state and the other is assumed to reside in its ground state. In the second, both atoms are assumed to reside in their excited states.

1. Initial state of only one atom excited: $|\Phi_0\rangle = |e_1\rangle|g_2\rangle$.

In the basis of the collective states of the system, the initial condition of only one atom excited corresponds to the initial condition with non-zero density matrix elements $\rho_{ss}(0) = \rho_{aa}(0) = \rho_{sa}(0) = \rho_{as}(0) = 1/2$. In Fig. 1, we plot the time evolution of the populations $\rho_{ss}(t)$, $\rho_{aa}(t)$ and the concurrence C , which we have found solving the equations of motion (6) with the condition that initially at $t = 0$ one of the atoms was prepared in its excited state and the other in the ground state. One can see from Fig. 1 that the concurrence is zero at $t = 0$; in other words there is no entanglement in the system at $t = 0$. The time evolution of the concurrence reflects the time evolution of the entanglement between the atoms. It is seen from Fig. 1 that the concurrence builds up as time develops indicating that spontaneous emission can create entanglement between the initially unentangled atoms. As time progresses, the concurrence rapidly increases and reaches a maximum at time close to the point where the symmetric state becomes depopulated. At later times, the concurrence slowly decreases and overlaps with the time evolution of the population of the antisymmetric state.

The physical understanding of the creation of the transient entanglement can be achieved by considering the time-dependent density matrix of the system, which for the initial condition of only one atom excited has the following form

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{ss}(t) & \rho_{sa}(t) & 0 \\ 0 & \rho_{as}(t) & \rho_{aa}(t) & 0 \\ 0 & 0 & 0 & \rho_{gg}(t) \end{pmatrix}, \quad (10)$$

where the non-zero time-dependent density matrix elements are given by

$$\begin{aligned}
\rho_{ss}(t) &= \frac{1}{2} \exp[-(\Gamma + \Gamma_{12})t] , \\
\rho_{aa}(t) &= \frac{1}{2} \exp[-(\Gamma - \Gamma_{12})t] , \\
\rho_{gg}(t) &= 1 - \exp(-\Gamma t) \cosh \Gamma_{12}t , \\
\rho_{as}(t) &= \rho_{sa}^*(t) = \frac{1}{2} \exp[-(\Gamma + 2i\Omega_{12})t] .
\end{aligned} \tag{11}$$

One can see that the density matrix is not diagonal due to the presence of coherences $\rho_{sa}(t)$ and $\rho_{as}(t)$. The density matrix can be re-diagonalized to give new "diagonal" states

$$\begin{aligned}
|\Psi_1\rangle &= \{[P_1(t) - \rho_{ss}(t)]|a\rangle + \rho_{as}(t)|s\rangle\} / \left\{ [P_1(t) - \rho_{ss}(t)]^2 + |\rho_{as}(t)|^2 \right\}^{\frac{1}{2}} , \\
|\Psi_2\rangle &= \{\rho_{as}(t)|a\rangle + [P_2(t) - \rho_{aa}(t)]|s\rangle\} / \left\{ [P_2(t) - \rho_{aa}(t)]^2 + |\rho_{as}(t)|^2 \right\}^{\frac{1}{2}} , \\
|\Psi_3\rangle &= |g\rangle , \\
|\Psi_4\rangle &= |e\rangle ,
\end{aligned} \tag{12}$$

where the diagonal probabilities (populations of the new states) are

$$\begin{aligned}
P_1(t) &= \exp(-\Gamma t) \cosh \Gamma_{12}t , \\
P_2(t) &= 0 , \\
P_3(t) &= 1 - \exp(-\Gamma t) \cosh \Gamma_{12}t , \\
P_4(t) &= 0 .
\end{aligned} \tag{13}$$

Thus, the coherences $\rho_{sa}(t)$ and $\rho_{as}(t)$ cause the system to evolve effectively only between two states: the ground product state $|g\rangle$ and the entangled state $|\Psi_1\rangle$, which is a linear combination of the states $|s\rangle$ and $|a\rangle$. In this case, the density matrix of the system is diagonal for all times t , and is given by

$$\rho(t) = P_1(t)|\Psi_1\rangle\langle\Psi_1| + \rho_{gg}(t)|g\rangle\langle g| . \tag{14}$$

It is easy to find from Eq. (11) that $|\rho_{as}(t)|^2 = \rho_{aa}(t)\rho_{ss}(t)$, and then the state $|\Psi_1\rangle$ can be written as

$$|\Psi_1\rangle = \left(\sqrt{\rho_{ss}(t)}|s\rangle + \sqrt{\rho_{aa}(t)}|a\rangle \right) / \sqrt{\rho_{aa}(t) + \rho_{ss}(t)} , \tag{15}$$

The state $|\Psi_1\rangle$ reduces to a nonentangled state when the states $|s\rangle$ and $|a\rangle$ have equal populations. On the other hand, the state $|\Psi_1\rangle$ reduces to a maximally entangled state when either $\rho_{ss}(t)$ or $\rho_{aa}(t)$ are equal to zero.

We are now in a position to understand quantitatively the method of creation of entanglement via spontaneous emission. The spontaneously induced entanglement results from unequal populations of the symmetric and antisymmetric states. This observation is supported by the Peres-Horodecki measure for entanglement. It is easy to show that the eigenvalues of the partial transposition of the density matrix (10) are

$$\begin{aligned}
\mu_1 &= \frac{1}{2} [\rho_{ss}(t) + \rho_{aa}(t) - \rho_{as}(t) - \rho_{sa}(t)] , \\
\mu_2 &= \frac{1}{2} [\rho_{ss}(t) + \rho_{aa}(t) + \rho_{as}(t) + \rho_{sa}(t)] , \\
\mu_3 &= \frac{1}{2} \left\{ \rho_{gg}(t) + \left[\rho_{gg}^2(t) + (\rho_{ss}(t) - \rho_{aa}(t))^2 - (\rho_{as}(t) - \rho_{sa}(t))^2 \right]^{\frac{1}{2}} \right\} , \\
\mu_4 &= \frac{1}{2} \left\{ \rho_{gg}(t) - \left[\rho_{gg}^2(t) + (\rho_{ss}(t) - \rho_{aa}(t))^2 - (\rho_{as}(t) - \rho_{sa}(t))^2 \right]^{\frac{1}{2}} \right\} .
\end{aligned} \tag{16}$$

It is obvious from Eqs. (16) and (11) that μ_1, μ_2 and μ_3 are always positive. The eigenvalue μ_4 becomes negative if and only if

$$|\rho_{ss}(t) - \rho_{aa}(t)| > |\rho_{as}(t) - \rho_{sa}(t)| , \tag{17}$$

which, according to Eq. (11), is satisfied if $\rho_{ss}(t) \neq \rho_{aa}(t)$.

Since the population of the symmetric state decays faster than the antisymmetric state (see Eq. (11)), at time when the state $|s\rangle$ becomes depopulated, the state $|\Psi_1\rangle$ reduces to the maximally entangled antisymmetric state $|a\rangle$. The above analysis give clear evidence that a transient entanglement created by spontaneous emission can appear only in spatially extended two-atom systems where the antisymmetric state fully participates in the dynamics of the system.

2. Initial state of both atoms excited: $|\Phi_0\rangle = |e_1\rangle|e_2\rangle$.

The role of the antisymmetric state in the entanglement creation is more evident when we choose the initial state of both atoms in their excited states. Here, $\rho_{ee}(0) = 1$ and the initial values of the remaining density matrix elements are zero. Note, that in this case there are no any initial coherences between the atoms, and also there are no any initial coherences between the collective states. Recent analysis of the entanglement creation in the Dicke model have shown that there is no entanglement creation in spontaneous emission when the atoms are initially prepared in their excited states [10,18]. In Fig. 2, we show the concurrence C and the populations $\rho_{ss}(t)$ and $\rho_{aa}(t)$ for the case of two spatially separated atoms prepared in their excited states at $t = 0$. Since, at $t = 0$, only the upper state $|e\rangle$ is occupied, one finds from Eq. (6) that the spontaneous emission populates the symmetric and antisymmetric states with different rates. Thus, at early times the symmetric state is more populated than the antisymmetric state. Again, there is no entanglement between the atoms at $t = 0$ as the initial state $|e\rangle$ is an unentangled state. However, the figure clearly demonstrates that contrary to the Dicke model, a transient entanglement can be generated in spontaneous emission from two spatially separated atoms. Note that the entanglement is not generated until the population $\rho_{ss}(t)$ becomes zero.

This effect can be explained by the Peres-Horodecki measure for entanglement. Consider the time evolution of the density operator of the system, with the initial condition of $\rho_{ee}(0) = 1$, has the diagonal form for all times

$$\rho(t) = \rho_{ee}(t)|e\rangle\langle e| + \rho_{ss}(t)|s\rangle\langle s| + \rho_{aa}(t)|a\rangle\langle a| + \rho_{gg}(t)|g\rangle\langle g| , \quad (18)$$

where $\rho_{ii}(t)$ ($i = e, s, a, g$) are the time dependent populations of the collective atomic states. According to the Peres-Horodecki criterion for entanglement, the two-atom system represented by the density matrix (18) is entangled when

$$|\rho_{ss}(t) - \rho_{aa}(t)| > 2\sqrt{\rho_{ee}(t)\rho_{gg}(t)} . \quad (19)$$

Thus, in contrast to the case of only one atom excited, it is not enough to produce an unbalanced population distribution between the states $|s\rangle$ and $|a\rangle$ to obtain an entanglement in the system. The reason is the presence of a population in the unentangled state $|e\rangle$. In order to analyse the dependence of the entanglement on the populations of the collective states, we solve Eq. (6) with the initial condition $\rho_{ee}(0) = 1$ and find that the time-dependent populations are given by

$$\begin{aligned} \rho_{ee}(t) &= \exp(-2\Gamma t) , \\ \rho_{ss}(t) &= \frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \{ \exp[-(\Gamma + \Gamma_{12})t] - \exp(-2\Gamma t) \} , \\ \rho_{aa}(t) &= \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \{ \exp[-(\Gamma - \Gamma_{12})t] - \exp(-2\Gamma t) \} . \end{aligned} \quad (20)$$

One can see from Eq. (20) that in general the populations decay with different decay rates. However, for small interatomic separations, $\Gamma_{12} \approx \Gamma$, and then the upper state $|e\rangle$ and the symmetric state $|s\rangle$ decay with the same rate (2Γ), whereas the antisymmetric state decays with a significantly reduced rate $\Gamma - \Gamma_{12}$. At early times the population is mostly in the state $|e\rangle$, and then the inequality (19) is not satisfied. The inequality (19) is not satisfied until $\rho_{ee}(t) \approx 0$. Since the population of the symmetric state decays with the same rate as $\rho_{ee}(t)$, at time where $\rho_{ee}(t) \approx 0$ the population $\rho_{ss}(t) \approx 0$. Thus, the lack of the entanglement for $\rho_{ss}(t) \neq 0$ can be attributed to a large population of the product state $|e\rangle$ which, in turn, decays on the same time scale as $\rho_{ss}(t)$.

At time t_s when $\rho_{ss}(t) = 0$, the upper state $|e\rangle$ is also depopulated, but there is still some population accumulated in the antisymmetric state, as a result of a slow decay of the population with the reduced rate $(\Gamma - \Gamma_{12})$. Therefore, for times larger than t_s , the system behaves effectively as a two-level system, whose the density matrix can be written as

$$\rho(t) = \rho_{aa}(t)|a\rangle\langle a| + \rho_{gg}(t)|g\rangle\langle g| . \quad (21)$$

Following the Peres-Horodecki measure for entanglement, we see that the atoms are entangled until the antisymmetric state is populated, i.e. when $\rho_{aa}(t) \neq 0$. The entanglement persists for a longer time and slowly decays to zero on a time scale of order $\sim (\Gamma - \Gamma_{12})^{-1}$, which for small interatomic separations is much longer than the single atom decay rate Γ^{-1} .

B. Non-identical atoms

Although most of the work on entanglement is concerned with identical atoms, there can be interesting information available on entanglement creation with non-identical atoms. In this section, we give illustrative examples of both the entanglement creation and entanglement transfer between two nonidentical atoms with different transition frequencies.

Figure 3 shows the concurrence C and the populations $\rho_{ss}(t)$ and $\rho_{aa}(t)$ as a function of time for the initial condition of only one atom excited. We take $\Delta = \Gamma$ that the atom "2" has a higher frequency (energy) than the atom "1", i.e. $\omega_2 > \omega_1$, and assume that initially the atom of the higher transition frequency was in its excited state while the other atom of lower transition frequency was in the ground state. As in the case of identical atoms, there is no initial entanglement between the atoms, and at early times the entanglement builds up rapidly to a maximum appearing at short time $\Gamma t < 1$. However, comparing with the entanglement for identical atoms, shown in Fig. 1, we see that the maximum of the entanglement obtained with nonidentical atoms is greater than that obtained with identical atoms. It is interesting to note that after passing through the maximum, the entanglement decays with two different time scales. This effect is more pronounced if we choose the atom of the lower frequency to be initially in its excited state and the other atom of higher frequency in the ground state. We illustrate this in Fig. 4, where we plot the concurrence C and the populations $\rho_{ss}(t)$ and $\rho_{aa}(t)$ as a function of time for the same parameters as in Fig. 3. The two decay time scales are well resolved. At early times the entanglement decays on a time scale of order Γ^{-1} , and next the time decay "jumps" into the time scale of order $(\Gamma - \Gamma_{12})^{-1}$, which coincides with the decay of the population of the antisymmetric state.

The enhanced entanglement and the two different decay time scales can be explained as a consequence of the coherent transfer of the population from the symmetric to the antisymmetric state, that is absent for identical atoms. In order to show this, consider the dynamics of the entangled states $|s\rangle$ and $|a\rangle$, described by Eq. (6), that in the case of initially only one atom excited simplify to the following equations of motion

$$\begin{aligned}\dot{\rho}_{ss} &= -(\Gamma + \Gamma_{12})\rho_{ss} + i\Delta(\rho_{as} - \rho_{sa}) , \\ \dot{\rho}_{aa} &= -(\Gamma - \Gamma_{12})\rho_{aa} - i\Delta(\rho_{as} - \rho_{sa}) , \\ \dot{\rho}_{as} &= -(\Gamma + 2i\Omega_{12})\rho_{as} + i\Delta(\rho_{ss} - \rho_{aa}) .\end{aligned}\tag{22}$$

Equations (18) are formally identical to the optical Bloch equations for a two-level atom driven by a coherent field [36]. Here, the detuning Δ plays the role of a Rabi frequency that coherently transfers the population between the symmetric and antisymmetric states. Note that the interaction between the states does not involve the ground state $|g\rangle$, and therefore is not accompanied by spontaneous emission. Thus, the coherent transfer of the population between the states is a decoherence free process. In this process, the population is efficiently transferred from the more populated symmetric state to the antisymmetric state before it decays to the ground state leading to the enhancement of the entanglement.

Finally, we point out one more difference between the entanglement creation in identical and nonidentical atoms. Figure 5 shows the concurrence C and the populations $\rho_{ss}(t)$ and $\rho_{aa}(t)$ for the same parameters as in Fig. 3, but with the new initial condition $\rho_{ss}(0) = 1$. In this case there is perfect ($C = 1$) initial entanglement between the atoms. Since, at $t = 0$, only the state $|s\rangle$ is occupied, one finds that at early times the initial entanglement decays with the enhanced rate $(\Gamma + \Gamma_{12})$ corresponding to the decay rate of the population of the symmetric state. The entanglement decays in time until the atoms become disentangled. This happens at time $\Gamma t \approx 2$, where the population is equally distributed over the entangled states, $\rho_{ss}(t) = \rho_{aa}(t)$. However, as time develops, the entanglement emerges again. This is remarkable as spontaneous emission is essentially an irreversible process, and one might expect that spontaneous emission merely degrades the initial entanglement. The revival effect is absent for identical atoms, and is due to the coherent transfer of the population from the symmetric to the antisymmetric state. The revival of the entanglement at time $\Gamma t \approx 2$ is more easily understood by reference to the density matrix of the system. It is seen from Fig. 5 that for times $\Gamma t > 2$ the population of the symmetric state is negligible, and therefore the system behaves as a two-level system whose the density matrix is in the diagonal form $\rho(t) = \rho_{aa}(t)|a\rangle\langle a| + \rho_{gg}(t)|g\rangle\langle g|$. Since $\rho_{aa}(t) \neq 0$ for $\Gamma t > 2$, the atoms are entangled until the population $\rho_{aa}(t)$ decays eventually to the ground state.

In contrast, if the atoms were initially prepared in the antisymmetric state, $\rho_{aa}(0) = 1$, the initial entanglement remains in the antisymmetric state for all times t even if a part of the population is transferred to the symmetric state. This feature is shown in Fig. 6, where we plot the time evolution of the concurrence and the population $\rho_{aa}(t)$ for the same parameters as in Fig. 5, but with the initial condition $\rho_{aa}(0) = 1$. It is clear from Fig. 6 that the entanglement decays with the reduced rate $(\Gamma - \Gamma_{12})$ corresponding to the decay rate of the population of the antisymmetric state, and the coherent coupling does not transfer much of the entanglement to the symmetric state.

Our calculations clearly demonstrate that the transient entanglement induced from initially unentangled atoms depends crucially on the presence of the antisymmetric state, that is characteristic of spatially extended atomic systems.

IV. SUMMARY

In this paper, we have analysed the role of the antisymmetric state in entanglement creation by spontaneous emission from two spatially separated atoms. The results show that the generation of the entanglement is strongly dependent upon the population distribution between the symmetric and antisymmetric states of the system. The entanglement is maximal when the population of the symmetric state becomes zero. Thus, our results show the participation of the symmetric state in the atomic dynamics has a destructive effect on the entanglement. We have also considered the entanglement creation in two non-identical atoms. We have found that the entanglement can be enhanced by the process of the coherent transfer of the population between the symmetric and antisymmetric states. In addition, we have shown that the entanglement can decay with two different time scales and exhibits a revival behavior due to the coherent transfer of the population from the rapidly decaying symmetric to the slowly decaying antisymmetric state before it decays to zero. Although the entanglement created by spontaneous emission appears only in the transient regime, it provides an useful information about entanglement sharing and entanglement transfer between two entangled states.

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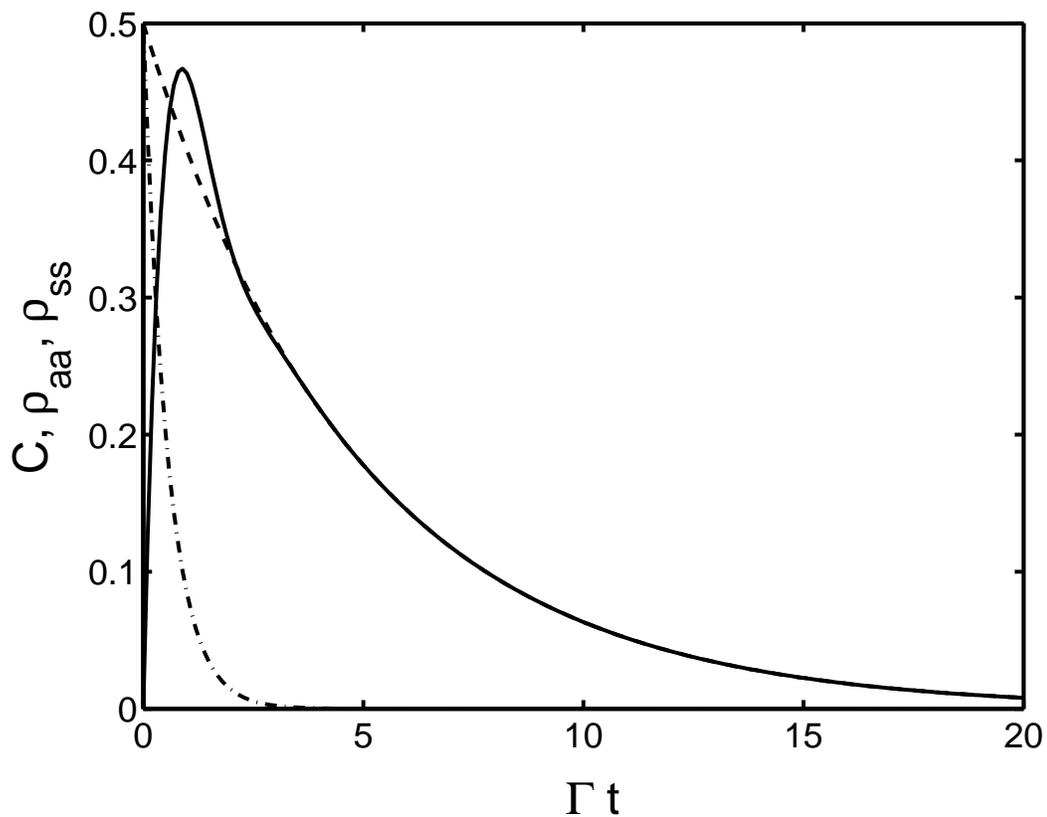


FIG. 1. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for the atoms prepared initially in the unentangled state $|\Phi_0\rangle = |e_1\rangle |g_2\rangle$, with $\bar{\mu} \perp \bar{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma$, $\Omega_{12} = 1.12\Gamma$).

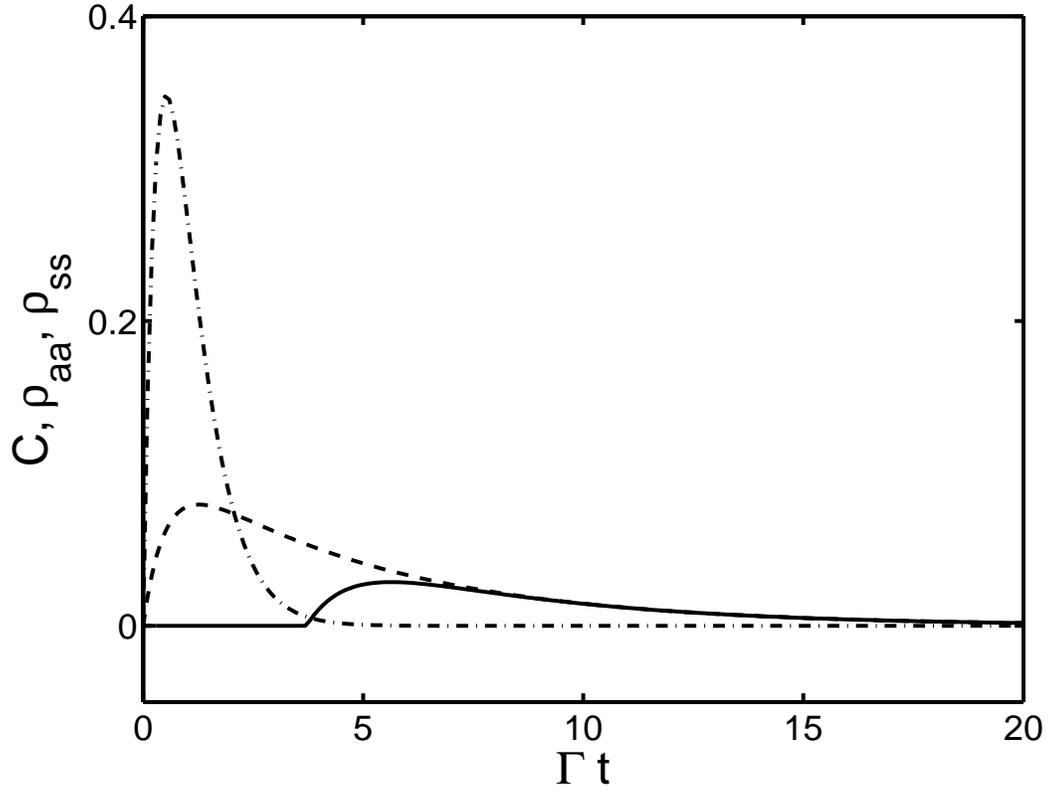


FIG. 2. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for the initial unentangled state of both atoms excited, $|\Phi_0\rangle = |e_1\rangle|e_2\rangle$, with $\vec{\mu} \perp \vec{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma$, $\Omega_{12} = 1.12\Gamma$).

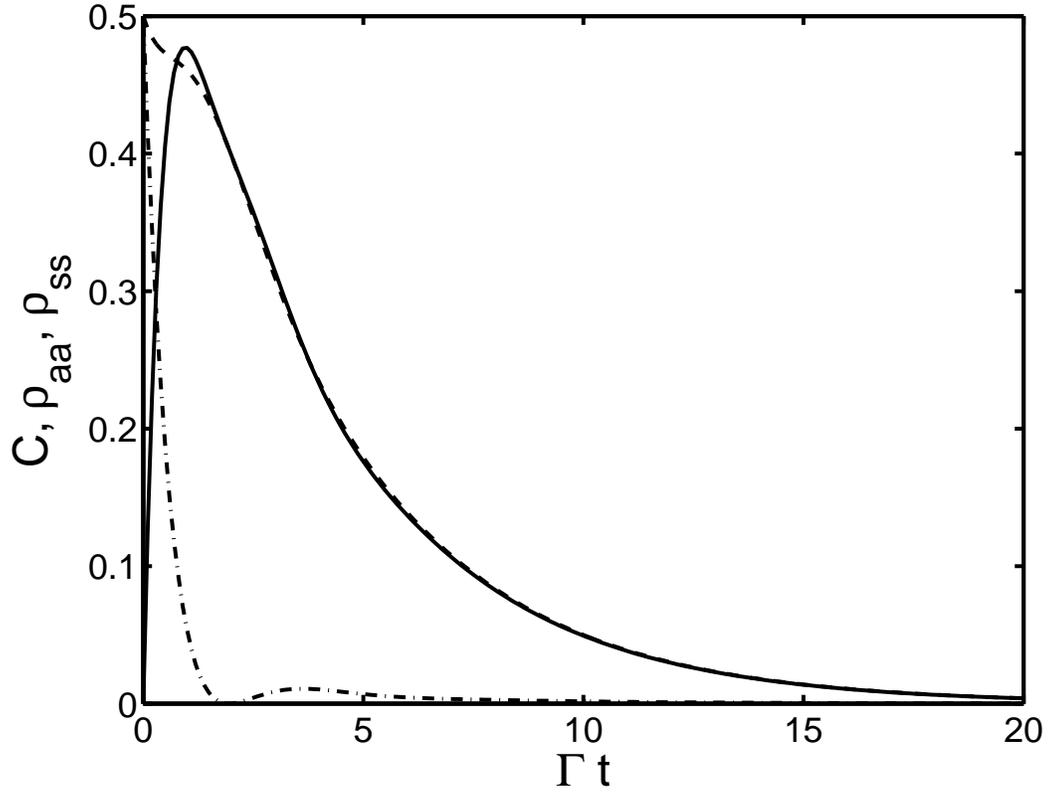


FIG. 3. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for two non-identical atoms with $\Delta = \Gamma$, $\bar{\mu} \perp \bar{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma, \Omega_{12} = 1.12\Gamma$). The atoms were initially in the unentangled state $|\Phi_0\rangle = |e_1\rangle|g_2\rangle$.

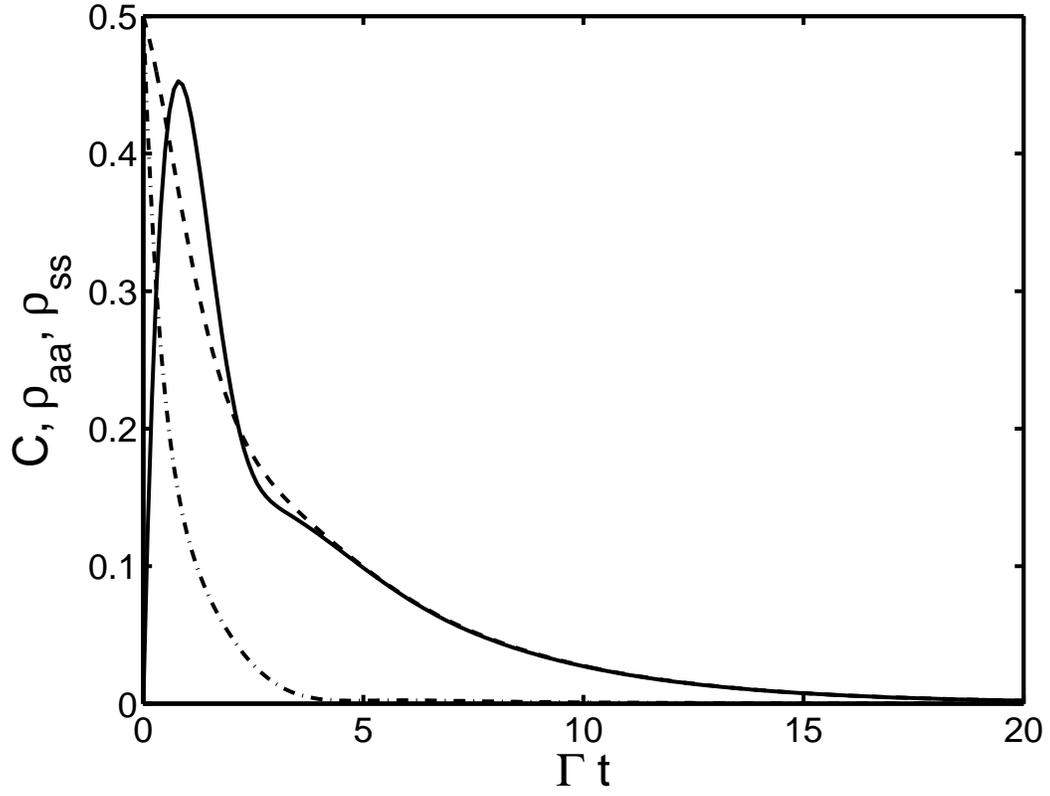


FIG. 4. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for two non-identical atoms with $\Delta = \Gamma$, $\bar{\mu} \perp \bar{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma, \Omega_{12} = 1.12\Gamma$). The atoms were initially in the unentangled state $|\Phi_0\rangle = |g_1\rangle|e_2\rangle$.

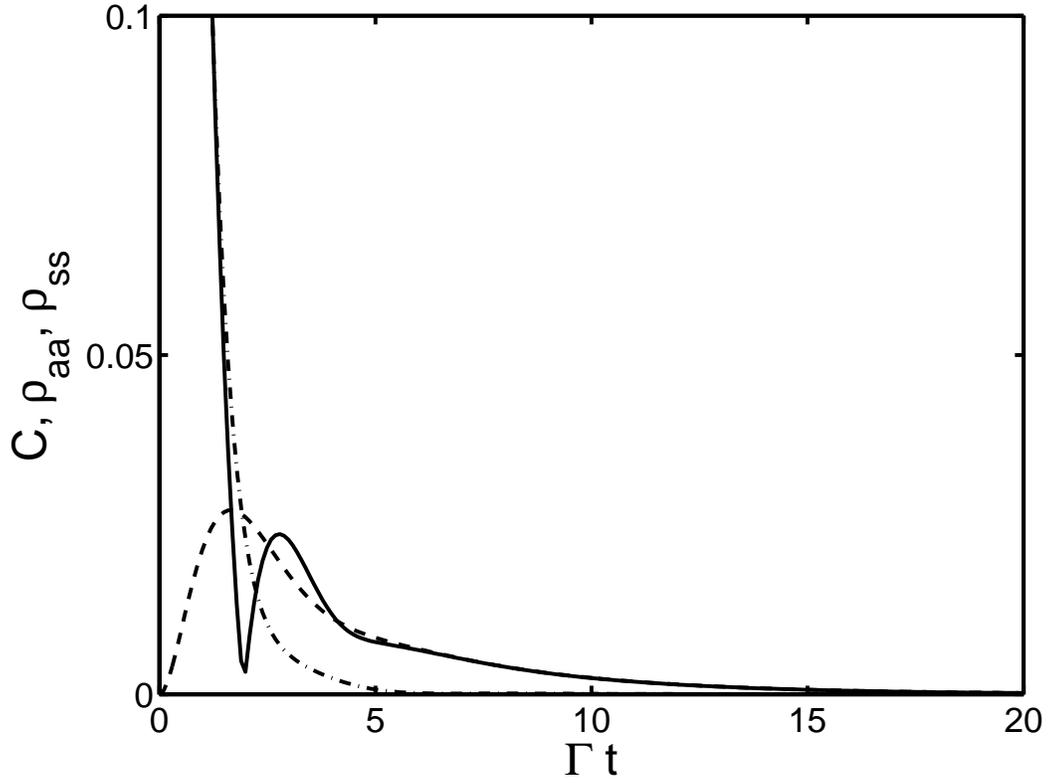


FIG. 5. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for two non-identical atoms with $\Delta = \Gamma$, $\bar{\mu} \perp \bar{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma, \Omega_{12} = 1.12\Gamma$). The atoms were initially in the maximally entangled state $|\Phi_0\rangle = |s\rangle$. For $t = 0$ both ρ_{ss} and concurrence start from unity, and the figure is cut to better visualize concurrence revival.

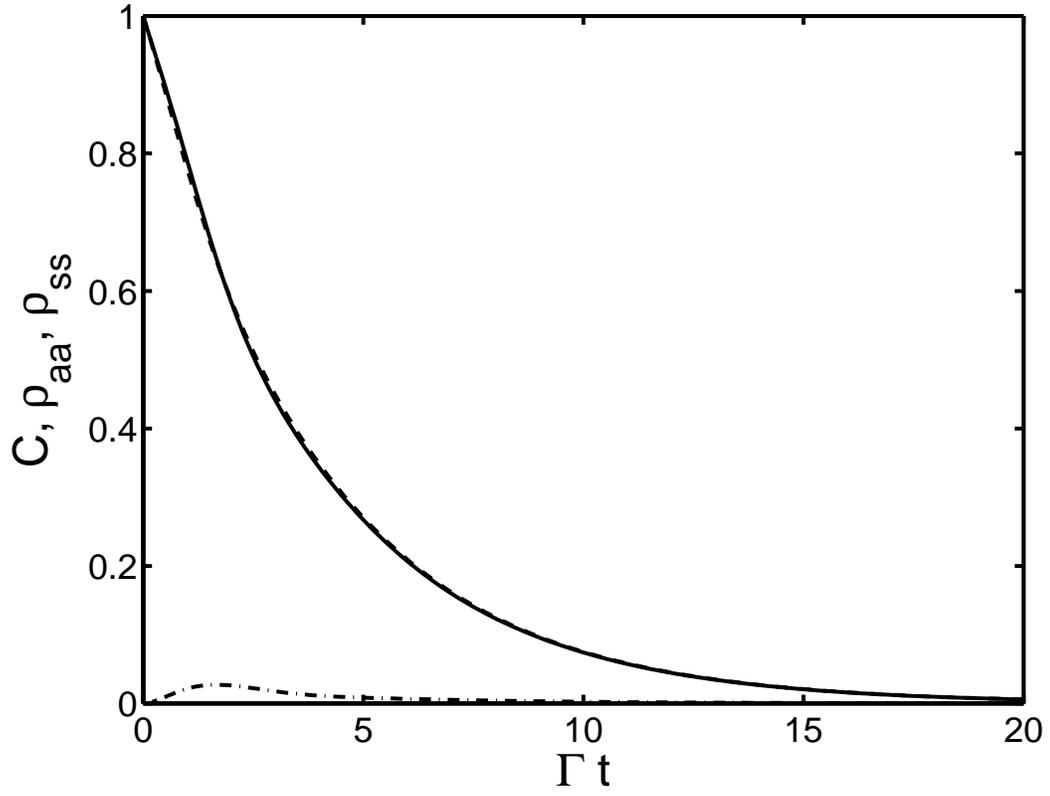


FIG. 6. Time evolution of the concurrence C (solid line) and the population of the antisymmetric state $\rho_{aa}(t)$ (dashed line) and the symmetric state $\rho_{ss}(t)$ (dashed-dotted line) for two non-identical atoms with $\Delta = \Gamma$, $\bar{\mu} \perp \bar{r}_{12}$, and $r_{12} = \lambda/6$ ($\Gamma_{12} = 0.79\Gamma$, $\Omega_{12} = 1.12\Gamma$). The atoms were initially in the maximally entangled state $|\Phi_0\rangle = |a\rangle$.