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Entanglement of two atoms

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We discuss the problem of creation of entangled states in a system of two two-level atoms which are separated by an arbitrary distance r_{12} and interact with each other via the dipole-dipole interaction and both are driven by a laser field. The entangled antisymmetric state of the system is included throughout, even for small inter-atomic separations. Different mechanisms leading to effective transfer of population to the antisymmetric state are identified. The steady-state values of concurrence which is a measure of entanglement are calculated showing that perfect entanglement can be reached in case of two non-identical atoms.

1 Introduction

Entangled states of collective quantum systems, which are states that cannot be factorized into product states of the subsystems, are of fundamental interest in quantum mechanics. Recently, there is growing interest in entangled states because their potential applicability in processing of quantum information. Several physical realizations of entangled atoms have been proposed involving trapping and cooling of a small number of ions or neutral atoms [1–3]. The reason for using cold trapped atoms or ions is twofold. On the one hand, it has been realized that the trapped atoms are essentially motionless and lie at a known and controllable distance from one another, permitting qualitatively new studies of inter-atomic interactions not accessible in a gas cell or an atomic beam. On the other hand it was discovered that cold trapped atoms can be prepared in maximally entangled states that are isolated from its environment [4–7].

One of the simplest scheme in which entanglement can be created is a system containing two two-level atoms. A number of theoretical studies have been performed recently on preparation of a two-atom system in an entangled state, and two-atom entangled states have already been demonstrated experimentally using ultra cold trap ions [2, 8] and cavity quantum electrodynamics (QED) schemes [9]. The theoretical studies, however, have suffered from a common handicap: their choice of the collective symmetric Dicke states as the basis states of the systems [10]. The difficulty is that the collective symmetric Dicke states do not correspond in general to the eigenstates of spatially extended atomic systems, such as trapped atoms. Thus, their choice as a basis corresponds to a very special geometrical configuration of the atoms that are confined to a volume much smaller compared with the atomic resonant wavelength. In this case, the antisymmetric state of the system is ignored. Moreover, many of these studies do not take into account the near-field or the dipole-dipole interaction among the atoms. This shortcoming renders questionable the applicability of these studies to physical systems.

In this paper, we propose the collective antisymmetric state of an extended atomic system as potential state for a large steady-state entanglement in a two-atom system. Our scheme is based on the observation

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that the antisymmetric state is an example of a decoherence-free entangled state. As it was first pointed out by Dicke [10], the influence on each atomic dipole of the electromagnetic field produced by the other atomic dipoles could cause the multiatom system to behave as a single system with the entangled collective symmetric and antisymmetric states. These states decay with two significantly different, the symmetric state with enhanced and the antisymmetric state with reduced, spontaneous emission rates. This reduced spontaneous emission rate of the antisymmetric state implies that the state is weakly coupled to the environment. Thus, a two-atom system prepared in the antisymmetric state can decohere slower than independent atoms. We therefore wish to investigate the possibilities of preparing the system in an entangled decoherence-free state based on radiative properties of the collective antisymmetric states.

2 Entangled states of two atoms

We consider two atoms, $i = 1, 2$, located at $\vec{r}_i$, having lower states $|g_i\rangle$ and upper states $|e_i\rangle$ separated by energy $\hbar\omega_i$, driven by a coherent laser field. In the absence of any coupling between the atoms and the field, the energy states of the two-atom system are given by four product states

$$|g_1\rangle|g_2\rangle \,, \quad |e_1\rangle|g_2\rangle \,, \quad |g_1\rangle|e_2\rangle \,, \quad |e_1\rangle|e_2\rangle \,, \tag{1}$$

with corresponding energies

$$E_{gg} = -\hbar\omega_0 \,, \quad E_{eg} = -\hbar\Delta \,, \quad E_{ge} = \hbar\Delta \,, \quad E_{ee} = \hbar\omega_0 \,, \tag{2}$$

where $\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$ and $\Delta = \frac{1}{2}(\omega_2 - \omega_1)$.

The product states $|e_1\rangle|g_2\rangle$ and $|g_1|e_2\rangle$ form a pair of nearly degenerated states. When we include the dipole-dipole interaction between the atoms, the product states combine into two linear superpositions (entangled states), with their energies shifted from $\pm\hbar\Delta$ by the dipole-dipole interaction energy Ω_{12}. To see this, we begin with the Hamiltonian of two atoms including the dipole-dipole interaction

$$\hat{H}_{aa} = \sum_{i=1}^{2} \hbar\omega_i S_i^z + \hbar \sum_{i \neq j} \Omega_{ij} S_i^+ S_j^- \,. \tag{3}$$

In the basis of the product states (1), the Hamiltonian (3) can be written in a matrix form as

$$\hat{H}_{aa} = \hbar \begin{pmatrix} -\omega_0 & 0 & 0 & 0 \\ 0 & -\Delta & \Omega_{12} & 0 \\ 0 & \Omega_{12} & \Delta & 0 \\ 0 & 0 & 0 & \omega_0 \end{pmatrix} . \tag{4}$$

Evidently, in the presence of the dipole-dipole interaction the matrix (4) is not diagonal, which indicates that the product states (1) are not the eigenstates of two interacting atoms. The matrix (4) can be diagonalized separately for the case of identical ($\Delta = 0$) and non-identical ($\Delta \neq 0$) atoms to find eigenstates of the systems and their energies.

2.1 Identical atoms

Consider first a system of two identical atoms ($\Delta = 0$). The diagonalisation of the matrix (4) leads to the following energies and corresponding eigenstates [10, 11]

$$\begin{aligned} E_g &= -\hbar\omega_0 \,, & |g\rangle &= |g_1\rangle|g_2\rangle \,, \\ E_s &= \hbar\Omega_{12} \,, & |s\rangle &= \frac{1}{\sqrt{2}}\left(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle\right) \,, \\ E_a &= -\hbar\Omega_{12} \,, & |a\rangle &= \frac{1}{\sqrt{2}}\left(|e_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle\right) \,, \\ E_e &= \hbar\omega_0 \,, & |e\rangle &= |e_1\rangle|e_2\rangle \,. \end{aligned} \tag{5}$$

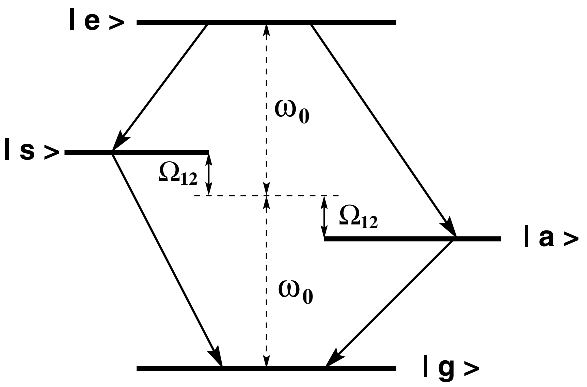

Fig. 1 Collective states of two identical atoms.

The eigenstates (5), first introduced by Dicke [10], are known as the collective states of two interacting atoms. The ground state $|g\rangle$ and the upper state $|e\rangle$ are product states of the atomic states, while the states $|s\rangle$ and $|a\rangle$ are linear superpositions of the product states that form maximally entangled states of the two-atom system. We show the collective states of two identical atoms in Fig. 1. It is seen that in the collective states representation, the two-atom system behaves as a single four-level system, with the ground state $|g\rangle$, the upper state $|e\rangle$, and two intermediate states: the symmetric state $|s\rangle$ and the antisymmetric state $|a\rangle$. The energies of the intermediate states depend on the dipole-dipole interaction and these states suffer a large shift when the inter-atomic separation is small. There are two transition channels $|e\rangle \rightarrow |s\rangle \rightarrow |g\rangle$ and $|e\rangle \rightarrow |a\rangle \rightarrow |g\rangle$, each with two cascade non-degenerate transitions. For two identical atoms, these two channels are uncorrelated, but the transitions in these channels are damped with significantly different rates. To illustrate these features, we can write the standard master equation of the system [11, 12] in the basis of the collective states as

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}_{as}, \hat{\rho}\right] + \left(\frac{\partial}{\partial t}\hat{\rho}\right)_s + \left(\frac{\partial}{\partial t}\hat{\rho}\right)_a \; , \tag{6}$$

where

$$\begin{aligned}
\hat{H}_{as} &= \hbar\left[\omega_0\left(A_{ee} - A_{gg}\right) + \Omega_{12}\left(A_{ss} - A_{aa}\right)\right] \\
&\quad -\frac{\hbar}{2\sqrt{2}}\left(\Omega_1 + \Omega_2\right)\left[\left(A_{es} + A_{sg}\right)\mathrm{e}^{\mathrm{i}(\omega_L t + \phi_L)} + \mathrm{H.c.}\right] \\
&\quad -\frac{\hbar}{2\sqrt{2}}\left(\Omega_2 - \Omega_1\right)\left[\left(A_{ea} - A_{ag}\right)\mathrm{e}^{\mathrm{i}(\omega_L t + \phi_L)} + \mathrm{H.c.}\right] \; ,
\end{aligned} \tag{7}$$

is the Hamiltonian of the dipole-dipole interacting atoms driven by a laser field of frequency ω_L and phase ϕ_L, $A_{ij} = |i\rangle\langle j|$, $(i, j = e, a, s, g)$, are collective operators that represent the energies $(i = j)$ of the collective states and coherences $(i \neq j)$, and $\Omega_i = \Omega \exp(i\vec{k}_L \cdot \vec{r}_i)$ is the Rabi frequency of the laser field of the propagation vector $\vec{k}_L$.

The dissipative part of the master equation (6) is composed of two terms. The term

$$\begin{aligned}
\left(\frac{\partial}{\partial t}\hat{\rho}\right)_s &= -\frac{1}{2}\left(\Gamma + \Gamma_{12}\right)\left\{\left(A_{ee} + A_{ss}\right)\hat{\rho} + \hat{\rho}\left(A_{ee} + A_{ss}\right)\right. \\
&\quad \left. -\, 2\left(A_{se} + A_{gs}\right)\hat{\rho}\left(A_{es} + A_{sg}\right)\right\} \; ,
\end{aligned} \tag{8}$$

describes dissipation through the cascade $|e\rangle \rightarrow |s\rangle \rightarrow |g\rangle$ channel involving the symmetric state $|s\rangle$, and

$$\begin{aligned}
\left(\frac{\partial}{\partial t}\hat{\rho}\right)_a &= -\frac{1}{2}\left(\Gamma - \Gamma_{12}\right)\left\{\left(A_{ee} + A_{aa}\right)\hat{\rho} + \hat{\rho}\left(A_{ee} + A_{aa}\right)\right. \\
&\quad \left. -\, 2\left(A_{ae} - A_{ga}\right)\hat{\rho}\left(A_{ea} - A_{ag}\right)\right\} \; ,
\end{aligned} \tag{9}$$

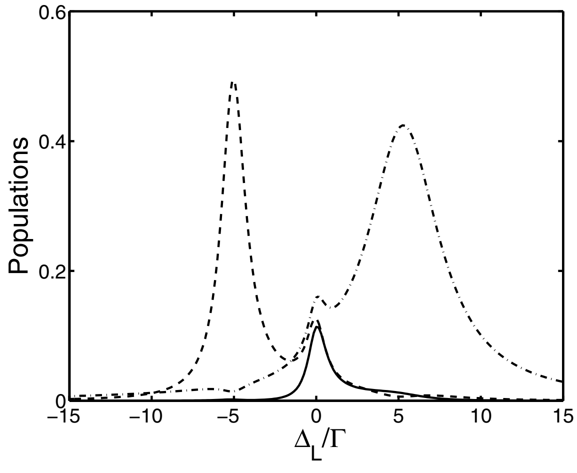

Fig. 2 The steady-state populations of the collective atomic states of two identical atoms as a function of Δ_L for the driving field propagating in the direction of the inter-atomic axis, $\Omega = 2.5\Gamma$, $r_{12}/\lambda = 0.08$: ρ_{ee} (solid line), ρ_{aa} (dashed line), ρ_{ss} (dashed-dotted line).

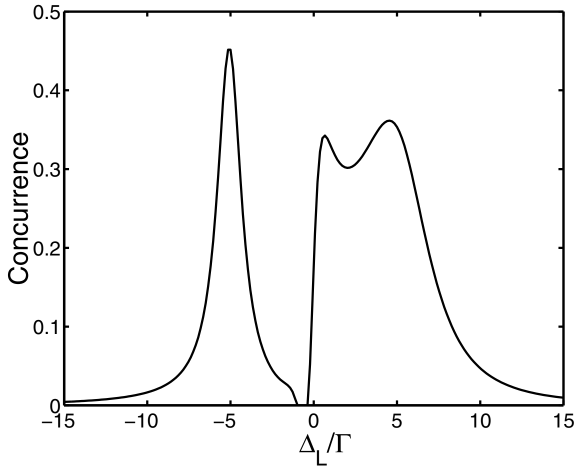

Fig. 3 Steady-state values of concurrence for the same parameters as in Fig. 2.

describes dissipation through the cascade $|e\rangle \to |a\rangle \to |g\rangle$ channel involving the antisymmetric state $|a\rangle$, where Γ_{ij} is the collective damping parameter [11] that depends on the inter-atomic separation $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$. For large separations $\Gamma_{12} \approx 0$, whereas $\Gamma_{12} \approx \Gamma$ for separations smaller than the resonant wavelength.

One can see from Eqs. (6)–(9) that the symmetric state is superradiant with all transition rates to or from the state being proportional to $(\Gamma + \Gamma_{12})$, whereas the antisymmetric state is subradiant with all transition rates to and from the state being proportional to $(\Gamma - \Gamma_{12})$. For $\Gamma = \Gamma_{12}$, which appears when the inter-atomic separation is much smaller than the resonant wavelength, the antisymmetric state decouples from the driving field and does not decay. In this case, the system decays only through the symmetric channel. Hence, for $\Gamma_{12} = \Gamma$ the system reduces to a three-level cascade system, referred to as the small-sample model or two-atom Dicke model [10, 11]. The model assumes that the atoms are close enough that we can ignore any effects resulting from different spatial positions of the atoms. In other words, the phase factors $\exp(i\vec{k} \cdot \vec{r}_i)$ are assumed to have the same value for all the atoms, and are set equal to one. This assumption may prove difficult in experimental realization as the present atom trapping and cooling techniques can trap two atoms at distances of the order of a resonant wavelength [1–3]. At these distances the collective damping parameter Γ_{12} differs significantly from Γ, and we cannot ignore the transitions to and from the antisymmetric state. We illustrate such a situation in Figs. 2 and 3, where we plot the populations of the collective excited states and concurrence as a function of $\Delta_L = \omega_L - \omega_0$ for the case of the laser field propagating in the direction of the inter-atomic axis. It is seen that by tuning the laser field to the appropriate transition, we can obtain considerable steady-state population of either symmetric or antisymmetric state. Since the two states are maximally entangled states one can also expect considerable amount of entanglement which is measured by the value of concurrence, which is zero for product states and unity for the maximally entangled state. The steady-state values of concurrence for the situation illustrated in Fig. 2 are shown in Fig. 3. It is seen that a considerable amount of entanglement can be achieved in the steady state when the laser is tuned to either $|s\rangle \to |g\rangle$ or $|a\rangle \to |g\rangle$ transition. The concurrence, however, is smaller than $1/2$, as it is not possible in this system to obtain ρ_{ss} or ρ_{aa} greater than $1/2$. This situation can be different if one considers two non-identical atoms.

2.2 Non-identical atoms

Non-identical atoms can differ both in their frequencies ($\omega_1 \neq \omega_2$ or $\Delta \neq 0$) and their damping rates ($\Gamma_1 \neq \Gamma_2$). In this case, the states (5) are no longer eigenstates of the Hamiltonian (3). The matrix (4) can be diagonalized with $\Delta \neq 0$ and new energies and states can be found [13]. However, the resulting states are not the maximally entangled states, as for identical atoms. A different choice of collective states is possible which leads to the uncorrelated dissipative parts of the symmetric and antisymmetric transitions [5]. Moreover, it is possible to create an entangled state in the system of two non-identical atoms which can be decoupled from the external environment and, at the same time, the state exhibits a strong coherent coupling with the remaining states. The transfer of population to the antisymmetric state $|a\rangle$ from the upper state $|e\rangle$ and the symmetric state $|s\rangle$, which does not appear when the atoms are identical, is possible for non-identical atoms.

We illustrate this effect in Fig. 4, where we plot the steady-state population of the maximally entangled state $|a\rangle$ as a function of Δ_L for a very small interatomic separation, such that $\Omega_1 = \Omega_2$, and two different types of non-identical atoms. In the first case the atoms have the same damping rates ($\Gamma_1 = \Gamma_2$) but different transition frequencies ($\Delta \neq 0$), while in the second case the atoms have the same frequencies ($\Delta = 0$) but different damping rates ($\Gamma_1 \neq \Gamma_2$). It is seen from Fig. 4 that in both cases the antisymmetric state can be populated even if it is not directly driven from the ground state. This becomes most effective for $\Delta_L = -\Omega_{12}$. For this detuning and strong driving field the atomic population can be transferred completely into the antisymmetric state $|a\rangle$, which is the maximally entangled state.

To illustrate the level of entanglement in the system, we plot in Fig. 5 the steady-state values of the concurrence as a function of the detuning Δ_L. As it is evident, for both types of atoms it is possible to obtain a large entanglement in the system when the laser field is detuned from resonance. For atoms with different transition frequencies driven by a strong laser field the value of concurrence close to unity can be achieved, which means perfect entanglement. This reflects the fact that for non-identical atoms it is possible,

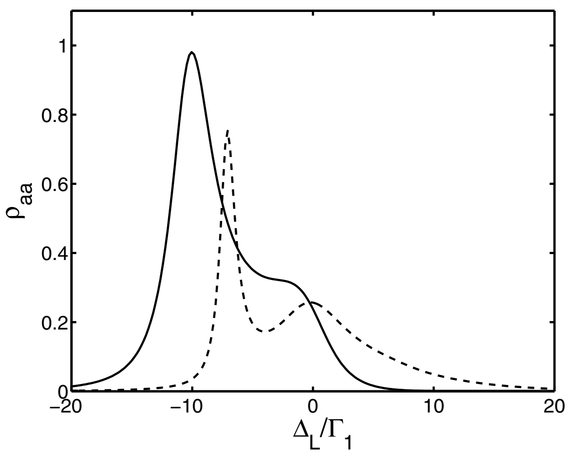

Fig. 4 The steady-state populations of maximally entangled antisymmetric state $|a\rangle$ for $\Omega = 10\Gamma_1$, $\Omega_{12} = 10\Gamma_1$ and $\Gamma_2 = \Gamma_1$, $\Delta = \Gamma_1$ (solid line), $\Gamma_2 = 2\Gamma_1$, $\Delta = 0$ (dashed line).

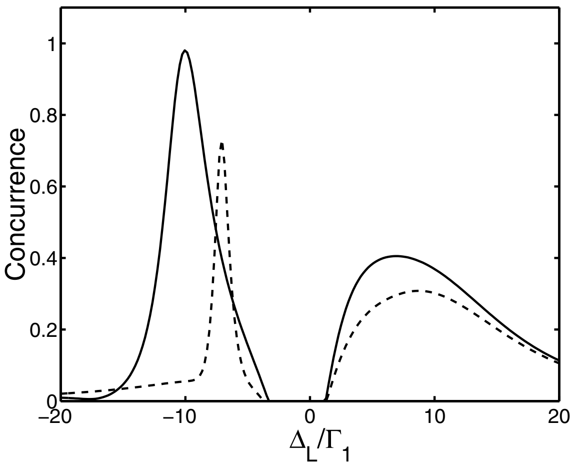

Fig. 5 Steady state values of concurrence for the same parameters as in Fig. 4.

contrary to the identical atoms, to collect total atomic population in the decoherence-free antisymmetric state $|a\rangle$.

3 Conclusion

In this paper we have investigated entanglement in a system of two two-level atoms. We have shown that considerable amount of entanglement can be produced in the system for both identical as well as non-identical atoms. The effective production of entanglement is related to preparation of the system in one of the maximally entangled states: the antisymmetric state or the symmetric state. The decoherence-free antisymmetric state which is most promising for quantum information processing can be effectively populated in steady-state situations. We have identified new mechanisms for the transfer of population to the antisymmetric state.

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