

The effect of a non-zero spontaneous decay rate on teleportation

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Abstract

We discuss the influence of the spontaneous decay rate of the excited state on teleportation of the atomic state via cavity decay in the scheme of Bose *et al* (Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158). We show that even a small but non-zero decay rate of the excited state leads to significant effects such as lowering the probability of successful teleportation if the product of the saturation parameter and the spontaneous decay rate is not negligible in comparison with the cavity mode decay rate. We compare analytical results obtained using adiabatic elimination with the results calculated numerically.

Keywords: Quantum teleportation, quantum information processing

1. Introduction

Spontaneous decay can play a destructive role in quantum information processing. The decay is responsible for decoherence and loss of information stored in a quantum system. However, spontaneous decay can also be helpful in quantum information processing. Detection of the decay allows entanglement, which is essential for many quantum applications. Researchers usually consider decays from cavities in this way, but many systems consist of optical cavities containing one atom [1–3] or two atoms [4]. Research is motivated by the fact that atomic states are ideal for storing quantum information. Spontaneous emission by the atoms, which is in effect unwanted decay leading to decoherence, has to be dealt with. One can avoid the problem by using a three-level atom in the Λ configuration. There are two stable ground states and one excited state of the atom, as shown in figure 1. The whole information processing procedure can be performed in such a way that the excited state will always have a small population. Under this condition, the excited atomic states can be eliminated adiabatically. If the population of the excited state is small enough, we can also neglect the spontaneous decay rate from this state. A teleportation scheme working in this way has recently been proposed [1]. However, one has to be very careful, because even a very small population can produce significant effects. This is because of the fact that the probability for an emission during the whole quantum

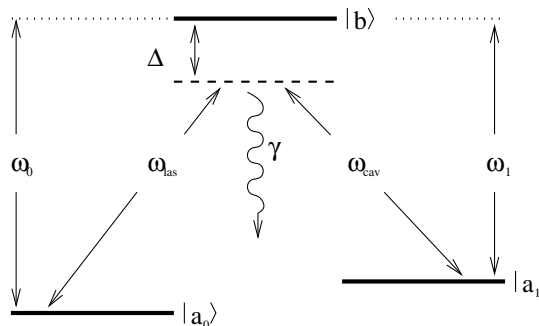


Figure 1. Atomic levels scheme. The $|a_0\rangle$ – $|b\rangle$ transition is driven by a classical laser field of frequency ω_{las} and the $|a_1\rangle$ – $|b\rangle$ transition is driven by the quantized cavity mode of frequency ω_{cav} . The laser and quantized fields are both detuned from the upper atomic state by Δ .

teleportation process is proportional to the product of the mean population of the excited state, the spontaneous decay rate and the total operation time. In the scheme of teleportation via cavity decay, discussed here, the process time has to be long compared with the inverse of the cavity decay rate κ^{-1} . If the time turns out to also be long compared with the inverse of the product of the atomic decay rate γ multiplied by the mean population in the upper level, the probability for the emission can take significant values and cannot be neglected even if the population of the excited state is very small.

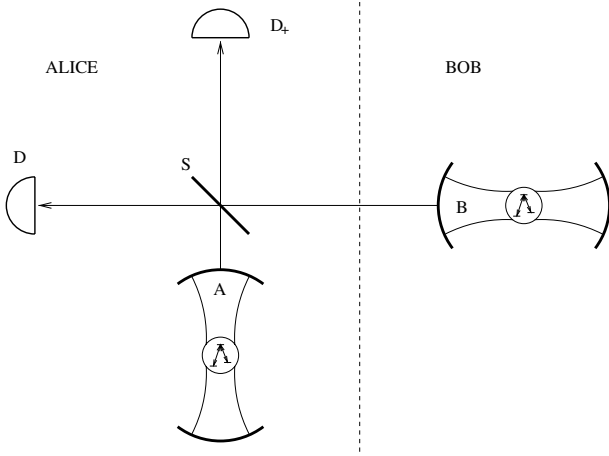


Figure 2. Schematic representation of the system. The atoms are coupled to cavities and are manipulated by lasers. The beamsplitter S , the atom-cavity system A and the detectors D_+ and D_- are at Alice's site. At Bob's site there is only the atom-cavity system B .

In this paper we study the effect of a non-zero spontaneous decay rate γ on the teleportation process. For this purpose we calculate analytically the evolution of the system determined by the adiabatically eliminated effective Hamiltonian. Moreover, we compare analytical results with the numerically calculated results for evolution of the system without applying adiabatic elimination. In the calculations we use a conditional time evolution approach [5, 6].

2. Model

We examine a model of a teleportation device proposed by Bose *et al* [1]. The device consists of two atom-cavity systems, a 50–50 beamsplitter and two detectors D_+ and D_- , as depicted in figure 2. One of the atom-cavity systems belongs to Bob. The other elements are at Alice's site. Either of the atom-cavity systems is a three-level Λ atom trapped in an optical cavity. The atom has two ground states $|a_0\rangle$, $|a_1\rangle$ and an excited state $|b\rangle$. Initially, Alice's atom is prepared in an unknown quantum superposition of the two ground states. Alice can teleport the unknown state to Bob. To perform this process, three stages are necessary. First, Alice and Bob need to prepare their atom-cavity systems. Next, Alice waits for a finite time detecting decays from the cavities. Alice informs Bob about her measurement result. Finally Bob performs a unitary operation on his atom-cavity system depending on the result obtained by Alice to recover Alice's unknown state. A preparatory stage is necessary because the device uses an atomic state for storage and photonic states to transfer quantum information. Therefore, Alice has to map her atomic state to her cavity state [7] to enable the transfer and Bob has to create a maximally entangled state of his atom and his cavity to store the transferred qubit in his atom. The most important parts of the device in the preparatory stage are the atom-cavity systems. Therefore, we now describe the model system in detail. There are two transitions in the Λ atom. The first, the $|a_0\rangle$ – $|b\rangle$ transition, is driven by a classical laser field with frequency ω_{las} . The coupling between the atom and the field for this transition is denoted by Ω . The second, the $|a_1\rangle$ – $|b\rangle$

transition, is coupled to the quantized mode with frequency ω_{cav} and coupling strength g . We assume that the couplings Ω and g are constants. This can be achieved by trapping the atom in a specific position in the cavity. Moreover, we assume that the laser and the cavity modes are detuned from the corresponding transitions by Δ . In the present context there are two decay mechanisms: spontaneous emission from the excited state $|b\rangle$ with rate γ and photons leaking out of the cavity at a rate κ . During the time intervals when no count is detected, the evolution of the quantum system is determined by the effective non-Hermitian Hamiltonian [2]

$$H_{eff} = H_0 + H_{int} \quad (1)$$

where

$$H_0 = -(\Delta + i\gamma)|b\rangle\langle b| - i\kappa c^\dagger c, \quad (2)$$

$$H_{int} = \Omega|b\rangle\langle a_0| + gc|b\rangle\langle a_1| + \Omega^*|a_0\rangle\langle b| + g^*c^\dagger|a_1\rangle\langle b|. \quad (3)$$

The annihilation and creation operators of the cavity-mode are denoted, respectively, by c and c^\dagger . The detuning is given by $\Delta = \omega_{las} - \omega_0$. The Hamiltonian is written in a frame rotating at the laser frequency. The evolution generated by (1) is interrupted by collapses with the collapse operator

$$S = \sqrt{2\kappa}c. \quad (4)$$

We want only the dark states of the atom to be effective. Thus, we choose such parameters that the population of the excited state is very small. Then the upper level can be eliminated adiabatically [8]. The Hamiltonian takes the new form, which makes analytical calculations possible [2]

$$H_{eff} = H_0 + H_1 + H_{int}, \quad (5)$$

where

$$H_0 = (\Delta - i\gamma)s_{00}|a_0\rangle\langle a_0|, \quad (6)$$

$$H_1 = (\Delta - i\gamma)s_{11}c^\dagger|a_1\rangle\langle a_1| - i\kappa c^\dagger c, \quad (7)$$

$$H_{int} = (\Delta - i\gamma)s_{01}c|a_0\rangle\langle a_1| + (\Delta - i\gamma)s_{10}c^\dagger|a_1\rangle\langle a_0|. \quad (8)$$

s_{00} , s_{01} , s_{10} and s_{11} are the saturation parameters:

$$s_{00} = \frac{|\Omega|^2}{\Delta^2 + \gamma^2}, \quad (9)$$

$$s_{11} = \frac{|g|^2}{\Delta^2 + \gamma^2}, \quad (10)$$

$$s_{01} = s_{10}^* = \frac{\Omega^*g}{\Delta^2 + \gamma^2}. \quad (11)$$

To apply adiabatic elimination the saturation parameters have to be much smaller than unity

$$s_{ii} \ll 1, \quad i = 0, 1. \quad (12)$$

The Hamiltonian (5) can take an even simpler form. First, we can avoid atom dissipative terms if we assume that the spontaneous emission rate γ is much smaller than the detuning Δ

$$\gamma \ll \Delta. \quad (13)$$

Thus, γ is set to zero in the following part of this section. Next, we assume that $g = \Omega$ (g is taken to be real), obtaining the simplest form of the Hamiltonian

$$H_{eff} = E|a_0\rangle\langle a_0| + Ec^+c|a_1\rangle\langle a_1| + E(c|a_0\rangle\langle a_1| + c^+|a_1\rangle\langle a_0|) - i\kappa c^+c, \quad (14)$$

where

$$E = \frac{g^2}{\Delta}. \quad (15)$$

Note the presence of the operator c^+c in the second term of (14) which has been omitted in [1]. The detection stage is responsible for quantum information transfer. Alice performs joint detection of Bob's and her own cavity fields. This time, the detectors and beamsplitter are the most important parts of the device. Photon decay detected by D_+ corresponds to action of the operator J_+ on the joint state vector of the two atom-cavity systems [9]

$$J_+ = \frac{1}{\sqrt{2}}(S_A + S_B) = \sqrt{\kappa}(c_A + c_B). \quad (16)$$

A click registered by D_- corresponds to action of the operator J_-

$$J_- = \frac{1}{\sqrt{2}}(S_A - S_B) = \sqrt{\kappa}(c_A - c_B). \quad (17)$$

Detection takes Alice a finite time t_D . Teleportation will be successful if Alice registers only one click during the time. Otherwise, when Alice does not register any click or registers two clicks, information is lost. Thus, the teleportation protocol is probabilistic. However, it can be converted to a reliable protocol [1, 10, 11]. Even if the transfer is successful, Bob's atom will not be in Alice's atom's initial state. Therefore the post-detection stage is necessary. In the last stage Bob uses the second local operation—Zeeman evolution to give $|a_1\rangle$ an extra phaseshift with respect to $|a_0\rangle$. We show in the next section how this phaseshift should be chosen.

3. Teleportation without a spontaneous decay rate of the excited state

In this section we show how to perform the whole teleportation process using the simplest form of the Hamiltonian (14). At the beginning of the teleportation protocol, both cavity fields are empty. The initial state of the Alice's atom-cavity system is

$$|\Psi\rangle_A = a|a_0\rangle_A|0\rangle_A + b|a_1\rangle_A|0\rangle_A, \quad (18)$$

where a and b are complex amplitudes that describe an unknown state Alice wants to teleport. Bob's atom is prepared in the ground state $|a_0\rangle_B$. Thus, the initial state of Bob's atom-cavity system is given by

$$|\Psi\rangle_B = |a_0\rangle_B|0\rangle_B. \quad (19)$$

First, Alice and Bob perform the preparatory stage. In this stage Alice has to map her atomic state to her cavity state and Bob has to create a maximally entangled state of his atom and his cavity system. They can realize their objectives by turning the lasers on for a definite period of time. The laser field driven atom-cavity system starts its evolution governed by the

operator e^{-iHt} . The effect of this operator on the initial states of Alice's and Bob's atom-cavity systems is

$$e^{-iHt}|\Psi\rangle_A = a\alpha(t)|a_1\rangle_A|1\rangle_A + b|a_1\rangle_A|0\rangle_A + a\theta(t)|a_0\rangle_A|0\rangle_A, \quad (20)$$

$$e^{-iHt}|\Psi\rangle_B = \alpha(t)|a_1\rangle_B|1\rangle_B + \theta(t)|a_0\rangle_B|0\rangle_B, \quad (21)$$

where

$$\alpha(t) = -2i\frac{E}{\Omega_\kappa}e^{-iEt}e^{-\frac{\kappa t}{2}}\sin\left(\frac{\Omega_\kappa t}{2}\right), \quad (22)$$

$$\theta(t) = e^{-iEt}e^{-\frac{\kappa t}{2}}\left[\cos\left(\frac{\Omega_\kappa t}{2}\right) + \frac{\kappa}{\Omega_\kappa}\sin\left(\frac{\Omega_\kappa t}{2}\right)\right]. \quad (23)$$

Alice turns off the laser after time t_A , where t_A is defined by:

$$e^{-iHt_A}|\Psi\rangle_A = a\alpha|a_1\rangle_A|1\rangle_A + b|a_1\rangle_A|0\rangle_A. \quad (24)$$

This implies that the values of t_A and α are given by:

$$t_A = \frac{2}{\Omega_\kappa}\left[\arctan\left(-\frac{\Omega_\kappa}{\kappa}\right) + k\pi\right], \quad (25)$$

$$\alpha = \alpha(t_A), \quad (26)$$

where

$$\Omega_\kappa = \sqrt{4E^2 - \kappa^2} \quad (27)$$

and k is an arbitrary integer. To achieve this state of Alice's atom-cavity system defined by (24), the evolution cannot be interrupted by collapses. Since the Hamiltonian (14) is non-Hermitian, the norm of the state vector (24) decreases with time. The probability that no collapse occurs during time t_A is given by the squared norm of the state vector:

$$P_{ND}(A) = |\alpha\alpha|^2 + |b|^2. \quad (28)$$

We want the fidelity of the teleportation to be close to unity and we want $P_{ND}(A) \approx 1$. Thus, we want $|\alpha|^2 \approx 1$, which leads to another condition

$$\Omega_\kappa \gg \kappa. \quad (29)$$

Bob switches off the evolution after shorter time t_B defined by

$$e^{-iHt_B}|\Psi\rangle_B = \beta(|a_0\rangle_B|0\rangle_B + i|a_1\rangle_B|1\rangle_B), \quad (30)$$

where β is a complex number. From (30) we find that

$$t_B = \frac{2}{\Omega_\kappa}\left[\arctan\left(-\frac{\Omega_\kappa}{2E + \kappa}\right) + k\pi\right], \quad (31)$$

$$\beta = -i\alpha(t_B). \quad (32)$$

The probability of no collapse during the time interval t_B is given by:

$$P_{ND}(B) = 2|\beta|^2 \quad (33)$$

Naturally, we want $P_{ND}(B) \approx 1$. This can be done provided that $\Omega_\kappa \gg \kappa$. The probability that the whole preparation stage is successful is

$$P_{ND}(\text{prep}) = P_{ND}(A)P_{ND}(B). \quad (34)$$

This is the end of the preparatory stage and the beginning of the detection stage. In this stage Alice waits for a finite time t_D registering decays from the cavities. The quantum transfer

will be successful if Alice registers only one collapse at time $t_j \leq t_D$. In the absence of a laser field ($\Omega = 0$), the evolution of the atom–cavity systems is governed by

$$H_{eff} = Ec^+c|a_1\rangle\langle a_1| - i\kappa c^+c. \quad (35)$$

Before time t_j , the states of Alice’s and Bob’s systems are given by the following unnormalized states

$$|\tilde{\Psi}(t)\rangle_A = \frac{1}{\sqrt{|\alpha\alpha|^2 + |b|^2}}(a\alpha e^{-iEt}e^{-\kappa t}|1\rangle_A + b|0\rangle_A), \quad (36)$$

$$|\tilde{\Psi}(t)\rangle_B = \frac{1}{\sqrt{2}}(|a_0\rangle_B|0\rangle_B + ie^{-iEt}e^{-\kappa t}|a_1\rangle_B|1\rangle_B). \quad (37)$$

The evolution is interrupted by a collapse at time t_j , which corresponds to the action of jump operator J_+ or J_- on the joint state of Alice’s and Bob’s systems

$$|\Phi(t)\rangle = |\Psi(t)\rangle_A \otimes |\Psi(t)\rangle_B. \quad (38)$$

After that the joint state becomes

$$|\tilde{\Phi}(t)\rangle^{J_{\pm}} = N(t_j)[(a\alpha|a_0\rangle_B \pm ib|a_1\rangle_B) \otimes |0\rangle_A|0\rangle_B + ia\alpha e^{-iEt}e^{-\kappa t}|a_1\rangle_B \otimes (|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B)], \quad (39)$$

where

$$N(t) = [|b|^2 + |\alpha\alpha|^2(1 + 2e^{-2\kappa t})]^{-\frac{1}{2}}. \quad (40)$$

From t_j until t_D the evolution is governed by (35). Finally, the joint state is given by

$$|\Phi(t_D)\rangle^{J_{\pm}} = N(t_D)[(a\alpha|a_0\rangle_B \pm ib|a_1\rangle_B) \otimes |0\rangle_A|0\rangle_B + ia\alpha e^{-iEt_D}e^{-\kappa t_D}|a_1\rangle_B \otimes (|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B)]. \quad (41)$$

We now can see that the joint state is close to Alice’s initial state for $t_D \gg \kappa^{-1}$. Those states will be almost the same if Bob gives $|a_1\rangle$ an extra phaseshift with respect to $|a_0\rangle$ in the post-detection stage. Taking into account the above assumptions, we have

$$\alpha \approx -1. \quad (42)$$

Thus, the phaseshift must be i if D_+ had clicked and $-i$ if D_- had clicked. This is the end of the whole teleportation process.

In the next step, we want to estimate the fidelity of the teleported state and the probability of success for teleportation. The average density matrix of Bob’s atom is given by

$$\rho = N^2(t_D)(|\alpha\alpha|^2|a_0\rangle_{BB}\langle a_0| + |b|^2|a_1\rangle_{BB}\langle a_1| - a\alpha b^*|a_0\rangle_{BB}\langle a_1| - a^*\alpha^*b|a_1\rangle_{BB}\langle a_0| + 2|a\alpha|^2e^{-2\kappa t_D}|a_1\rangle_{BB}\langle a_1|), \quad (43)$$

and the fidelity is

$$F(t_D, a, b) = N^2(t_D)[|\alpha\alpha|^2(|\alpha|^2 + 2|b|^2e^{-2\kappa t_D}) + |b|^4 - 2\text{Re}(\alpha)|\alpha|^2|b|^2]. \quad (44)$$

The fidelity (44) depends on the moduli of the amplitudes a and b of the initial state which are in general unknown. Therefore, we estimate the average fidelity of teleportation which is taken over all possible input states. This average fidelity takes the form

$$\bar{F}(t_D) = \frac{1}{C} \left[\frac{A}{3} + B - \frac{A}{C} + \left(C - B + \frac{A}{C} \right) \times \frac{1}{\sqrt{C}} \arctan(\sqrt{C}) \right], \quad (45)$$

where

$$A = |\alpha|^2 + 1 + 2\text{Re}(\alpha) - 2|\alpha|^2e^{-2\kappa t_D}, \quad (46)$$

$$B = 2|\alpha|^2e^{-2\kappa t_D} - 2\text{Re}(\alpha) - 2, \quad (47)$$

$$C = |\alpha|^2 - 1 + 2|\alpha|^2e^{-2\kappa t_D}. \quad (48)$$

The probability of a successful teleportation process is then given by

$$P(t_D, a, b) = P_{ND}(\text{prep})P_{1D}(\text{detec}), \quad (49)$$

where $P_{1D}(\text{detec})$ is the probability of observing only one detection during the detection time. The probability $P_{1D}(\text{detec})$ is equal to

$$P_{1D}(\text{detec}) = \int_0^{t_D} P_{ND}(0, t_j)P_{1D}(t_j)P_{ND}(t_j, t_D) dt_j, \quad (50)$$

where the probability for a photon emission to occur in the interval $(t_j, t_j + dt_j)$ is given by the formula

$$P_{1D}(t_j) dt_j = \langle \Phi(t_j)|J_+^+J_+|\Phi(t_j)\rangle dt_j + \langle \Phi(t_j)|J_-^+J_-|\Phi(t_j)\rangle dt_j. \quad (51)$$

The probability for no photon emission to occur in the interval $(0, t_j)$ is then

$$P_{ND}(0, t_j) = \langle \tilde{\Phi}(t_j)|\tilde{\Phi}(t_j)\rangle \quad (52)$$

and the probability for no photon emission to occur in the interval (t_j, t_D) takes the form

$$P_{ND}(t_j, t_D) = J_{\pm}^{\pm} \langle \tilde{\Phi}(t_D)|\tilde{\Phi}(t_D)\rangle^{J_{\pm}}. \quad (53)$$

Using the above relations, we obtain

$$P(t_D, a, b) = \frac{1}{2}P_{ND}(B)(1 - e^{-2\kappa t_D})(P_{ND}(A) + 2|a\alpha|^2e^{-2\kappa t_D}). \quad (54)$$

One can see that the probability (54) also depends on the input state. Thus, we again take the average over all input states of the probability of success and arrive at the formula

$$\bar{P}(t_D) = \frac{1}{6}P_{ND}(B)(1 - e^{-2\kappa t_D})(|\alpha|^2 + 2|\alpha|^2e^{-2\kappa t_D} + 2). \quad (55)$$

In order to obtain the results given by (45) and (55), we have assumed $\gamma = 0$. In the next section we will show that this simplification leads to essential differences in the results.

4. Teleportation with a spontaneous decay rate of the excited state

Let us plot the average fidelity of teleportation (45) and the average probability (55) as functions of the detection time t_D . We choose the parameters in such a way that all assumptions (12), (13) and (29) are satisfied [1]

$$(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:1:100) \text{ MHz}. \quad (56)$$

Figure 3 shows that the fidelity increases with the length of the detection stage and it does not differ significantly from unity for $t_D = 50 \mu\text{s}$. For reference, we also calculate numerically the fidelity of teleportation using the fourth-order Runge–Kutta integration of the non-unitary Schrödinger equation with the non-Hermitian Hamiltonian (1). Moreover, we use the Monte Carlo technique to estimate the average fidelity taken over all possible input states. As one can see from figure 3, the numerical results (points) differ significantly from the analytical results (solid curve). The average fidelity calculated numerically increases faster than the analytical one.

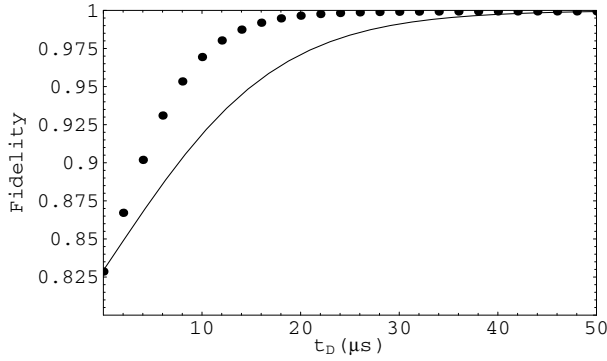


Figure 3. The average fidelity of teleportation given by (45) (solid line) as a function of the detection time t_D for $(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:1:100)$ MHz. The points show the average fidelity over 10 000 random initial states calculated numerically using the most general Hamiltonian (1).

Likewise, the average probability, as given by (55), increases with the length of the detection period as shown in figure 4. We find that the probability is about 0.48 for $t_D = 50 \mu\text{s}$. Thus, the probability is lower than the success probability for the ideal teleportation which is 0.5. This is related to the fact that the average probability of success in the preparatory stage is only 0.97. Nevertheless, the results are close to the ideal teleportation case. In order to verify the results obtained analytically we compute the average probability numerically using the non-Hermitian Hamiltonian (1). We estimate the integral in formula (50) by the sum, rewriting the dt_j as a finite step Δt_j . We compute the average probability using the trapezoidal rule. As can be seen from figure 4, the more general Hamiltonian gives the average probability (points) about two times smaller than the Hamiltonian (14) (solid curve). In order to find the reason for this behaviour we generalize the procedure to the case of a non-zero spontaneous decay rate using Hamiltonian (5) in our derivations. To simplify the following, we assume that $g = \Omega$ (g is assumed to be real). Thus, we can introduce only one saturation parameter s . Performing the teleportation process in the same way as in the previous section, we find more general forms for all the results. Now, Alice's state after time t is given by

$$e^{-iHt}|\Psi\rangle_A = a\alpha(t)|a_1\rangle_A|1\rangle_A + b|a_1\rangle_A|0\rangle_A + a\theta(t)|a_0\rangle_A|0\rangle_A, \quad (57)$$

where

$$\alpha(t) = -\frac{(\Delta - i\gamma)s}{\Omega_{\kappa,\gamma}} e^{-i\Delta st} e^{-(\frac{1}{2}\kappa + \gamma s)t} (e^{\frac{1}{2}i\Omega_{\kappa,\gamma}t} - e^{-\frac{1}{2}i\Omega_{\kappa,\gamma}t}), \quad (58)$$

$$\theta(t) = \frac{e^{-i\Delta st} e^{-(\frac{1}{2}\kappa + \gamma s)t}}{2} \times \left[e^{\frac{1}{2}i\Omega_{\kappa,\gamma}t} + e^{-\frac{1}{2}i\Omega_{\kappa,\gamma}t} - \frac{i\kappa}{\Omega_{\kappa,\gamma}} (e^{\frac{1}{2}i\Omega_{\kappa,\gamma}t} - e^{-\frac{1}{2}i\Omega_{\kappa,\gamma}t}) \right], \quad (59)$$

and

$$\Omega_{\kappa,\gamma} = \sqrt{4(\Delta - i\gamma)^2 s^2 - \kappa^2}. \quad (60)$$

The probability that no collapse takes place during time t_A is given by

$$P_{ND}(A) = |a\alpha|^2 + |b|^2 + |a\theta|^2. \quad (61)$$

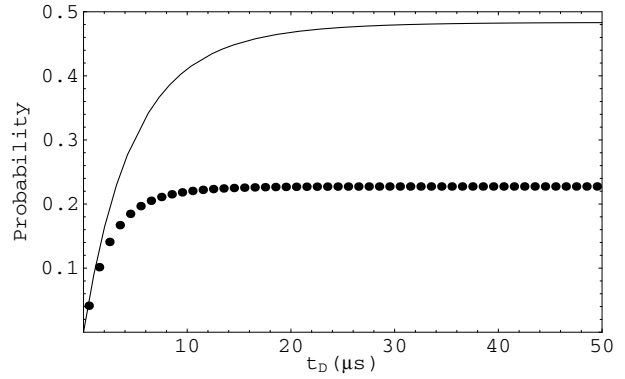


Figure 4. The average probability of success as a function of the detection time t_D for $(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:1:100)$ MHz given by (55) (solid curve) and calculated numerically using Hamiltonian (1).

The mapping efficiency of the atomic state onto the light state is close to unity if θ vanishes, therefore we need such t_A and the parameters which make θ as small as possible. The modulus squared of θ is given by

$$|\theta|^2 = \frac{1}{2(r^2 + u^2)} e^{-(\kappa + 2\gamma s)t_A} [2\kappa r \sin(rt_A) + 2\kappa u \sinh(ut_A) + (-\kappa^2 + r^2 + u^2) \cos(rt_A) + (\kappa^2 + r^2 + u^2) \cosh(ut_A)], \quad (62)$$

where r and u are the real and imaginary parts respectively of $\Omega_{\kappa,\gamma}$. One can see that the following conditions are required to make $|\theta|^2 \approx 0$

$$rt_A \approx \pi, \quad ut_A \ll 1, \quad \kappa^2 \ll r^2 + u^2. \quad (63)$$

The conditions (63) are equivalent to the condition

$$\Delta s \gg \gamma s + \kappa. \quad (64)$$

In the above limits, r and u can be well approximated by the zeroth-order expansion in γ/Δ and $\kappa/\Delta s$

$$r = 2\Delta s, \quad (65)$$

$$u = -2\gamma s. \quad (66)$$

Since we want the probability and the fidelity of teleportation to be close to unity, $|\alpha|^2$ has to be close to unity. The expression for $|\alpha|^2$ is given by

$$|\alpha|^2 = \frac{2(\Delta^2 + \gamma^2)s^2}{r^2 + u^2} e^{-(\kappa + 2\gamma s)t_A} (\cosh(ut_A) - \cos(rt_A)). \quad (67)$$

One can see that the conditions (63), if satisfied, give the value of the modulus squared of α close to unity, as expected. In order to estimate t_A we can use the following relations

$$\frac{d}{dt}|\theta|^2 = 0, \quad (68)$$

$$\frac{d^2}{dt^2}|\theta|^2 > 0. \quad (69)$$

Taking into account the limits (64), we get for t_A the following expression

$$t_A = \frac{\Delta(8\Delta g^2 \kappa + 8g^4 \pi + \Delta^2 \kappa^2 \pi)}{16g^6} + \frac{\gamma^2 \kappa(5\Delta \kappa \pi + g^2(\pi^2 - 4))}{8g^6} + \frac{\gamma^3(16\Delta g^2 \kappa \pi + 4g^4 \pi^2 + \Delta^2 \kappa^2(\pi^2 + 16))}{16g^6 \Delta^2}. \quad (70)$$

Bob switches on his laser for time t_B obtaining the quantum state given by

$$e^{-iHt} |\Psi\rangle_B = \beta_1(t) |a_0\rangle_B |0\rangle_B + i\beta_2(t) |a_1\rangle_B |1\rangle_B, \quad (71)$$

where

$$\beta_1(t) = \theta(t), \quad (72)$$

$$\beta_2(t) = -i\alpha(t). \quad (73)$$

Bob has to create a maximally entangled state, thus t_B should be calculated under the conditions

$$\frac{d}{dt} |\beta_1 - \beta_2|^2 = 0, \quad (74)$$

$$\frac{d^2}{dt^2} |\beta_1 - \beta_2|^2 > 0. \quad (75)$$

The probability for no collapse during the time t_B is then given by

$$P_{ND}(B) = |\beta_1|^2 + |\beta_2|^2. \quad (76)$$

In order to simplify the expressions for $|\beta_1 - \beta_2|^2$ and $P_{ND}(B)$, we use approximate forms (65) and (66) for r and u respectively. The expressions are given by

$$|\beta_1 - \beta_2|^2 = \frac{1}{\delta} e^{-(\kappa+2\gamma s)t_B} [\kappa^2 \cosh(2\gamma s t_B) + (4\Delta s \kappa + \delta) (\sin(2\Delta s t_B) + \cosh(2\gamma s t_B)) - \kappa(\kappa + 4\Delta s) \cos(2\Delta s t_B) + 4\gamma s \kappa \sinh(2\gamma s t_B)], \quad (77)$$

$$|\beta_1|^2 + |\beta_2|^2 = \frac{1}{\delta} e^{-(\kappa+2\gamma s)t_B} [4\gamma s \kappa \sinh(2\gamma s t_B) - \kappa^2 \cos(2\Delta s t_B) + (\kappa^2 + \delta) \cosh(2\gamma s t_B) + 4\Delta s \kappa \sin(2\Delta s t_B)], \quad (78)$$

where $\delta = 8(\Delta^2 + \gamma^2)s^2$. We want $|\beta_1 - \beta_2|^2 \approx 0$ and $P_{ND}(B) \approx 1$, therefore the following conditions are necessary:

$$2\Delta s t_A \approx \frac{3}{2}\pi \quad 2\gamma s t_A \ll 1 \quad \kappa^2 \ll (\Delta^2 + \gamma^2)s^2. \quad (79)$$

The conditions (79) are consistent with the assumptions (64), which we use to solve equation (74). The final result for t_B is then given by

$$t_B = \frac{\Delta(8\Delta g^2 \kappa + 24g^4 \pi + 3\Delta^2 \kappa^2 \pi)}{32g^6} + \frac{\gamma^2 \kappa(\Delta \kappa(9\pi + 2) + g^2(9\pi^2 - 8))}{32g^6} + \frac{\gamma^3(96\Delta g^2 \kappa \pi + 72g^4 \pi^2 + \Delta^2 \kappa^2(45\pi^2 + 32))}{128\Delta^2 g^6}. \quad (80)$$

The average probability of success at the preparation stage is given by

$$P_{ND}(\text{prep}) = P_{ND}(B) \frac{1}{3} (|\alpha|^2 + |\theta|^2 + 2). \quad (81)$$

Propagation without the laser field ($\Omega = 0$) in the detection stage is governed by

$$H_{eff} = (\Delta - i\gamma) s c^+ c |a_1\rangle \langle a_1| - i\kappa c^+ c, \quad (82)$$

hence the evolution of Alice's and Bob's states takes the form

$$|\tilde{\Psi}(t)\rangle_A = \frac{\alpha \alpha e^{-i\Delta s t} e^{-(\kappa+\gamma s)t} |a_1\rangle_A |1\rangle_A + b |a_1\rangle_A |0\rangle_A + a \theta |a_0\rangle_A |0\rangle_A}{\sqrt{|\alpha|^2 + |b|^2 + |a\theta|^2}}, \quad (83)$$

$$|\tilde{\Psi}(t)\rangle_B = \frac{\beta_1 |a_0\rangle_B |0\rangle_B + i e^{-i\Delta s t} e^{-(\kappa+\gamma s)t} \beta_2 |a_1\rangle_B |1\rangle_B}{\sqrt{|\beta_1|^2 + |\beta_2|^2}}. \quad (84)$$

After collapse detection at time t_j , the joint state of Alice's and Bob's systems becomes

$$|\tilde{\Phi}(t)\rangle^{j\pm} = N'(t_j) [(a\alpha\beta_1 |a_0\rangle_B \pm i b\beta_2 |a_1\rangle_B) \otimes |a_1\rangle_A |0\rangle_A |0\rangle_B + i a\alpha\beta_2 e^{-i\Delta s t} e^{-(\kappa+\gamma s)t} |a_1\rangle_A |a_1\rangle_B \otimes (|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B) \pm i a\theta\beta_2 |a_0\rangle_A |a_1\rangle_B |0\rangle_A |0\rangle_B], \quad (85)$$

where

$$N'(t) = (|\alpha|^2 |\beta_1|^2 + |b|^2 |\beta_2|^2 + |a\theta|^2 |\beta_2|^2 + 2|\alpha|^2 |\beta_2|^2 e^{-2(\kappa+\gamma s)t})^{-\frac{1}{2}}. \quad (86)$$

Finally, the joint state is given by $|\Phi(t_D)\rangle^{j\pm}$. After the detection stage Bob has to give $|a_1\rangle$ an extra phaseshift with respect to $|a_0\rangle$. We find that $\alpha \approx -1$, hence this phaseshift is i if D_+ had clicked and $-i$ if D_- had clicked. Now, we are ready for estimation of the fidelity of the teleported state and estimation of the probability of success for teleportation. In order to estimate the fidelity we have to derive the average density matrix of Bob's atom. The density matrix in this more general case takes the form

$$\rho = N'^2(t_D) [|\alpha|^2 |\beta_1|^2 |a_0\rangle_{BB} \langle a_0| + |b|^2 |\beta_2|^2 |a_1\rangle_{BB} \langle a_1| - \alpha \alpha \beta_1 b^* \beta_2^* |a_0\rangle_{BB} \langle a_1| - a^* \alpha^* \beta_1^* b \beta_2 |a_1\rangle_{BB} \langle a_0| + 2|\alpha|^2 |\beta_2|^2 e^{-2(\kappa+\gamma s)t_D} |a_1\rangle_{BB} \langle a_1| + |a\theta|^2 |\beta_2|^2 |a_1\rangle_{BB} \langle a_1|]. \quad (87)$$

Thus, the fidelity is given by

$$F(t_D, a, b) = N'^2(t_D) |\beta_2|^2 [|\alpha|^4 |\alpha|^2 |\beta_{12}|^2 + |b|^4 + |\alpha|^2 |b|^2 (|\theta|^2 - 2\text{Re}(\alpha\beta_{12}) + 2|\alpha|^2 e^{-2(\kappa+\gamma s)t_D})], \quad (88)$$

where $\beta_{12} = \beta_1/\beta_2$. As pointed out earlier, we should plot the average fidelity, taken over all possible input states, instead of the state-dependent fidelity. The average fidelity is now given by

$$\bar{F}(t_D) = \frac{1}{C} \left[\frac{A}{3} + B - \frac{A}{C} + \left(C - B + \frac{A}{C} \right) \times \frac{1}{\sqrt{C}} \arctan(\sqrt{C}) \right], \quad (89)$$

where

$$A = |\alpha|^2 |\beta_{12}|^2 + 1 + 2\text{Re}(\alpha\beta_{12}) - 2|\alpha|^2 e^{-2(\kappa+\gamma s)t_D} - |\theta|^2, \quad (90)$$

$$B = 2|\alpha|^2 e^{-2(\kappa+\gamma s)t_D} - 2\text{Re}(\alpha\beta_{12}) - 2 + |\theta|^2, \quad (91)$$

$$C = |\alpha|^2 |\beta_{12}|^2 - 1 + 2|\alpha|^2 e^{-2(\kappa+\gamma s)t_D} + |\theta|^2. \quad (92)$$

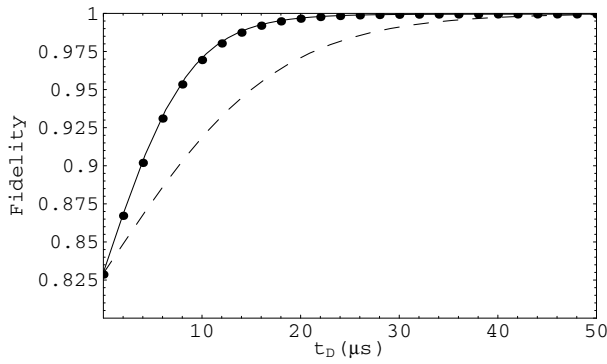


Figure 5. The average fidelity of teleportation for $(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:1:100)$ MHz using Hamiltonian (5) (solid curve) and Hamiltonian (14) (dashed curve). The points correspond to the average fidelity over 10000 random initial states calculated numerically for the same choice of parameters.

The probability of a successful teleportation process is given by

$$P(t_D, a, b) = \frac{\kappa(1 - e^{-2(\kappa+\gamma s)t_D})|\beta_2|^2}{\kappa + \gamma s} \times (|\alpha\alpha|^2(|\beta_{12}|^2 + 2e^{-2(\kappa+\gamma s)t_D}) + |b|^2 + |a\theta|^2). \quad (93)$$

We also need the average probability taken over all possible input states, which is now given by

$$\bar{P}(t_D) = \frac{\kappa(1 - e^{-2(\kappa+\gamma s)t_D})|\beta_2|^2}{3(\kappa + \gamma s)} (|\alpha|^2|\beta_{12}|^2 + |\theta|^2 + 2|\alpha|^2e^{-2(\kappa+\gamma s)t_D} + 2). \quad (94)$$

Now, we choose the same parameters (56) as in the previous section. It is easy to prove that all the assumptions (12), (13) and (64) are satisfied. Figure 5 presents the situation when Hamiltonian (5) is used in calculations (solid curve) and compares this with the situation when we take the simplest form of Hamiltonian (14) (dashed curve), and when we compute the fidelity numerically using the most general Hamiltonian (1) (points). As can be seen, in contrast to the analytical results from the last section, the results given by the Hamiltonian (5) are in a remarkable agreement with the numerical solution. This confirms that the atomic decay rate cannot be generally neglected.

It is interesting to know the importance of the effect of a non-zero spontaneous decay rate on the average probability for successful teleportation. In order to illustrate the difference between the probability with zero and with non-zero γ , we plot in figure 6 the probability given by (55) and the probability given by (94) as functions of the detection time t_D . As is evident from the figure, a non-zero spontaneous decay rate leads to a lower probability of successful teleportation. We see that it does not exceed 0.25.

It is reasonable, then, to ask: how large should γ be to make the probability close to 0.5? The answer comes from the ratio of the average probability with zero γ to the average probability with non-zero γ . Considering the limit $t_D \rightarrow \infty$, the ratio is given by

$$\frac{P_{suc}(\gamma = 0)}{P_{suc}(\gamma)} \approx \frac{\kappa}{\kappa + \gamma s}. \quad (95)$$

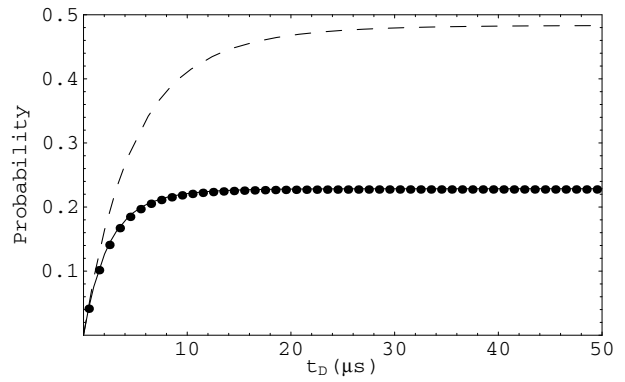


Figure 6. Comparison between the cases when the spontaneous decay rate of the excited state is neglected and when it is not. The solid line shows the average probability of success obtained using Hamiltonian (5) with non-zero γ . The dashed line corresponds to the average probability of success obtained using Hamiltonian (14) without γ . The points refer to numerical results obtained using Hamiltonian (1). The parameters regime is $(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:1:100)$ MHz.

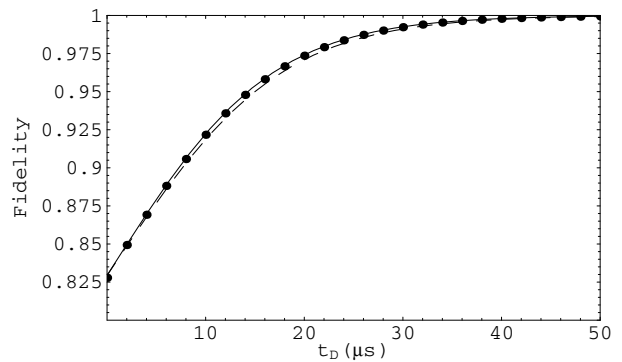


Figure 7. The average fidelity of teleportation given by (94) (solid curve), given by (55) (dashed curve) and computed numerically using the most general Hamiltonian (1) (points) for parameters satisfying assumption (96) $(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:0.05:100)$ MHz.

It is about 0.5 for the parameters which we use. To bring the teleportation probability closer to the ideal success probability, the ratio has to be close to unity. This can be done provided that

$$\kappa \gg \gamma s. \quad (96)$$

Let us choose another parameters which satisfy all the above assumptions ($s_{ii} \ll 1$, $\Delta s \gg \gamma s + \kappa$, $\kappa \gg \gamma s$):

$$(g:\Omega:\kappa:\gamma:\Delta)/2\pi = (10:10:0.01:0.05:100) \text{ MHz}. \quad (97)$$

We expect that the teleportation will be close to the ideal teleportation case for parameters (97). In order to check this expectation we plot in figure 7 the average fidelity of teleportation as a function of the detection time t_D for Hamiltonian (1) (points), Hamiltonian (5) (solid curve) and Hamiltonian (14) (dashed curve). As can be seen, it is hard to distinguish the points, showing the most general result calculated numerically, from the solid and dashed curves. A comparison between the average probability of teleportation with γ neglected (dashed curve) and with γ included (solid curve) shown in figure 8 is a final proof that the parameters (97) are chosen properly. Additionally, we plot the results obtained

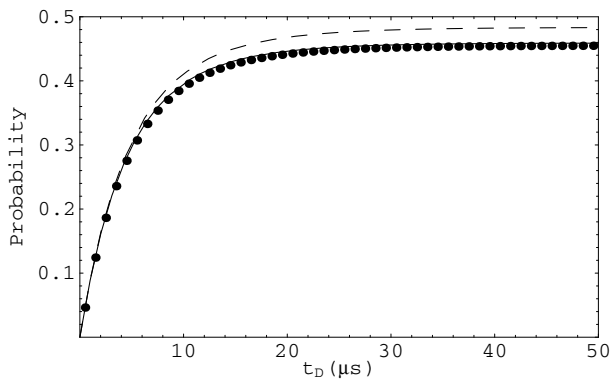


Figure 8. The average probability of success given by (94) (solid curve), given by (55) (dashed curve) and computed numerically using the Hamiltonian (1) (points) for parameters satisfying assumption (96) ($g:\Omega:\kappa:\gamma:\Delta/2\pi = (10:10:0.01:0.05:100)$ MHz).

numerically for Hamiltonian (1) (points). As is evident, the average probability of teleportation is close to 0.5 in all cases. Thus, we see that assumption (96) is necessary for the teleportation scheme to be effective. Note that we can make the undesired effect of non-zero γ even smaller by decreasing γs with respect to κ .

5. Conclusions

In this paper we have generalized the results implicit in the work of Bose *et al* [1]. We have proved that the effect of spontaneous atomic decay on the teleportation process cannot be neglected if the product of the saturation parameter s and the spontaneous decay rate γ is not much smaller than the decay rate κ of the cavity mode. Otherwise, the probability of successful teleportation is drastically lowered. This means

that the teleportation scheme does not work properly for the parameter regime suggested by Bose *et al* [1], and more restrictive conditions must be satisfied. On the other hand for sufficiently long detection times the fidelity of the teleported state can still be very good. Taking the spontaneous decay rate into account leads to a faster increase in the fidelity. Therefore, if higher fidelity is required at shorter detection times, this can be achieved by accepting lower success rates.

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