Invited Paper

Two-level atom in a structured reservoir

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ABSTRACT

A two-level atom driven by a strong resonant field and coupled to a structured reservoir is studied. The generalized master equation describing the evolution of the atom is derived under the Born-Markov approximation. To derive the master equation the dressing transformation on the atomic operators is performed first and next the dressed operators are coupled to the reservoir and the corresponding damping rates are calculated. The modifications introduced by a strong field and/or by the reservoir with non-flat density of modes lead to non-standard terms in the master equation, some of which are reminiscent of terms known for squeezed vacuum reservoirs although the reservoir is not a squeezed vacuum. The optical Bloch equations based on this generalized master equation are obtained and solved for the steady state. The resonance fluorescence spectra of the atom are shortly discussed.

Keywords: two-level atom, master equation, optical Bloch equations, optical spectra

1. INTRODUCTION

An exited atom interacting with a reservoir of electromagnetic field modes emits irreversibly in the spontaneous emission process its energy into the reservoir. Conventionally, it is supposed that the damping rate, at which the atom looses its energy by radiating photons into the reservoir, is an inherent property of the atom and can be expressed as the Einstein A coefficient for spontaneous emission. This is, however, only true when the atom is radiating to the ordinary vacuum with flat density of modes and is excited by the external field which is not too strong and does not affect the level structure of the atom. As a matter of fact, when the atom is placed inside a cavity, the atomic damping rate depends on the cavity mode structure [1–3]. The modifications are quite essential when the density of modes of the reservoir considerably depends on frequency (structured reservoir) and can be ignored when the density of modes is flat.

For the driving field which is extremely strong the energy structure of the atom is changed in such a way that also damping rates will depend on the intensity of the driving field as shown by Kocharovskaya *et al.* [4,5]. The effect of the mode structure on the atomic properties were investigated by Lewenstein *et al.* [3,6] within a non-Markovian approach. Recently, Kowalewska-Kudłaszyk and Tanaś [7,8] have studied the interaction of a two-level atom with a strong resonant field and a reservoir with a nontrivial structure of the density of modes using Markovian approach.

In this paper we present our derivation of the master equation for a two-level atom driven by a strong laser field and damped into a structured reservoir. The master equation has been derived under the Born and Markov approximations, however, it takes into account the dependence of the atomic relaxation rates on the strength of the driving field as well as on the density of modes of the reservoir with a non flat mode structure. Our master equation has an operator form similar to that known for ordinary vacuum, and it is a generalization of earlier results. A very simple and transparent form of the master equation allows, for example, an easy identification of new squeezing-like terms, which are reminiscent of the real squeezing terms in the master equation for an atom in a squeezed vacuum. Some consequences of the new terms in the master equation are discussed here.

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2. DERIVATION OF THE MASTER EQUATION

A model we study here includes a two-level atom driven by a strong monochromatic laser field of frequency ω_L with the Rabi frequency Ω and detuned by $\Delta = \omega_L - \omega_A$ from the atomic transition frequency ω_A. The atom is placed in a reservoir with a density of modes structure that depends non-trivially on frequency. The traditional approach to include damping into the evolution of an atom driven by a strong laser field assumes that the damping rates do not depend on the strength of the field and are the same as for spontaneous emission. An alternative approach takes into account the interaction of the strong field with the atom first to derive the "dressed states" of the atom, and next takes the coupling of the dressed atomic states to the reservoir in order to calculate the damping rates [9]. The order in which the two couplings are invoked is important and leads to different results [10]. In our calculations we first perform the dressing transformation on the atomic operators and only after that we couple the dressed atomic operators to the reservoir. This leads to a quite simple and transparent, operator form of the master equation which is a generalization of the standard form of the master equation.

We start with the Hamiltonian of the system which has the form

$$H = H_A + H_R + H_L + H_I\,, \tag{1}$$

where

$$H_A = \frac{1}{2}\hbar\omega_A\,\sigma_z = -\frac{1}{2}\hbar\Delta\,\sigma_z + \frac{1}{2}\hbar\,\omega_L\,\sigma_z \tag{2}$$

is the Hamiltonian of the atom, which we split into two parts for further convenience,

$$H_R = \hbar\int_0^\infty \omega\, b^+(\omega)\, b(\omega)\,\mathrm{d}\omega\,, \tag{3}$$

is the Hamiltonian of the reservoir field,

$$H_L = \frac{1}{2}\hbar\Omega\left[\sigma_+\exp(-i\omega_L t - i\varphi) + \sigma_-\exp(i\omega_L t + i\varphi)\right] \tag{4}$$

is the interaction between the atom and the classical laser field, and

$$H_I = i\hbar\int_0^\infty K(\omega)\left[\sigma_+\, b(\omega) - b^+(\omega)\,\sigma_-\right]\mathrm{d}\omega \tag{5}$$

describes the interactions of the atom and the reservoir. Operators $b(\omega)$ and $b^+(\omega)$ are the annihilation and creation operators of the reservoir field, respectively, satisfying the bosonic commutation relation, $\sigma_\pm$ are the atomic raising and lowering operators, σ_z is the atomic inversion operator, and $K(\omega)$ is the coupling between the atom and the reservoir which is related to the natural atomic linewidth γ (FWHM) through the relation

$$K^2(\omega) = \frac{\gamma}{2\pi}\left(\frac{\omega}{\omega_A}\right)^3\eta(\omega)\,. \tag{6}$$

In further calculations, to simplify notation, the phase of the driving field ϕ will be absorbed in the atomic operators, $\sigma_-\exp(i\phi)\to\sigma_-$, $\sigma_+\exp(-i\phi)\to\sigma_+$.

As a first step we transform the Hamiltonian into the frame rotating with the laser frequency ω_L, which gives us

$$H_0 = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(\sigma_+ + \sigma_-)\,, \tag{7}$$

$$\begin{aligned} H_I^r(t) = {} & i\hbar\int_0^\infty K(\omega)\{\sigma_+ b(\omega)\exp[i\phi + i(\omega_L-\omega)t] \\ & - b^+(\omega)\sigma_-\exp[-i\phi - i(\omega_L-\omega)t]\}\,\mathrm{d}\omega \end{aligned} \tag{8}$$

The second step is the unitary dressing transformation performed with the Hamiltonian (7)

$$\sigma_\pm(t) = \exp\left[-\frac{i}{\hbar}H_0 t\right]\sigma_\pm\exp\left[\frac{i}{\hbar}H_0 t\right], \tag{9}$$

which leads to the following time-dependent atomic raising and lowering operators

$$\sigma_{\pm}(t) \;=\; \frac{1}{2}\left[\mp(1\pm\tilde{\Delta})\tilde{\sigma}_{-}\exp(-i\Omega' t) \pm (1\mp\tilde{\Delta})\tilde{\sigma}_{+}\exp(i\Omega' t) + \tilde{\Omega}\tilde{\sigma}_{z}\right], \tag{10}$$

where

$$\begin{aligned}\tilde{\sigma}_{-} &= \frac{1}{2}\left[(1-\tilde{\Delta})\sigma_{-} - (1+\tilde{\Delta})\sigma_{+} - \tilde{\Omega}\sigma_{z}\right],\\ \tilde{\sigma}_{+} &= \frac{1}{2}\left[-(1+\tilde{\Delta})\sigma_{-} + (1-\tilde{\Delta})\sigma_{+} - \tilde{\Omega}\sigma_{z}\right],\\ \tilde{\sigma}_{z} &= \tilde{\Omega}(\sigma_{-}+\sigma_{+}) - \tilde{\Delta}\sigma_{z},\end{aligned} \tag{11}$$

and

$$\Omega' \;=\; \sqrt{\Omega^2+\Delta^2},\quad \tilde{\Omega} = \frac{\Omega}{\Omega'},\quad \tilde{\Delta} = \frac{\Delta}{\Omega'}. \tag{12}$$

After the dressing transformation (9) the interaction Hamiltonian (5) takes the form

$$\begin{aligned}H_I(t) &= i\hbar\int_0^{\infty} K(\omega)\{\sigma_{+}(t)b(\omega)\exp[i\phi + i(\omega_L-\omega)t]\\ &\quad - b^{+}(\omega)\sigma_{-}(t)\exp[-i\phi - i(\omega_L-\omega)]\}\mathrm{d}\omega,\end{aligned} \tag{13}$$

where $\sigma_{+}(t)$ and $\sigma_{-}(t)$ are given by (10).

Using standard methods [11], we derive the master equation, in the Born approximation, for the reduced atomic density matrix

$$\frac{\partial\rho^{(d)}}{\partial t} \;=\; -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{[H_I(t),[H_I(t-\tau),\rho_R(0)\rho^{(d)}(t-\tau)]]\right\}\mathrm{d}\tau, \tag{14}$$

where the superscript (d) stands for the dressed picture, $\rho_R(0)$ is the density operator for the field reservoir. Tr_R is the trace over the reservoir states and the Hamiltonian $H_I(t)$ is given by (13). At this stage we make the Markov approximation [11] by replacing $\rho^{(d)}(t-\tau)$ in (14) by $\rho^{(d)}(t)$, substitute the Hamiltonian (13) and take the trace over the reservoir variables, assuming that

$$\begin{aligned}\mathrm{Tr}_R\{b(\omega)b^{+}(\omega)\rho_R(0)\} &= [N(\omega)+1]\delta(\omega-\omega'),\\ \mathrm{Tr}_R\{b^{+}(\omega)b(\omega)\rho_R(0)\} &= N(\omega)\delta(\omega-\omega'),\end{aligned} \tag{15}$$

where $N(\omega)$ is the mean number of photons at frequency ω. In the Markov approximation we extend the upper limit of the integral over τ to infinity and perform necessary integrations using the formula

$$\int_0^{\infty}\exp(\pm i\epsilon\tau)\,\mathrm{d}\tau \;=\; \pi\delta(\epsilon) \pm i\,\mathcal{P}\frac{1}{\epsilon}, \tag{16}$$

where $\mathcal{P}$ means the Cauchy principal value.

After performing all the calculations and transforming back from the dressed picture to the original density operator, in the frame rotating with the laser frequency, we finally arrive at the following master equation

$$\begin{aligned}\frac{\partial}{\partial t}\rho &= \frac{i}{2}\Delta'\,[\sigma_z,\rho] - \frac{i}{2}\Omega\,[\sigma_{+}+\sigma_{-},\rho]\\ &\quad + \frac{1}{2}N\,(2\sigma_{+}\rho\sigma_{-} - \sigma_{-}\sigma_{+}\rho - \rho\sigma_{-}\sigma_{+})\\ &\quad + \frac{1}{2}(N+a)\,(2\sigma_{-}\rho\sigma_{+} - \sigma_{+}\sigma_{-}\rho - \rho\sigma_{+}\sigma_{-})\\ &\quad - M\sigma_{+}\rho\sigma_{+} - M^{\star}\sigma_{-}\rho\sigma_{-}\\ &\quad + \frac{1}{2}L\,[\sigma_{+},\rho\sigma_z] - \frac{1}{2}L^{\star}\,[\sigma_{-},\sigma_z\rho]\\ &\quad + \frac{1}{2}(L+b)\,[\sigma_{-},\rho\sigma_z] - \frac{1}{2}(L+b)^{\star}\,[\sigma_{+},\sigma_z\rho],\end{aligned} \tag{17}$$

where

$$
\begin{aligned}
\Delta^{'} &= \Delta + \Delta_p\,, \\
\Delta_p &= \frac{\gamma}{8}\Big[(1+\tilde{\Delta})^2(1+2N_-)b_- + (1-\tilde{\Delta})^2(1+2N_+)b_+ \\
&\quad +2(1-\tilde{\Delta}^2)(1+2N_0)b_0\Big]\,, \\
N &= \frac{\gamma}{4}\Big[(1+\tilde{\Delta})^2N_-a_- + (1-\tilde{\Delta})^2N_+a_+ + 2(1-\tilde{\Delta}^2)N_0a_0\Big]\,, \\
a &= \frac{\gamma}{4}\Big[(1+\tilde{\Delta})^2a_- + (1-\tilde{\Delta})^2a_+ + 2(1-\tilde{\Delta}^2)a_0\Big]\,, \\
M &= \frac{\gamma}{8}(1-\tilde{\Delta}^2)\left[(1+2N_-)(a_- - ib_-) + (1+2N_+)(a_+ - ib_+)\right. \\
&\quad \left. - 2(1+2N_0)(a_0 - ib_0)\right]\,, \\
L &= \frac{\gamma}{4}\tilde{\Omega}\Big[(1+\tilde{\Delta})N_-(a_- + ib_-) - (1-\tilde{\Delta})N_+(a_+ + ib_+) \\
&\quad - 2\tilde{\Delta}N_0(a_0 + ib_0)\Big]\,, \\
b &= \frac{\gamma}{4}\tilde{\Omega}\Big[(1+\tilde{\Delta})(a_- + ib_-) - (1-\tilde{\Delta})(a_+ + ib_+) - 2\tilde{\Delta}(a_0 + ib_0)\Big]\,,
\end{aligned}
\tag{18}
$$

where $\eta(\omega)$ describes the deviation of the reservoir density of modes from the vacuum density of modes (for vacuum $\eta(\omega) = 1$). The other quantities are defined by

$$
\begin{aligned}
N_0 &= N(\omega_L)\,, \quad N_\pm = N(\omega_L \pm \Omega^{'})\,, \\
a_0 &= \left(\frac{\omega_L}{\omega_A}\right)^3 \eta(\omega_L)\,, \quad a_\pm = \left(\frac{\omega_L \pm \Omega^{'}}{\omega_A}\right)^3 \eta(\omega_L \pm \Omega^{'})\,, \\
b_0 &= -\frac{1}{\gamma}\mathcal{P}\int_0^\infty \frac{K^2(\omega)}{\omega_L - \omega}\mathrm{d}\omega\,, \quad b_\pm = -\frac{1}{\gamma}\mathcal{P}\int_0^\infty \frac{K^2(\omega)}{\omega_L - \omega \pm \Omega^{'}}\mathrm{d}\omega\,,
\end{aligned}
\tag{19}
$$

where $N(\omega)$ is the mean number of photons at frequency ω. Lamb shifts are included via the redefinition of the atomic transition frequency. We have also included in (18) the shifts coming from the principal value terms. They contribute essentially to the atomic evolution in the cavity situation when the density of modes $\eta(\omega)$ has a non-trivial ω dependence.

We model the reservoir density of modes by a dimensionless Lorentzian $\eta(\omega)$ centered at frequency ω_c with the width γ_c

$$
\eta(\omega) = \frac{\gamma_c^2}{(\omega - \omega_c)^2 + \gamma_c^2}\,. \tag{20}
$$

For very broad reservoir, ($\gamma_c \to \infty$), the Lorentzian (20) becomes constant ($\eta(\omega) \to 1$), and our results reproduce the results for ordinary vacuum.

For the reservoir described by the Lorentzian (20), parameters b_0 and $b_\pm$ take the following form

$$
\begin{aligned}
b_0 &= -\frac{1}{2}\left(\frac{\omega_L}{\omega_A}\right)^3 \frac{\delta_c\gamma_c}{\delta_c^2 + \gamma_c^2}\,, \\
b_\pm &= -\frac{1}{2}\left(\frac{\omega_L \pm \Omega^{'}}{\omega_A}\right)^3 \frac{(\delta_c \pm \Omega^{'})\gamma_c}{(\delta_c \pm \Omega^{'})^2 + \gamma_c^2}\,.
\end{aligned}
\tag{21}
$$

It is clear from (21) that the shifts coming from the principal value terms give nonzero contributions for moderately intense laser fields and reservoirs with finite bandwidth. The most interesting situations appear when the peak of the Lorentzian is centered at the laser frequency ($\delta_c = 0$) or the Rabi sidebands ($\delta_c = \pm\Omega^{'}$).

From the form of the master equation (17) one can easily identify the new terms coming from the interaction of the atom with a very strong laser field, seen via the ω^3 terms in $a_\pm$, and/or from the interaction with the reservoir, seen via the density of modes $\eta(\omega_L \pm \Omega')$ at the dressed atom transitions and via the principal value terms b_0 and $b_\pm$. Our master equation is a generalization, on the one hand, of the Bloch equations introduced by Kocharovskaya *et al.* [4] and, on the other hand, the Bloch equations introduced by Keitel *et al.* [10] for the cavity situation in the secular approximation.

For weak driving fields and thermal reservoirs, for which $a_\pm = a_0 = 1$ and $N_\pm = N_0$, our master equation (17) has the well known standard form, but for stronger fields and/or structured reservoirs it is easy, from the transparent form of the equation (17), to identify the new terms proportional to M, L and b. Terms proportional to M are reminiscent of the terms appearing when the atom is coupled to squeezed reservoirs [12,13]. These terms, however, are not the real squeezing terms, because, despite the fact that they depend of the phase of the driving field, they do not give the phase dependence of the fluorescence or absorption spectra, which is a characteristic feature for squeezed reservoirs.

3. GENERALIZED BLOCH EQUATIONS

Having the generalized master equation (17) at hand, it is immediate to derive the generalized Bloch equations describing the evolution of the expectation values of the atomic operators. The Bloch equations obtained from (17) can be written in the following matrix form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \langle\sigma_-(t)\rangle \\ \langle\sigma_+(t)\rangle \\ \langle\sigma_z(t)\rangle \end{pmatrix} = \mathbf{A}\begin{pmatrix} \langle\sigma_-(t)\rangle \\ \langle\sigma_+(t)\rangle \\ \langle\sigma_z(t)\rangle \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b_r - i\Lambda_i \\ b_r + i\Lambda_i \\ -2a \end{pmatrix}, \tag{22}$$

where the matrix $\mathbf{A}$ has the form

$$\mathbf{A} = \begin{pmatrix} i\Delta' - \Gamma & -M & \frac{i}{2}\Omega \\ -M^\star & -i\Delta' - \Gamma & -\frac{i}{2}\Omega \\ i(\Omega + b_i) + \Lambda_r & -i(\Omega + b_i) + \Lambda_r & -2\Gamma \end{pmatrix}. \tag{23}$$

To shorten the notation we denote the real part of any complex number Q by Q_r and the imaginary part by Q_i, and we have also used the replacements

$$\Gamma = \frac{1}{2}(a + 2N), \quad \Lambda = b + 2L. \tag{24}$$

When we introduce the Hermitian operators representing the two quadrature components of the atomic dipole moment

$$\sigma_x = \frac{1}{2}(\sigma_- + \sigma_+), \quad \sigma_y = \frac{1}{2i}(\sigma_- - \sigma_+), \tag{25}$$

the Bloch equations (17) can be rewritten in terms of the new operators as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \langle\sigma_x(t)\rangle \\ \langle\sigma_y(t)\rangle \\ \langle\sigma_z(t)\rangle \end{pmatrix} = \mathbf{B}\begin{pmatrix} \langle\sigma_x(t)\rangle \\ \langle\sigma_y(t)\rangle \\ \langle\sigma_z(t)\rangle \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b_r \\ -\Lambda_i \\ -2a \end{pmatrix} \tag{26}$$

with the matrix $\mathbf{B}$ given by

$$\mathbf{B} = \begin{pmatrix} -\Gamma - M_r & -\Delta' + M_i & 0 \\ \Delta' + M_i & -\Gamma + M_r & \frac{1}{2}\Omega \\ \Lambda_r & -2(\Omega + b_i) & -2\Gamma \end{pmatrix}. \tag{27}$$

The generalized Bloch equations (17) and (26) are different from the standard Bloch equations. The relaxation rates have been obtained by coupling the dressed atom rather than the bare atom to the reservoir, so they take into account the dependence of the relaxation rates on the strength of the laser field and the structure of the reservoir modes including the shifts which are non-zero when the density of modes is not flat. From the form of the matrix $\mathbf{B}$ it is easily seen that two components of the atomic dipole moment, $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ decay with different rates when M_r is different from zero. This effect is well known for squeezed reservoirs [14], but here it is associated with the modification of the damping rates by very strong fields and/or the non-flat density of modes of the reservoir. A new feature of the Bloch equations (26) is the presence of the shifts b_0 and $b_\pm$, given by (21), which do not appear in other Markovian approaches, but they do appear in the non-Markovian approach [6]. On neglecting the shifts, the Bloch equations (22) are equivalent of the Bloch equations obtained by Kocharovskaya *et al.* [4].

The steady state solutions to equations (26) have the following general form

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{1}{2d}\{(a\,\Omega + 2\Gamma\Lambda_i)(\Delta' + M_i) + b_r\,[\Omega(\Omega + b_i) + 2\Gamma(\Gamma - M_r)]\}\,, \\
\langle\sigma_y\rangle_{ss} &= -\frac{1}{2d}\{(a\,\Omega + 2\Gamma\Lambda_i)(\Gamma + M_r) - b_r\,[\Omega\Lambda_r + 2\Gamma(\Delta' + M_i)]\}\,, \\
\langle\sigma_z\rangle_{ss} &= -\frac{1}{d}\{a\,(\Gamma^2 - |M|^2 + \Delta'^2) + b_r\,[(\Omega + b_i)(\Delta' - M_i) - \Lambda_r(\Gamma - M_r)] \\
&\quad -\Lambda_i[(\Omega + b_i)(\Gamma + M_r) + \Lambda_r(\Delta' + M_i)]\}\,,
\end{aligned} \tag{28}$$

where

$$d = \Omega(\Omega + b_i)(\Gamma + M_r) + \Lambda_r\Omega(\Delta' + M_i) + 2\Gamma(\Gamma^2 - |M|^2 + \Delta'^2)\,. \tag{29}$$

In particular, for very strong laser fields and equal number of photons, at each frequency, $N(\omega) = N_0 = N_\pm$, the steady state solutions for the atomic operators take the much simpler approximate form

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{\tilde{\Omega}}{2(1+2N_0)}\,\frac{(1+\tilde{\Delta})^2\,a_- - (1-\tilde{\Delta})^2\,a_+}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,, \\
\langle\sigma_y\rangle_{ss} &= -\frac{\tilde{\Omega}}{4\Omega'}\,\frac{2(1-\tilde{\Delta}^2)\,a_-\,a_+ + [(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+]\,a_0}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,, \\
\langle\sigma_z\rangle_{ss} &= -\frac{\tilde{\Delta}}{1+2N_0}\,\frac{(1+\tilde{\Delta})^2\,a_- - (1-\tilde{\Delta})^2\,a_+}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,.
\end{aligned} \tag{30}$$

From solutions (30), it is seen that for thermal reservoir and very strong laser fields the steady state inversion between the atomic states for the specific range of the laser field detuning can be realized, the effect predicted by Kocharovskaya and Radeonychev [5] and attributed to $((\omega \pm \Omega')/\omega_A)^3$ factors. Moreover, a nonzero solution for $\langle\sigma_x\rangle_{ss}$ component in the resonant case can be found. It is also obvious that placing the atom inside a cavity, where there is a peak in the density of modes at some characteristic frequency, may increase the values of $\langle\sigma_x\rangle_{ss}$ and the steady state atomic inversion [6,15].

For not too strong laser fields the exact solutions have to be used. In this laser field intensity regime it is possible to observe changes of the atomic behavior coming from the nonzero principal value terms in the case of structured reservoirs. The possibility of creating the steady state atomic inversion by tuning the cavity was discussed in the non-Markovian approach by Lewenstein and Mossberg [6]. One can see that also the much simpler Markovian approach, presented here, leads to similar effects.

4. RESONANCE FLUORESCENCE

When dealing with very strong fields and/or structured reservoirs a special care must be taken when calculating the resonance fluorescence spectrum of the atom. We define the fluorescence spectrum of light emitted into the structured reservoir modes as a rate at which the mean number of photons $b^+(\omega)b(\omega)$ of the reservoir mode at frequency ω changes in time for the steady state conditions. It is given by

$$\mathcal{F}(\omega) = \lim_{t\to\infty}\frac{d}{dt}\langle b^+(\omega,t)b(\omega,t)\rangle = \lim_{t\to\infty}\langle\frac{d}{dt}b^+(\omega,t)b(\omega,t) + b^+(\omega,t)\frac{d}{dt}b(\omega,t)\rangle \tag{31}$$

Using the Heisenberg equations of motion for the bosonic reservoir operators, we obtain the following formula for the fluorescence spectrum emitted into the cavity modes

$$\mathcal{F}(\omega) = 2K^2(\omega)\,\mathrm{Re}\int_0^\infty d\tau\langle\sigma_+(0)\sigma_-(\tau)\rangle e^{i(\omega-\omega_L)\tau}\,. \tag{32}$$

Formula (32) differs from the standard definition of the resonance fluorescence spectrum, as the Fourier transform of the atomic correlation function, by the frequency dependent factor $K^2(\omega)$, which is important here. The standard definition assumes that the atomic rate is constant. The equations of motion for the atomic correlation function appearing in (32) can be obtained from the generalized Bloch equations (22) with the use of the quantum regression theorem [16]. Taking the Laplace transform of the evolution equations for the atomic correlation functions with the appropriate initial conditions, we finally arrive at the following expression for the Laplace transform of the correlation function $\langle\sigma_+(0)\sigma_-(\tau)\rangle$, which enters the definition of the resonance fluorescence spectrum

$$
\begin{aligned}
F(z) &= \frac{1}{2zd(z)}\left\{\frac{z}{2}(1+\langle\sigma_z\rangle_{ss})\left[2(z+2\Gamma)(z+\Gamma+i\Delta')+\Omega(\Omega+b_i+i\Lambda_r)\right]\right.\\
&+ \langle\sigma_+\rangle_{ss}\Big[-i\left[\Omega(z+a)+\Lambda_i(z+2\Gamma)\right](z+\Gamma+M+i\Delta')\\
&+ \left.b_r\left[\Omega(\Omega+b_i+i\Lambda_r)+(z+2\Gamma)(z+\Gamma-M+i\Delta')\right]\Big]\right\}. \qquad (33)
\end{aligned}
$$

where

$$
d(z) = d+z\left[(z+2\Gamma)^2+\Omega(\Omega+b_i)+\Gamma^2-|M|^2+\Delta'^2\right], \qquad (34)
$$

and $d=d(0)$ is given by (29).

The Laplace transform (33) contains both the coherent and incoherent contributions to the spectrum. Coherent part of the spectrum is the delta function $\delta(\omega-\omega_L)$ centered on the laser frequency, the amplitude of which is defined by the residuum for $z=0$

$$
\mathcal{F}_{coh}(\omega) = \gamma\left(\frac{\omega}{\omega_A}\right)^3\eta(\omega)\lim_{z\to 0} zF(z)\,\delta(\omega-\omega_L). \qquad (35)
$$

The incoherent part of the spectrum can be calculated from

$$
\mathcal{F}_{inc}(\omega) = \frac{\gamma}{\pi}\left(\frac{\omega}{\omega_A}\right)^3\eta(\omega)\,\mathrm{Re}\left\{\left[F(z)-\frac{1}{z}\lim_{z\to 0} zF(z)\right]_{z=-i(\omega-\omega_L)}\right\}, \qquad (36)
$$

where we have used the expression (6) for the frequency dependent coupling constant $K(\omega)$. We would like to emphasize that the presence of this factor is necessary when one wish to derive the fluorescence spectrum into the structured reservoir modes. When the fluorescent light is emitted to the structure-less background modes the traditional definition is applicable and $K(\omega)$ can be omitted. This factor is crucial for structured reservoirs and/or very strong laser fields. However, the expressions (33) and (36) are quite general and they are applicable for both strong and weak driving fields and all reservoirs with sufficiently broad linewidth, which is much broader than the atomic linewidth to justify the Markovian approximation used to derive the master equation.

In the strong field limit the approximate roots of the denominator $d(z)$ can be found easily. In zeroth approximation in γ/Ω' they take the form [8]

$$
z_0 = -\Gamma_{pop}, \quad z_\pm = \mp i\Omega' - \Gamma_{coh}, \qquad (37)
$$

where Γ_{pop}, Γ_{coh} are given by

$$
\begin{aligned}
\Gamma_{coh} &= \Gamma+\frac{1}{2}\left[(1-\tilde{\Delta}^2)(\Gamma-M_r)-\tilde{\Delta}\tilde{\Omega}\Lambda_r\right],\\
\Gamma_{pop} &= 2\left\{\Gamma-\frac{1}{2}\left[(1-\tilde{\Delta}^2)(\Gamma-M_r)-\tilde{\Delta}\tilde{\Omega}\Lambda_r\right]\right\}. \qquad (38)
\end{aligned}
$$

The damping rates Γ_{pop} and Γ_{coh}, which describe the relaxations of the dressed populations and coherences, define the linewidths of the central line and the sidebands of the Mollow triplet, respectively. From the form of (38) we see that when M_r can take negative values, which is possible for the reservoir tuned to the central line ($\omega_c=\omega_L$), the value of Γ_{pop} can be smaller than in the standard case ($M_r=\Lambda_r=0$). This means narrowing of the central line, but a simultaneous broadening of the sidebands because Γ_{coh} becomes larger. This effect is known for the squeezed reservoirs [14]. The approximate roots (37) allow for getting simple analytical formulas for the resonance fluorescence spectrum [8], which in the secular approximation reproduces the spectra obtained by Keitel *et al.* [10].

5. CONCLUSION

In this paper we have presented the generalized master equation for the reduced density matrix within the Born and Markov approximations for a two-level atom driven by a strong laser field and damped by a structured reservoir. The master equation is valid for a wide range of laser intensities (for very strong laser fields it includes the dependence of the damping rates on the laser intensity) and different types of Markovian reservoirs. It includes the shifts coming from the nonzero principal value terms which appear for structured reservoirs. We have identified some new terms in the master equation that are reminiscent of the well known terms occurring when atom is interacting with the squeezed vacuum. Such terms lead to similar effects as if the atom were damped by a squeezed vacuum such as narrowing of the central line of the Mollow triplet or the different damping rates for the two quadrature components of the atomic dipole.We have also shortly discussed the resonance fluorescence spectra.

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