Optics and Spectroscopy, Vol. 91, No. 3, 2001, pp. 470–476. From Optika i Spektroskopiya, Vol. 91, No. 3, 2001, pp. 499–505.

QUANTUM COMPUTATIONS AND OPTICAL PROCESSES

Markovian Master Equation for a Two-Level Atom in a Strong Field and/or a Tailored Reservoir[1]

A. Kowalewska-Kudłaszyk and R. Tanaś

Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland

e-mail: annakow@kielich.amu.edu.pl

e-mail: tanas@kielich.amu.edu.pl

Received November 17, 2000

Abstract—The master equation for a two-level atom driven by a strong classical field and damped into a tailored reservoir with nonflat density of modes is derived under the Born–Markov approximation. To derive the master equation, the dressing transformation on the atomic operators is performed first, and, next, the dressed operators are coupled to the reservoir and the corresponding damping rates are calculated. The effects of a strong field and/or structured reservoir are seen as nonstandard terms in the master equation, some of which are reminiscent of terms known for squeezed vacuum reservoirs. The master equation leads to the generalized optical Bloch equations that can easily be solved for the steady state and, together with the quantum regression theorem, allow for analytical expressions for the fluorescence, as well as absorption spectra. © *2001 MAIK "Nauka/Interperiodica".*

INTRODUCTION

The coupling of an excited two-level atom to a reservoir of vacuum modes results in spontaneous emission; i.e., the atom loses its energy at the rate A being the Einstein coefficient. However, it has long been known that atomic damping rates depend on the mode structure of the atomic environment [1–3]. When the atom is driven on resonance by a strong monochromatic laser beam, the structure of atomic levels changes dramatically. For very strong fields, when the Rabi frequency becomes much larger than the spontaneous emission rate, the dressed atom picture can be used to describe atomic dynamics [4, 5]. In many cases, a strong laser field can be treated as a classical field, and the "semiclassical dressed states" can be used to describe atomic radiative properties [6–8]. Also, in the dressed atom description, the damping rates are usually treated as constants that do not depend on the strength of the applied field and the structure of the reservoir. The situation is quite different, however, when the driven atom is placed in an environment with the density of modes that appreciably depend on frequency [3, 9, 10]. Lewenstein and Mossberg [9] have analyzed the spectral and statistical properties of atoms driven by a strong, single-mode light field and atoms coupled to a reservoir of electromagnetic field modes with strong frequency dependence. They used the non-Markovian approach leading to a complicated set of equations describing the atomic dynamics. Their theory predicted a number of interesting features of the atomic spectra; one of them was an asymmetry of the fluorescence spectrum radiated to the background modes, which has been measured by Lezama *et al.* [11]. Recently, the master equation has been derived [12, 13] for the reduced atomic density matrix under the Born–Markov approximation, which takes into account the dependence of the relaxation rates on the strength of the laser field. In this master equation, even for flat reservoirs like ordinary vacuum, the relaxation rates depend on the strength of the field through the ω^3 factor in the vacuum density of modes. Keitel *et al.* [14, 15] have shown that in the secular limit, the resonance fluorescence spectra should be symmetric even for tailored reservoirs with asymmetric density of modes, despite the fact that the populations of the dressed states are not equal. The reason for this is that the difference in populations is compensated for by the difference in the transition rates between the dressed states. They emphasized that it is important for strong fields to perform the dressing operation first, and only after that to consider the coupling of the dressed atom to the reservoir modes. Results obtained in this way differ from results obtained in the conventional treatment.

In this paper, we present the master equation derived in the Born and Markov approximation for the two-level atom driven by a strong, classical laser field and placed in a reservoir that can be tailored at will [16]. Our master equation explicitly includes the dependence of the relaxation rates on the Rabi frequency of the driving field, as well as the structure of the reservoir modes, which is modeled by a Lorentzian function. In our derivation, we first perform the dressing transformation and, next, couple the dressed atomic operators to the reservoir. The order in which the two interactions are introduced into the calculations is not an obvious matter, and the results can depend on the order in which the

[1] This article was submitted by the authors in English.

calculation is carried out. This problem has been addressed by Keitel *et al.* [14]. They have provided arguments for the order in which the dressing transformation is performed before the system is coupled to the reservoir, which is important when the driving field is strong. We accept their arguments here and perform the dressing transformation first, but we do not make the secular approximation from the beginning. This allows us to make an *a posteriori* statement that this order leads to the results that correctly reproduce the traditional results in a weak field limit. Since we treat the laser field as classical, our approach is in the spirit of the semiclassical dressed states recently used by Berman [17] to calculate the resonance fluorescence and absorption spectra, except that we do not actually use the dressed states but rather work with the dressed operators. This allowed us to obtain the master equation in the operator form, which is fairly simple and yet sufficiently general to predict effects that usually require more elaborate techniques. We also derive the optical Bloch equations based on this master equation and discuss their steady-state solutions.

MARKOVIAN MASTER EQUATION

We consider a two-level atom driven by a strong monochromatic laser field of frequency ω_L with the Rabi frequency Ω and detuned by $\Delta = \omega_L - \omega_A$ from the atomic transition frequency ω_A. We derive the master equation that explicitly takes into account the dependence of atomic relaxation rates on the strength of the field, as well as the structure of the reservoir. The idea of this approach was proposed by Carmichael and Walls [18] and Cresser [19], and recently used by Yeoman and Barnett [20] and Tanaś *et al*. [21] to derive the master equation for a two-level atom damped by a squeezed vacuum with finite bandwidth. In this approach, we first perform the dressing transformation to include the interaction of the atom with the driving field, and then we couple the resulting dressed atom to the reservoir. We derive the master equation under the Markov approximation, which requires the reservoir bandwidth to be much greater than the atomic line width, but not necessarily greater than the Rabi frequency of the driving field and detuning.

We start from the Hamiltonian of the system, which in the rotating-wave and electric-dipole approximations is given by

$$H = H_A + H_R + H_L + H_I, \tag{1}$$

where

$$H_A = \frac{1}{2}\hbar\omega_A\sigma_z = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\omega_L\sigma_z \tag{2}$$

is the Hamiltonian of the atom;

$$H_R = \hbar\int_0^\infty \omega b^+(\omega)b(\omega)d\omega \tag{3}$$

is the Hamiltonian of the reservoir field;

$$H_L = \frac{1}{2}\hbar\Omega[\sigma_+\exp(-i\omega_L t - i\varphi) + \sigma_-\exp(i\omega_L t + i\varphi)] \tag{4}$$

is the interaction between the atom and the classical laser field; and

$$H_I = i\hbar\int_0^\infty K(\omega)[\sigma_+ b(\omega) - b^+(\omega)\sigma_-]d\omega \tag{5}$$

is the interaction of the atom with the reservoir. In (1)–(4), $K(\omega)$ is the coupling of the atom to the reservoir modes, $\Delta = \omega_L - \omega_A$ is the detuning of the driving laser field frequency ω_L from the atomic resonance ω_A, and σ_+, σ_-, and σ_z are the Pauli pseudo-spin operators describing the two-level atom. The laser driving field strength is given by the Rabi frequency Ω (assumed real) and its phase is φ, while the operators $b(\omega)$ and $b^+(\omega)$ are the annihilation and creation operators for the reservoir modes satisfying the commutation relation

$$[b(\omega), b^+(\omega')] = \delta(\omega - \omega'). \tag{6}$$

To derive the master equation, we perform a two-step unitary transformation. In the first step, we use the second part of the atomic Hamiltonian (1) and the free field Hamiltonian (2) to transform to the frame rotating with the laser frequency ω_L and to the interaction picture, with respect to the reservoir modes. The rotating frame is also shifted in phase by φ; i.e., we introduce new raising and lowering operators, which absorb the phase factor according to the relations

$$\sigma_- e^{i\varphi} \longrightarrow \sigma_-, \quad \sigma_+ e^{-i\varphi} \longrightarrow \sigma_+. \tag{7}$$

After these transformations, our system is described by the Hamiltonian

$$H_0 + H_I^r(t), \tag{8}$$

where

$$H_0 = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(\sigma_+ + \sigma_-) \tag{9}$$

and

$$H_I^r(t) = i\hbar\int_0^\infty K(\omega)[\sigma_+ b(\omega)\exp[i\varphi + i(\omega_L - \omega)t] - b^+(\omega)\sigma_-\exp[-i\varphi - i(\omega_L - \omega)t]]d\omega. \tag{10}$$

The second step is the unitary dressing transformation performed with the Hamiltonian H_0, given by (9). The transformation

$$\sigma_\pm(t) = \exp\left[-\frac{i}{\hbar}H_0 t\right]\sigma_\pm \exp\left[\frac{i}{\hbar}H_0 t\right] \tag{11}$$

leads to the following time-dependent atomic raising and lowering operators

$$\begin{aligned}\sigma_\pm(t) = \frac{1}{2}[&\mp(1\pm\tilde{\Delta})\tilde{\sigma}_-\exp(-i\Omega' t)\\ &\pm(1\mp\tilde{\Delta})\tilde{\sigma}_+\exp(i\Omega' t)+\tilde{\Omega}\tilde{\sigma}_z],\end{aligned} \tag{12}$$

where

$$\begin{aligned}\tilde{\sigma}_- &= \frac{1}{2}[(1-\tilde{\Delta})\sigma_- - (1+\tilde{\Delta})\sigma_+ - \tilde{\Omega}\sigma_z],\\ \tilde{\sigma}_+ &= \frac{1}{2}[-(1+\tilde{\Delta})\sigma_- + (1-\tilde{\Delta})\sigma_+ - \tilde{\Omega}\sigma_z],\\ \tilde{\sigma}_z &= \tilde{\Omega}(\sigma_- + \sigma_+) - \tilde{\Delta}\sigma_z\end{aligned} \tag{13}$$

are the dressed operators oscillating at frequencies $-\Omega'$, Ω', and 0, respectively, and

$$\tilde{\Omega} = \frac{\Omega}{\Omega'},\quad \tilde{\Delta} = \frac{\Delta}{\Omega'},\quad \Omega' = \sqrt{\Omega^2+\Delta^2}. \tag{14}$$

Since we assume $\Omega' > 0$, as $\Omega \longrightarrow 0$, the dressed operators $\tilde{\sigma}_\pm \longrightarrow \sigma_\pm$, $\tilde{\sigma}_z \longrightarrow \sigma_z$ for $\Delta < 0$, and $\tilde{\sigma}_\pm \longrightarrow -\sigma_\mp$, $\tilde{\sigma}_z \longrightarrow -\sigma_z$ for $\Delta > 0$.

Under the transformation (12), the interaction Hamiltonian takes the form

$$\begin{aligned}H_I(t) = i\hbar\int_0^\infty K(\omega)[&\sigma_+(t)b(\omega)\exp[i\varphi + i(\omega_L-\omega)t]\\ &- b^+(\omega)\sigma_-(t)\exp[-i\varphi - i(\omega_L-\omega)t]]d\omega.\end{aligned} \tag{15}$$

The master equation for the reduced density operator ρ of the system can be derived using standard methods [22]. In the Born approximation, the equation of motion for the reduced density operator is given by [22]

$$\begin{aligned}&\frac{\partial\rho^D}{\partial t}\\ &= -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\{[H_I(t),[H_I(t-\tau),\rho_R(0)\rho^D(t-\tau)]]\}d\tau,\end{aligned} \tag{16}$$

where the superscript D stands for the dressed picture, $\rho_R(0)$ is the density operator for the field reservoir, Tr_R is the trace over the reservoir states, and the Hamiltonian $H_I(t)$ is given by (15). We next make the Markov approximation [22] by replacing $\rho^D(t-\tau)$ in (16) by $\rho^D(t)$, substitute the Hamiltonian (15), and take the trace over the reservoir variables. We assume that the reservoir operators satisfy the relations

$$\begin{aligned}\mathrm{Tr}_R[b(\omega)b^+(\omega')\rho_R(0)] &= [N(\omega)+1]\delta(\omega-\omega'),\\ \mathrm{Tr}_R[b^+(\omega)b(\omega')\rho_R(0)] &= N(\omega)\delta(\omega-\omega'),\end{aligned} \tag{17}$$

where $N(\omega)$ is the mean number of photons at frequency ω. In the Markov approximation, we can extend the upper limit of the integration over τ to infinity and perform the necessary integrations using the formula

$$\int_0^\infty \exp(\pm i\epsilon\tau)d\tau = \pi\delta(\epsilon) \pm i\mathcal{P}\frac{1}{\epsilon}, \tag{18}$$

where $\mathcal{P}$ means the Cauchy principal value. In our case, ϵ takes the values $\omega_L-\omega$, $\omega_L-\omega\pm\Omega'$, so there are three different spectral contributions if the field is strong. In the traditional approach, there is only one contribution at $\omega_L = \omega$.

Performing lengthy, but straightforward calculations, including the principal value contributions, after back transformation from the dressed picture to the original operators in the frame rotating with the laser frequency ω_L and shifted in phase by φ, we obtain the master equation, which has the following form

$$\begin{aligned}\frac{\partial\rho}{\partial t} &= \frac{i}{2}\Delta'[\sigma_z,\rho] - \frac{i}{2}\Omega[\sigma_+ + \sigma_-,\rho]\\ &+ \frac{1}{2}N(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+)\\ &+ \frac{1}{2}(N+a)(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)\\ &- M\sigma_+\rho\sigma_+ - M^*\sigma_-\rho\sigma_- + \frac{1}{2}L[\sigma_+,\rho\sigma_z]\\ &- \frac{1}{2}L^*[\sigma_-,\sigma_z\rho] + \frac{1}{2}(L+b)[\sigma_-,\rho\sigma_z]\\ &- \frac{1}{2}(L+b)^*[\sigma_+,\sigma_z\rho],\end{aligned} \tag{19}$$

where

$$\begin{aligned}\Delta' &= \Delta + \Delta_p,\\ \Delta_p &= \frac{\gamma}{8}[(1+\tilde{\Delta})^2(1+2N_-)b_-\\ &+ (1-\tilde{\Delta})^2(1+2N_+)b_+ + 2(1-\tilde{\Delta}^2)(1+2N_0)b_0],\\ N &= \frac{\gamma}{4}[(1+\tilde{\Delta})^2 N_- a_-\\ &+ (1-\tilde{\Delta})^2 N_+ a_+ + 2(1-\tilde{\Delta}^2)N_0 a_0],\end{aligned}$$

$$a = \frac{\gamma}{4}[(1+\tilde{\Delta})^2 a_- + (1-\tilde{\Delta})^2 a_+ + 2(1-\tilde{\Delta}^2)a_0],$$
$$M = \frac{\gamma}{8}(1-\tilde{\Delta}^2)[(1+2N_-)(a_- - ib_-) + (1+2N_+)(a_+ - ib_+) - 2(1+2N_0)(a_0 - ib_0)], \tag{20}$$
$$L = \frac{\gamma}{4}\tilde{\Omega}[(1+\tilde{\Delta})N_-(a_- + ib_-) - (1-\tilde{\Delta})N_+(a_+ + ib_+) - 2\tilde{\Delta}N_0(a_0 + ib_0)],$$
$$b = \frac{\gamma}{4}\tilde{\Omega}[(1+\tilde{\Delta})(a_- + ib_-) - (1-\tilde{\Delta})(a_+ + ib_+) - 2\tilde{\Delta}(a_0 + ib_0)]$$

with γ being the natural atomic linewidth (FWHM), which is related to the coupling constant $K(\omega)$ (assumed to be real) by the following relation

$$K(\omega)^2 = \frac{\gamma}{2\pi}\left(\frac{\omega}{\omega_A}\right)^3 \eta(\omega), \tag{21}$$

where $\eta(\omega)$ describes the deviation of the reservoir density of modes from the vacuum density of modes for the vacuum $\eta(\omega) = 1$ and $K(\omega_A) = \sqrt{\gamma/2\pi}$ (we take into account only the frequency dependence assuming that the integration over angular variables has already been performed). The remaining quantities are defined by

$$N_0 = N(\omega_L), \quad N_\pm = N(\omega_L \pm \Omega'),$$
$$a_0 = \left(\frac{\omega_L}{\omega_A}\right)^3 \eta(\omega_L), \quad a_\pm = \left(\frac{\omega_L \pm \Omega'}{\omega_A}\right)^3 \eta(\omega_L \pm \Omega'),$$
$$b_0 = -\frac{1}{\gamma}\mathcal{P}\int_0^\infty \frac{K(\omega)^2}{\omega_L - \omega}d\omega, \tag{22}$$
$$b_\pm = -\frac{1}{\gamma}\mathcal{P}\int_0^\infty \frac{K(\omega)^2}{\omega_L - \omega \pm \Omega'}d\omega,$$

where $N(\omega)$ is the mean number of the reservoir photons at frequency ω. In the derivation of equation (19), we have included the divergent frequency shifts (the Lamb shift) to the redefinition of the atomic transition frequency [22], and we have explicitly calculated the shifts that come from the principal value terms in (22). These shifts can give contributions to the master equation when the atom is placed in a cavity with frequency-dependent density of modes and $\eta(\omega)$ has essential ω dependence. We will consider the influence of both a strong laser field and the density of modes on the two-level atom spectral properties.

The principal value terms in (22) can be evaluated when $\eta(\omega)$ is known. In our calculations, we model the mode structure by the dimensionless Lorentzian functions. Let us assume that $\eta(\omega)$ is a Lorentzian

$$\eta(\omega) = \frac{\gamma_c^2}{(\omega - \omega_c)^2 + \gamma_c^2} \tag{23}$$

with the width $\gamma_c (\gamma_c \gg \gamma)$ and centered at some frequency ω_c (for $\gamma_c \longrightarrow \infty$ $\eta(\omega) \longrightarrow 1$). Physically, this can be considered, for example, as a cavity situation. More realistic modeling of the cavity will introduce some flat background modes and cavity modes with a Lorentzian peak at the cavity resonance [3, 9]. In that case, instead of being simply a Lorentzian, our $\eta(\omega)$ would be a constant independent of ω representing the background modes plus the Lorentzian describing the cavity modes. Since we are mainly interested in structured reservoirs, in our calculations we use only the Lorentzian function to describe the non-flat reservoir, although adding a constant part would be straightforward (the constant part does not contribute to the shifts). The width γ_c should be much greater than the atomic linewidth γ so as not to violate the Markovian approximation made in the derivation of the master equation. From the definitions (22), using (23) we can calculate the parameters b_0 and $b_\pm$ in the following way

$$b_0 = -\frac{1}{\gamma}\mathcal{P}\int_0^\infty \frac{K(\omega)^2}{\omega_L - \omega}d\omega$$
$$= -\frac{1}{2\pi}\mathcal{P}\int_0^\infty \left(\frac{\omega}{\omega_A}\right)^3 \frac{\gamma_c^2}{(\omega-\omega_c)^2 + \gamma_c^2}\frac{1}{\omega_L - \omega}d\omega$$
$$\approx -\left(\frac{\omega_L}{\omega_A}\right)^3 \frac{1}{2\pi}\mathcal{P}\int_0^\infty \frac{\gamma_c^2}{(\omega-\omega_c)^2 + \gamma_c^2}\frac{1}{\omega_L - \omega}d\omega \tag{24}$$
$$= \left(\frac{\omega_L}{\omega_A}\right)^3 \frac{1}{2\pi}\mathcal{P}\int_{-\infty}^\infty \frac{\gamma_c^2}{(x^2+\gamma_c^2)(x-\delta_c)}dx$$
$$= -\frac{1}{2}\left(\frac{\omega_L}{\omega_A}\right)^3 \frac{\delta_c\gamma_c}{\delta_c^2 + \gamma_c^2},$$

where $\delta_c = \omega_L - \omega_c$, and after changing variables we have extended the integration from $-\omega_c$ to $-\infty$. Proceeding in the same manner, we get

$$b_\pm = -\frac{1}{2}\left(\frac{\omega_L \pm \Omega'}{\omega_A}\right)^3 \frac{(\delta_c \pm \Omega')\gamma_c}{(\delta_c \pm \Omega')^2 + \gamma_c^2}. \tag{25}$$

The values of the shifts depend on the width γ_c and the position of the mode density peak. The most interesting cases are when the peak is centered at the laser frequency ($\delta_c = 0$), or at the Rabi sidebands ($\delta_c = \pm\Omega'$).

Our master equation (19), in operator form, is a generalization of the standard master equation known for

the two-level atom. The generalization takes into account the dependence of the relaxation rates on the strength of the driving field, described by the dependence of $a_\pm$ on the Rabi frequency Ω' through the ω^3 terms, as well as the difference of the reservoir mode density $\eta(\omega)$ from the ordinary vacuum mode density. N_0 and $N_\pm$ are the mean number of reservoir photons at the laser frequency ω_L and at the sidebands $\omega_L \pm \Omega'$, respectively. On neglecting the shift terms, our master equation (19), although different in form, is equivalent to the generalized Bloch equations introduced by Kocharovskaya *et al.* [12]. The difference is that we have performed the dressing transformation on the operators rather than on the atomic states. We believe the advantage of our approach is a strikingly simple and transparent form of the master equation (19), which allows for easy identification of the standard terms known for ordinary vacuum and recognizing the new, nonstandard terms that appear due to the strong-field modification of the damping rates and/or tailoring of the reservoir.

For weak driving fields and thermal reservoirs ($\eta = 1$), we have $a_0 = a_\pm = 1$, and $N_0 = N_\pm$ is the mean number of photons of the reservoir, which means that $N = \gamma N_0$ and $a = \gamma$, while $M = L = b = 0$ and master equation (19) take the well-known standard form. For nonthermal or tailored reservoirs, however, for which $\eta(\omega)$ is different from unity, the new terms become important, and the atomic evolution is changed in an essential way. It is particularly interesting that the new terms, which are proportional to M and that are well known for the atom damped to the squeezed vacuum reservoir, appear in the master equation (19) despite the fact that the reservoir does not exhibit nondiagonal, phase dependent correlations. These terms appear for ordinary vacuum because of the asymmetry introduced to the system by the strong field and/or the nonflat mode structure. Other nonstandard terms are those proportional to b and L.

Since the atomic operators $\sigma_\pm$ contain, according to (7), the phase factors $\exp(\mp i\varphi)$; the terms proportional to Ω, M, b, and L in the master equation (19) are phase dependent. Their phase dependence stems solely from the phase of the driving field, and, therefore, the phase will appear in the steady-state mean values of the atomic dipole moment $\langle\sigma_\pm\rangle_{ss}$, for example, but not in the resonance fluorescence and absorption spectra, in which the phase factors cancel. This is an important difference between the squeezing-like terms, proportional to M, in our master equation and the real squeezing terms coming from the squeezed vacuum reservoir. For the squeezing reservoir, the phase dependence of these terms is $\exp[\pm i(2\varphi - \varphi_s)]$, where φ_s is the phase of the squeezed vacuum field, and even if the phase factors stemming from the driving field cancel in the resonance fluorescence spectrum, the dependence on the squeezing phase remains, and the fluorescence spectrum is sensitive to the squeezing phase. However, as it will later become clear, the phase-sensitive terms that appear in our master equation lead to some effects that are known for squeezing reservoirs, e.g., the difference in the damping rates of the two quadrature components of the atomic dipole.

BLOCH EQUATIONS

From the master equation (19), it is easy to derive the generalized Bloch equations describing the time evolution of the expectation values of the atomic operators, which take the form

$$\frac{d}{dt}\begin{pmatrix}\langle\sigma_-(t)\rangle\\ \langle\sigma_+(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} = \mathbf{A}\begin{pmatrix}\langle\sigma_-(t)\rangle\\ \langle\sigma_+(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} + \frac{1}{2}\begin{pmatrix}b_r - i\Lambda_i\\ b_r + i\Lambda_i\\ -2a\end{pmatrix}, \quad (26)$$

$$\mathbf{A} = \begin{pmatrix} i\Delta' - \Gamma & -M & \frac{i}{2}\Omega \\ -M^* & -i\Delta' - \Gamma & -\frac{i}{2}\Omega \\ i(\Omega + b_i) + \Lambda_r & -i(\Omega + b_i) + \Lambda_r & -2\Gamma \end{pmatrix}, \quad (27)$$

and we have used the convention that for any complex quantity Q

$$Q = Q_r + iQ_i, \quad (28)$$

where Q_r and Q_i denote the real and imaginary parts, respectively. For brevity of notation, we have made the substitutions

$$\Gamma = \frac{1}{2}(a + 2N), \quad \Lambda = b + 2L. \quad (29)$$

Introducing the Hermitian operators

$$\sigma_x = \frac{1}{2}(\sigma_- + \sigma_+), \quad \sigma_y = \frac{1}{2i}(\sigma_- - \sigma_+), \quad (30)$$

we get from (26) the following equations of motion for the atomic polarization quadratures

$$\frac{d}{dt}\begin{pmatrix}\langle\sigma_x(t)\rangle\\ \langle\sigma_y(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} = \mathbf{B}\begin{pmatrix}\langle\sigma_x(t)\rangle\\ \langle\sigma_y(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} + \frac{1}{2}\begin{pmatrix}b_r\\ -\Lambda_i\\ -2a\end{pmatrix} \quad (31)$$

with the matrix $\mathbf{B}$ given by

$$\mathbf{B} = \begin{pmatrix} -\Gamma - M_r & -\Delta' + M_i & 0 \\ \Delta' + M_i & -\Gamma + M_r & \frac{1}{2}\Omega \\ \Lambda_r & -2(\Omega + b_i) & -2\Gamma \end{pmatrix}. \quad (32)$$

The generalized Bloch equations (26) and (31) are different from the standard Bloch equations. The relaxation rates have been obtained by coupling the dressed atom rather than the bare atom to the reservoir, so they take into account the dependence of the relaxation rates

on the strength of the laser field and the structure of the reservoir modes including the shifts, which are nonzero when the density of modes is not flat. If we ignore the shift terms coming from the principal value contributions, our Bloch equations are equivalent to the Bloch equations obtained earlier by Kocharovskaya *et al.* [12].

It is interesting to note in the generalized Bloch equations the presence of the M terms, which have a character known from the squeezed vacuum reservoir. They introduce the coupling of $\langle\sigma_\pm\rangle$ to their complex conjugates, and as it is seen from (32), the two quadrature components have different damping rates similarly to the squeezed vacuum reservoirs [23], but the physical origin of this effect is quite different. There are also free b_r and Λ_i terms in the equations (31), which give, for example, a nonzero steady state solution for $\langle\sigma_x\rangle$. Another important feature is the presence of additional terms describing the shifts arising from the principal value contributions. They should manifest themselves in situations of moderately intense laser fields and atoms in reservoirs with frequency-dependent density of modes.

The steady state solutions to equations (31) are the following

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{1}{2d}\{(a\Omega + 2\Gamma\Lambda_i)(\Delta' + M_i) \\
&\quad + b_r[\Omega(\Omega + b_i) + 2\Gamma(\Gamma - M_r)]\}, \\
\langle\sigma_y\rangle_{ss} &= -\frac{1}{2d}\{(a\Omega + 2\Gamma\Lambda_i)(\Gamma + M_r) \\
&\quad - b_r[\Omega\Lambda_r + 2\Gamma(\Delta' + M_i)]\}, \\
\langle\sigma_z\rangle_{ss} &= -\frac{1}{d}\{a(\Gamma^2 - |M|^2 + \Delta'^2) \\
&\quad + b_r[(\Omega + b_i)(\Delta' - M_i) - \Lambda_r(\Gamma - M_r)] \\
&\quad - \Lambda_i[(\Omega + b_i)(\Gamma + M_r) + \Lambda_r(\Delta' + M_i)]\},
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
d &= \Omega(\Omega + b_i)(\Gamma + M_r) \\
&\quad + \Lambda_r\Omega(\Delta' + M_i) + 2\Gamma(\Gamma^2 - |M|^2 + \Delta'^2).
\end{aligned} \tag{34}$$

In the strong field limit, when Ω' is much greater than all the damping terms, the steady-state solutions (33) take a much simpler, approximate form (we keep only the lowest nonvanishing terms)

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{1}{2}\frac{\tilde\Delta\tilde\Omega a + (1-\tilde\Delta^2)b_r}{(1-\tilde\Delta^2)(\Gamma + M_r) + \tilde\Delta(2\tilde\Delta\Gamma + \tilde\Omega\Lambda_r)}, \\
\langle\sigma_y\rangle_{ss} &= -\frac{1}{2\Omega'}\frac{\tilde\Omega(\Gamma + M_r)a - (2\tilde\Delta\Gamma + \tilde\Omega\Lambda_r)b_r}{(1-\tilde\Delta^2)(\Gamma + M_r) + \tilde\Delta(2\tilde\Delta\Gamma + \tilde\Omega\Lambda_r)}, \\
\langle\sigma_z\rangle_{ss} &= -\frac{\tilde\Delta(\tilde\Delta a + \tilde\Omega b_r)}{(1-\tilde\Delta^2)(\Gamma + M_r) + \tilde\Delta(2\tilde\Delta\Gamma + \tilde\Omega\Lambda_r)}.
\end{aligned} \tag{35}$$

For thermal reservoirs, for which the mean number of photons does not appreciably depend on frequency, $N(\omega) = N_0 = N_\pm$, Eqs. (35) go into

$$\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{\tilde\Omega}{2(1+2N_0)}\frac{(1+\tilde\Delta)^2 a_- - (1-\tilde\Delta)^2 a_+}{(1+\tilde\Delta)^2 a_- + (1-\tilde\Delta)^2 a_+}, \\
\langle\sigma_y\rangle_{ss} &= -\frac{\tilde\Omega}{4\Omega'} \\
&\quad \times \frac{2(1-\tilde\Delta^2)a_- a_+ + [(1+\tilde\Delta)^2 a_- + (1-\tilde\Delta)^2 a_+]a_0}{(1+\tilde\Delta)^2 a_- + (1-\tilde\Delta)^2 a_+}, \\
\langle\sigma_z\rangle_{ss} &= -\frac{\tilde\Delta}{1+2N_0}\frac{(1+\tilde\Delta)^2 a_- - (1-\tilde\Delta)^2 a_+}{(1+\tilde\Delta)^2 a_- + (1-\tilde\Delta)^2 a_+}.
\end{aligned} \tag{36}$$

From Eqs. (36), it is evident that $\langle\sigma_y\rangle_{ss}$ is of the order of $1/\Omega'$ and becomes zero in the secular limit. It is also clear that the steady-state values of $\langle\sigma_x\rangle_{ss}$ and $\langle\sigma_z\rangle_{ss}$ depend on the density of photon modes at the sidebands only. Moreover, upon making an appropriate choice of the detuning $\tilde\Delta$ and choosing different mode densities at the two sidebands ($a_- \neq a_+$), steady-state atomic inversion can be realized. This effect, called vacuum-field dressed-state pumping, has been predicted by Lewenstein and Mossberg [9] and observed by Zhu *et al.* [10]. On resonance, $\tilde\Delta = 0$, $\tilde\Omega = 1$, the steady-state solutions simplify even further, and the steady state value of $\langle\sigma_z\rangle_{ss}$ becomes zero, meaning equal populations in the two atomic levels.

Another important feature of the solutions (36) is that the dispersion component of the atomic dipole $\langle\sigma_x\rangle_{ss}$ is nonzero if $a_- \neq a_+$. This can happen because of the difference in the mode density at the two sidebands and/or the dependence of the damping rate on the field intensity through the $((\omega \pm \Omega')/\omega_A)^3$ factor. The nonzero solution for $\langle\sigma_x\rangle_{ss}$ means the nonzero steady-state atomic dipole moment, which has a dramatic effect on the resonance fluorescence spectrum [9] in the frequency-dependent photon reservoirs.

Even for flat reservoirs, but very strong fields, $\langle\sigma_x\rangle_{ss}$ can be nonzero due to the dependence of the relaxation rates on the strength of the field (Rabi frequency). This effect, which was called symmetry breaking of the two-level atomic response due to field-dependent relaxation, has been extensively discussed by Kocharovskaya and Radeonychev [13].

CONCLUSIONS

In this paper, we have presented the generalized master equation for the reduced atomic density matrix, obtained within the Born and Markov approximations, for the atom driven on resonance by a strong, classical electromagnetic field. The derivation is based on the idea that the dressing transformation is performed first, and, next, the coupling of such a dressed atom to the reservoir with a nonflat density of modes is turned on. The master equation, in the operator form, which takes into account the influence of the high-intensity laser field and the structure of the mode density of the reservoir, has a simple and transparent structure allowing for easy identification of different physical contributions to the atomic evolution. No secular approximation has been made in the derivation, so the results obtained from the master equation are quite general and should also be valid for weak fields.

In our master equation, we have identified terms that are similar to terms arising from nondiagonal field correlations in squeezed vacuum reservoirs. These squeezing-like terms proportional to M cause an effect similar to the squeezed vacuum; i.e., the two quadrature components of the atomic dipole decay at different rates. This leads to the narrowing of spectral lines similar to that in the squeezed vacuum. Other nonstandard terms that we have identified are those with b and L. Although all the nonstandard terms are phase dependent, their phase dependence stems solely from the phase of the driving field, and it disappears in the resonance fluorescence and absorption spectra. This differentiates the squeezing-like terms of our master equation from the real squeezing terms that lead to the spectra that are sensitive to the squeezing phase.

The generalized optical Bloch equations, together with the quantum regression theorem, allow for calculations of the fluorescence and absorption spectra for the atom driven by a strong field and coupled to a tailored reservoir. The exact analytical formulas for the spectra have been obtained and discussed elsewhere [16].

ACKNOWLEDGMENTS

We thank Poznań Supercomputing and Networking Center for access to the computing facilities.

REFERENCES

1. E. M. Purcell, Phys. Rev. **69**, 681 (1946).
2. D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).
3. M. Lewenstein, T. W. Mossberg, and R. J. Glauber, Phys. Rev. Lett. **59**, 775 (1987).
4. C. Cohen-Tannoudji and S. Reynaud, J. Phys. B **10**, 345 (1977).
5. C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
6. E. Courtens and A. Szoke, Phys. Rev. A **15**, 1588 (1977).
7. P. L. Knight and P. W. Milloni, Phys. Rep. **66**, 21 (1980).
8. P. R. Berman and R. Salomaa, Phys. Rev. A **25**, 2667 (1981).
9. M. Lewenstein and T. W. Mossberg, Phys. Rev. A **37**, 2048 (1988).
10. Y. Zhu, A. Lezama, T. W. Mossberg, and M. Lewenstein, Phys. Rev. Lett. **61**, 1946 (1988).
11. A. Lezama, Y. Zhu, S. Morin, and T. W. Mossberg, Phys. Rev. A **39**, 2754 (1989).
12. O. Kocharovskaya *et al.*, Phys. Rev. A **49**, 4928 (1994).
13. O. Kocharovskaya and Y. V. Radeonychev, Quantum Semiclassic. Opt. **8**, 7 (1996).
14. C. H. Keitel, P. L. Knight, L. M. Narducci, and M. O. Scully, Opt. Commun. **118**, 143 (1995).
15. C. H. Keitel, J. Mod. Opt. **43**, 1555 (1996).
16. A. Kowalewska-Kudłaszyk and R. Tanaś, J. Mod. Opt. **48**, 347 (2001).
17. P. R. Berman, Phys. Rev. A **53**, 2627 (1996).
18. H. Carmichael and D. F. Walls, J. Phys. A **6**, 1552 (1973).
19. J. Cresser, J. Mod. Opt. **39**, 2187 (1992).
20. G. Yeoman and S. M. Barnett, J. Mod. Opt. **43**, 2037 (1996).
21. R. Tanaś, Z. Ficek, A. Messikh, and T. El-Shahat, J. Mod. Opt. **45**, 1859 (1998).
22. W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).
23. C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).