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# Two-level Atom in a Squeezed Vacuum with Finite Bandwidth: Master Equation versus Coupled-systems Approach

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## Abstract

We address the question: how broad must be the squeezed vacuum to make the Markov approximation still applicable? We compare the resonance fluorescence spectra obtained using the Markovian master equation with the spectra calculated from the coupled-systems approach. We show that both approaches give very similar spectra up to realistic values of the squeezed vacuum bandwidth ($\sim 10\gamma$).

Broadband squeezed vacuum can be treated as a reservoir to an atom and a master equation in the Born and Markov approximation can be derived for the reduced density matrix of the atomic system. Realistic sources of squeezed field, such as the degenerate parametric oscillator (DPO), produce a squeezed vacuum with finite bandwidth. However, when the bandwidth of the DPO cavity is much larger than the atomic linewidth, one can still treat the squeezed vacuum as a reservoir to the atom and derive the Markovian master equation that describes the dynamics of the atomic variables only. If, moreover, the atom is driven by a classical coherent laser field one can perform the dressing transformation first and next apply the standard perturbation procedure to derive the master equation [1]. We have derived such a master equation [2,3], which in the frame rotating with the laser frequency $\omega_L$ can be written as

$$\begin{aligned}\dot{\rho} = & \frac{1}{2}\, i\, [(\delta\, \sigma_z - \Omega\, (\sigma_+ + \sigma_-)), \rho] \\ & + \frac{1}{2}\, \gamma\, \tilde{N}\, \left(2\, \sigma_+ \rho\, \sigma_- - \sigma_- \sigma_+ \rho - \rho\, \sigma_- \sigma_+\right) \\ & + \frac{1}{2}\, \gamma\, (\tilde{N} + 1)\, \left(2\, \sigma_- \rho\, \sigma_+ - \sigma_+ \sigma_- \rho - \rho\, \sigma_+ \sigma_-\right) \\ & - \gamma\, \tilde{M}\, \sigma_+ \rho\, \sigma_+ - \gamma\, \tilde{M}^*\, \sigma_- \rho\, \sigma_-\end{aligned} \tag{1}$$

$$+\frac{1}{4}i\,\left(\beta\,[\sigma_+,[\sigma_z,\rho]]-\beta^*[\sigma_-,[\sigma_z,\rho]]\right)\,,$$

where

$$\begin{aligned}
\tilde{N} &= N(\omega_L+\Omega')+\frac{1}{2}(1-\tilde{\Delta}^2)\,\mathrm{Re}\Gamma_-\,,\\
\tilde{M} &= M(\omega_L+\Omega')-\frac{1}{2}(1-\tilde{\Delta}^2)\Gamma_-+i\,\tilde{\Delta}\delta_M\,\mathrm{e}^{i\phi}\,,\\
\delta &= \Delta+\frac{\gamma}{2}(1-\tilde{\Delta}^2)\,\mathrm{Im}\Gamma_-+\gamma\,\tilde{\Delta}\,\delta_N\,,\\
\beta &= \gamma\tilde{\Omega}\left[\delta_N+\delta_M\,\mathrm{e}^{i\phi}-i\,\tilde{\Delta}\,\Gamma_-\right]\,,\\
\Gamma_- &= N(\omega_L)-N(\omega_L+\Omega')-[M(\omega_L)-M(\omega_L+\Omega')]\,,\\
\delta_N &= \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{N(x)}{x+\Omega'}\,\mathrm{d}x\,,\quad \delta_M=\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{|M(x)|}{x+\Omega'}\,\mathrm{d}x\,,\\
\tilde{\Omega} &= \frac{\Omega}{\Omega'}\,,\quad \tilde{\Delta}=\frac{\Delta}{\Omega'}\,,\quad \Omega'=\sqrt{\Omega^2+\Delta^2}\,,
\end{aligned}\tag{2}$$

and the principal value terms for DPO have the form

$$\begin{aligned}
\delta_N &= \Omega'\,\frac{\lambda^2-\mu^2}{4}\left[\frac{1}{\mu\,(\Omega'^2+\mu^2)}-\frac{1}{\lambda\,(\Omega'^2+\lambda^2)}\right]\,,\\
\delta_M &= \Omega'\,\frac{\lambda^2-\mu^2}{4}\left[\frac{1}{\mu\,(\Omega'^2+\mu^2)}+\frac{1}{\lambda\,(\Omega'^2+\lambda^2)}\right]\,,\\
\lambda &= \frac{\kappa}{2}+\epsilon\qquad \mu=\frac{\kappa}{2}-\epsilon\,,
\end{aligned}\tag{3}$$

where $\kappa$ is the bandwidth of the DPO cavity and $\epsilon$ the amplitude of the pump field.

In the derivation of equation (1) we have included the divergent frequency shifts (the Lamb shift) to the redefinition of the atomic transition frequency. Moreover, we have assumed that the squeezed vacuum is symmetric about the central frequency $\omega_L$, so that $N(\omega_L-\Omega')=N(\omega_L+\Omega')$, and a similar relation holds for $M(\omega)$. The atomic natural linewidth is $\gamma$, $\Delta=\omega_L-\omega_A$ is the detuning of the laser frequency of the atomic resonance, and $\Omega$ is the Rabi frequency of the coherent driving field.

The master equation (1) has the standard form known from the broadband squeezing approaches with the new effective squeezing parameters $\tilde{N}$ and $\tilde{M}$. There are also new terms, proportional to $\beta$ which are essentially narrow bandwidth modifications to the master equation. All the narrow bandwidth modifications are determined by the parameter $\Gamma_-$ and the shifts $\delta_N$ and $\delta_M$. These parameters become zero when the squeezing bandwidth goes to infinity.

As a reference for testing our master equation (1) we use the coupled-system (or cascaded-system) approach [4,5] in which output of the first system (DPO) drives the second system (atom) without any coupling back from the the second system to the first. In our case of an atom driven by a squeezed light from DPO and a coherent field with the Rabi frequency $\Omega$, the corresponding master equation has the form [6].

$$\dot{\rho} = \frac{1}{2} i \left[ (\Delta \sigma_z - \Omega (\sigma_+ + \sigma_-) + (\epsilon a^{\dagger 2} - \epsilon^* a^2)), \rho \right] + \frac{\kappa}{2} \left\{ 2 a \rho a^\dagger - \rho a^\dagger a - a^\dagger a \rho \right\}$$
$$- \sqrt{\eta \kappa \gamma} \left\{ [\sigma_+, a \rho] + [\rho a^\dagger, \sigma_-] \right\} + \frac{\gamma}{2} \left\{ 2 \sigma_- \rho \sigma_+ - \rho \sigma_+ \sigma_- - \sigma_+ \sigma_- \rho \right\} , \tag{4}$$

where the parameter $\eta$ ($0 < \eta \leq 1$) describes the matching of the incident squeezed vacuum to the modes surrounding the atom. For perfect matching $\eta = 1$, whereas $\eta < 1$ for an imperfect matching.

Since in (4) DPO is not treated as a reservoir but as a part of the system, equation (4) is applicable for any bandwidth of the squeezed vacuum produced by DPO. The advantage of the Markovian master equation (1) over equation (4), however, is the fact that the former allows for analytical solutions while the latter does not. When the DPO cavity bandwidth $\kappa \gg \gamma$, both equations are expected to give the same results. There is a question, however, how big really must be $\kappa$ with respect to $\gamma$ to make equation (1) still applicable. To

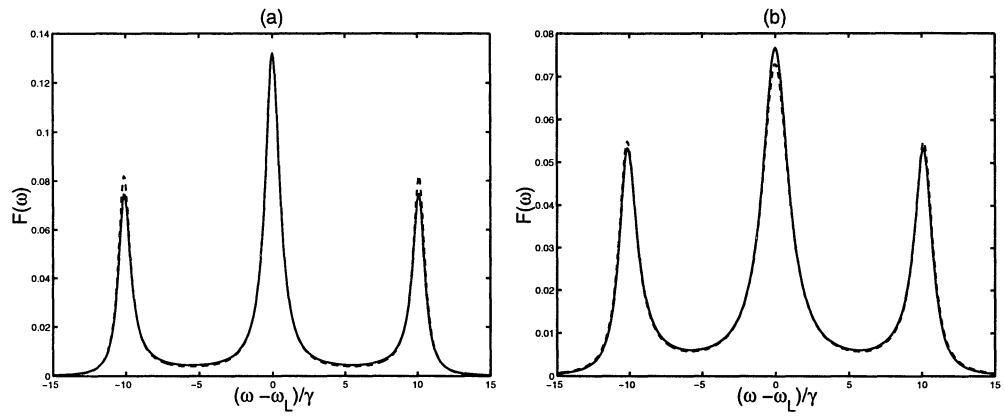


FIG. 1. Fluorescence spectra — coupled-systems (solid) and Markovian master equation (dash): $\epsilon = \kappa/8$ ($N = 0.26$, $M = 0.57$), $\Omega = 10$, $\varphi = 0$, $\Delta = 0$, $\eta = 0.98$ and (a) $\kappa = 10$, (b) $\kappa = 40$

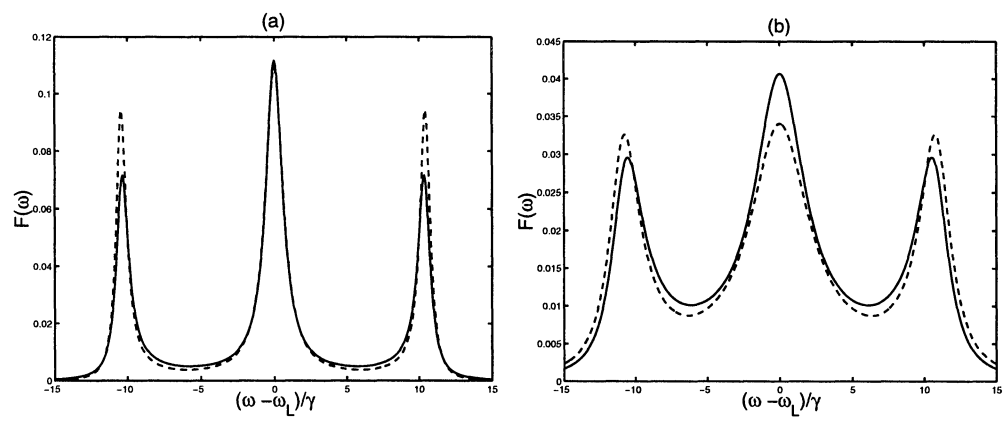


FIG. 2. Fluorescence spectra — coupled-systems (solid) and Markovian master equation (dash): $\epsilon = \kappa/4$ ($N = 1.78$, $M = 2.22$), $\Omega = 10$, $\varphi = 0$, $\Delta = 0$, $\eta = 0.98$ and (a) $\kappa = 10$, (b) $\kappa = 40$

answer this question we show in Figs. 1-3 few examples of the resonance fluorescence spectra obtained using both approaches. The width of the squeezed light $\kappa$ as well as the amplitude of the pump field $\epsilon$ are given in units of the atomic linewidth of $\gamma$. In Figs. 1 and 2 there

are examples of the resonance fluorescence spectra for strong field ($\Omega = 10$ in units of $\gamma$). In Fig. 1 the squeezing is smaller than in Fig. 2. The well known squeezing parameters $N$ and $M$ (mean number of photons and the field correlation) are: for Fig. 1 $N = 0.26$, $M = 0.57$, and for Fig. 2 $N = 1.78$, $M = 2.22$. As it is seen from Fig. 1, for weak squeezing the agreement between the two approaches is perfect even for the squeezing bandwidth $\kappa$ as small as 10. When the squeezing becomes more pronounced the agreement is worse for the same bandwidths of the squeezed vacuum, but it is still pretty good. We would like to emphasize that for $\kappa = \Omega = 10$ the squeezing bandwidth is the same as the Rabi frequency of the field, which shows explicitly that the Markovian approximation works well when the squeezing bandwidth is much broader than the atomic linewidth, but not necessarily larger than the Rabi frequency. This is an advantage of our master equation (1), which was derived by performing the dressing transformation first, and next coupling the atom to the reservoir.

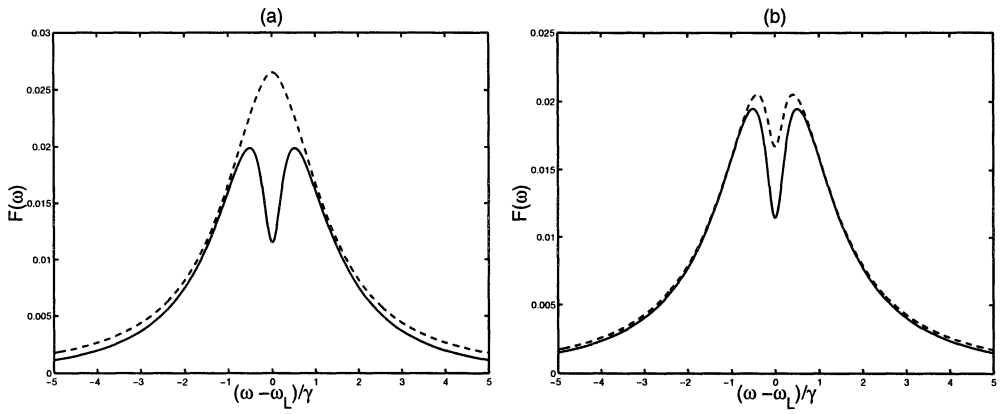


FIG. 3. Fluorescence spectra — coupled-systems (solid) and Markovian master equation (dash): $\epsilon = \kappa/8$ ($N = 0.26$, $M = 0.57$), $\Omega = 0.35$, $\varphi = 0$, $\Delta = 0$, $\eta = 0.98$ and (a) $\kappa = 20$, (b) $\kappa = 40$

In Fig. 3 we show examples of the spectra for a weak field ($\Omega = 0.35$). In this figure the Rabi frequency is chosen as to show a possibility to burn a hole in the spectrum. It is seen that for $\kappa = 10$ in this case the Markovian master equation does not reproduce the hole, and broader squeezing is needed to reproduce the feature, but for $\kappa = 40$ agreement is already quite good.

The results shown here convince us that the Markovian master equation (1) works quite well for the squeezing bandwidth which is ten times bigger than the atomic linewidth.

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