Elektrodynamika

Część 10

Promieniowanie

Ryszard Tanaś

Zakład Optyki Nieliniowej, UAM

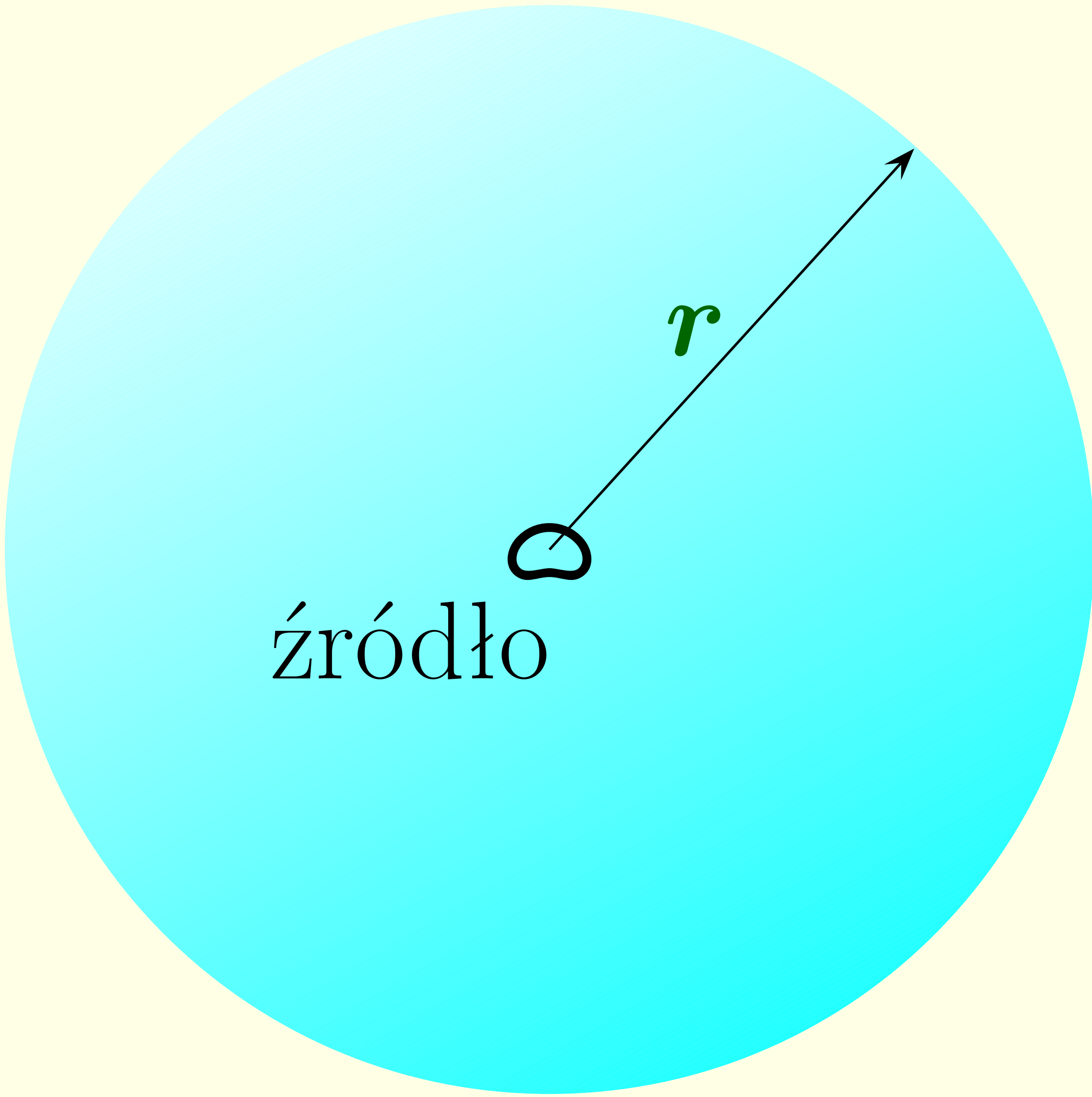
http://zon8.physd.amu.edu.pl/~tanas

Spis treści

11 Promieniowanie

11.1 Promieniowanie dipolowe

11.1.1 Czym jest promieniowanie?



$$P(r) = \oint \boldsymbol{S} \cdot \, \mathrm{d}\boldsymbol{a} = \frac{1}{\mu_0} \oint (\boldsymbol{E} \times \boldsymbol{B}) \cdot \, \mathrm{d}\boldsymbol{a}$$

moc przechodząca
przez powierzchnię

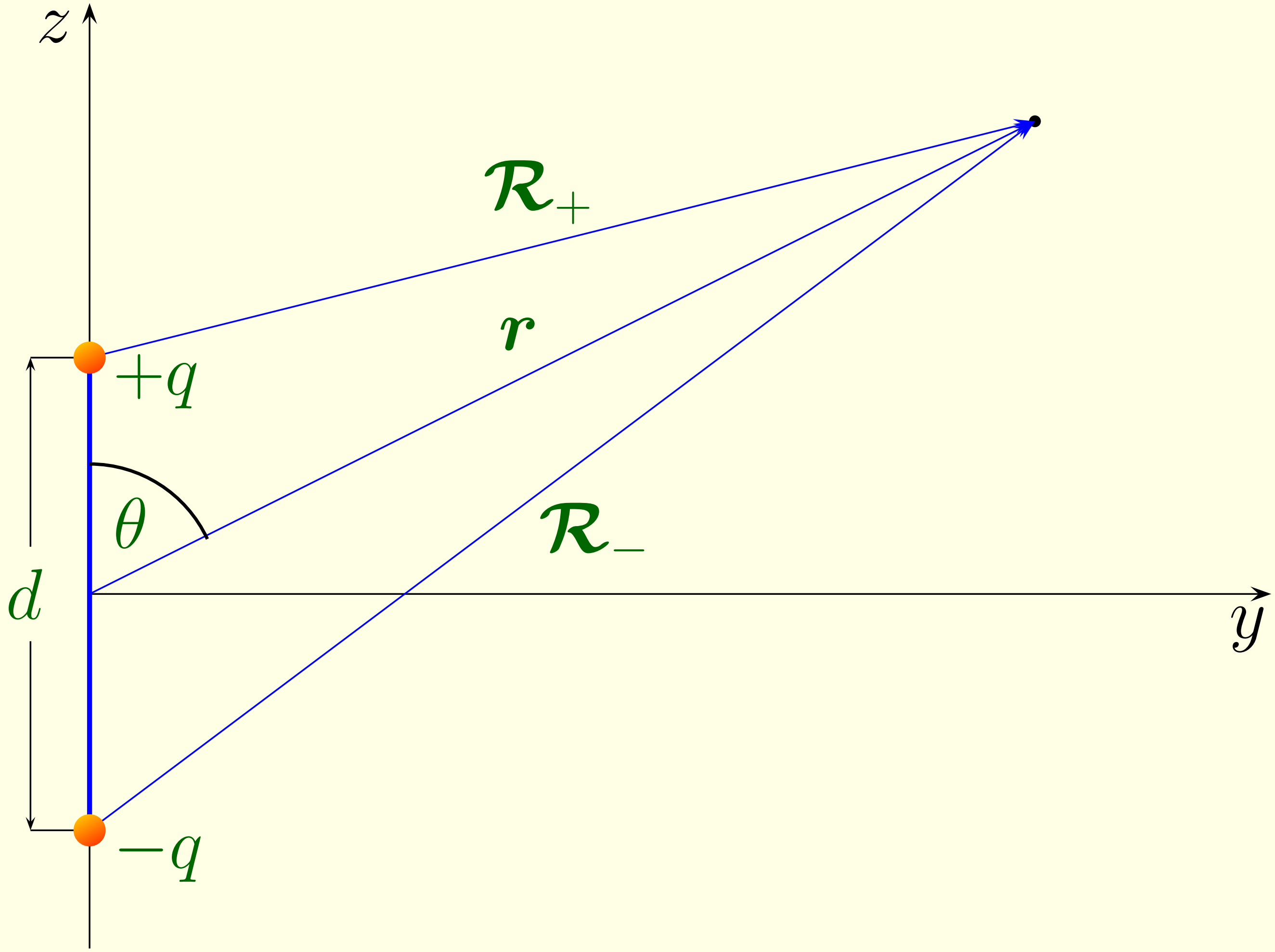
$$P(r) = \oint \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = \frac{1}{\mu_0} \oint (\boldsymbol{E} \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{a}$$

moc przechodząca przez powierzchnię

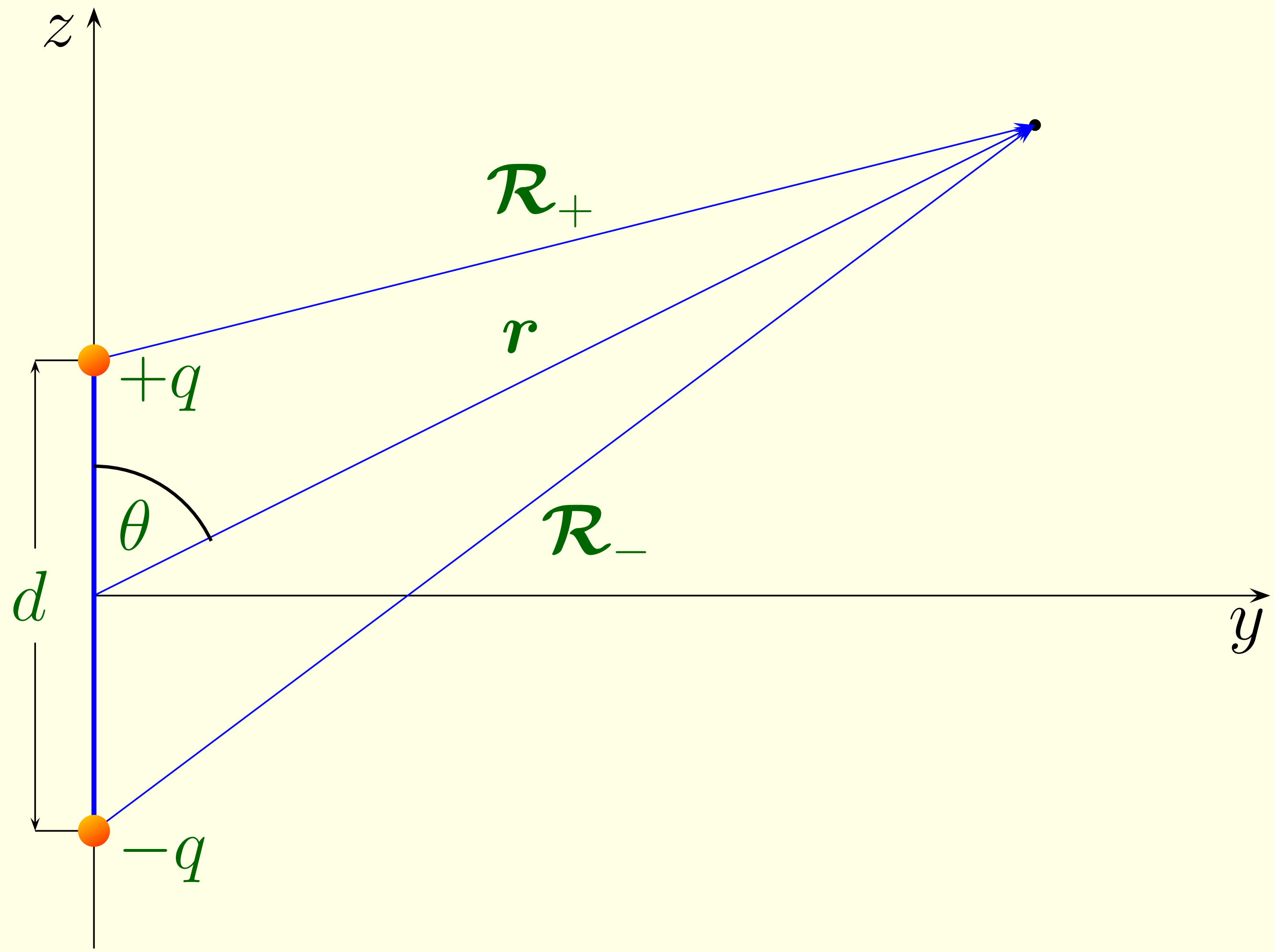
$$P_{\mathrm{rad}} \equiv \lim_{r \to \infty} P(r)$$

moc wypromieniowana

11.1.2 Promieniowanie elektryczne dipolowe

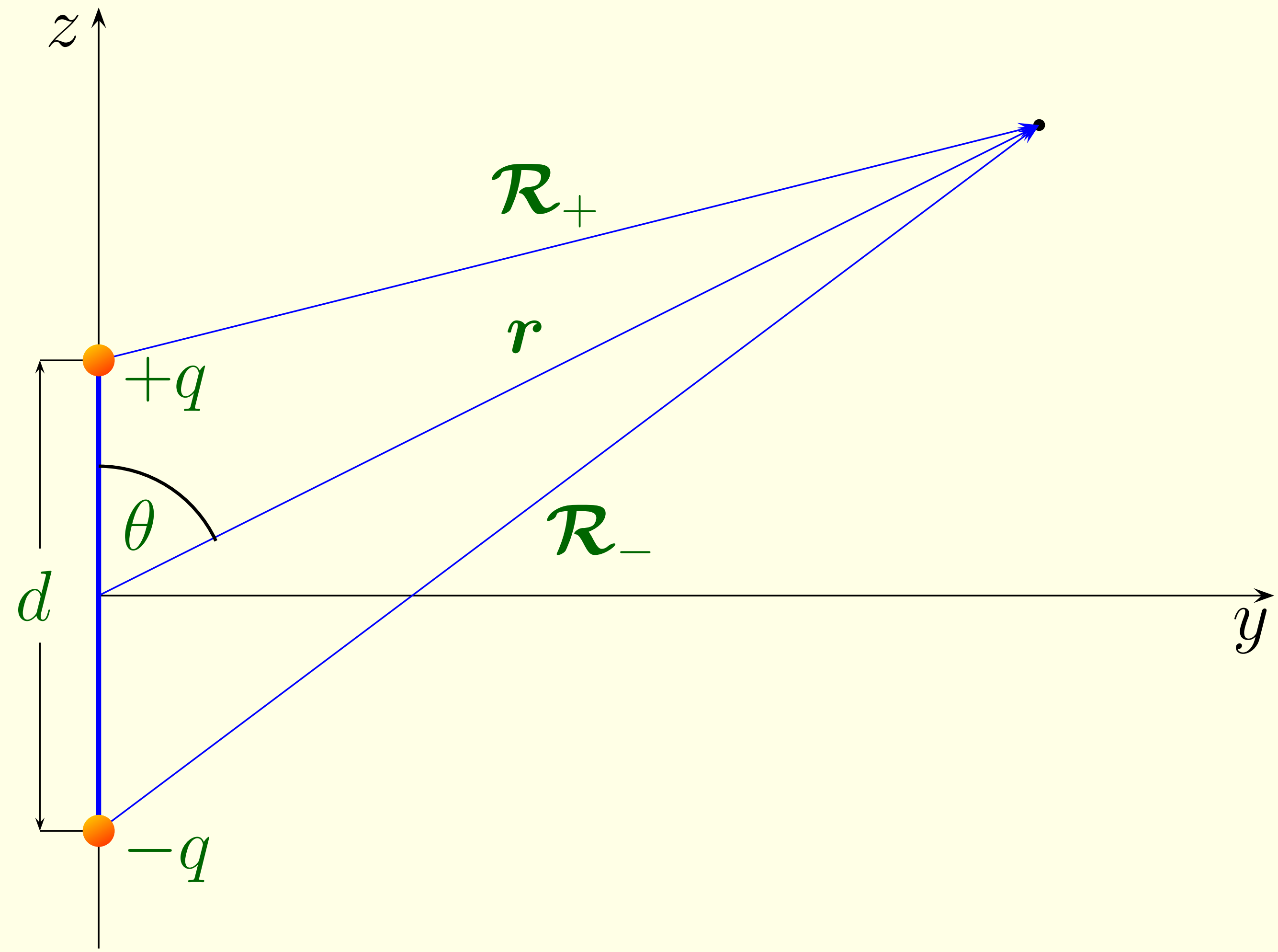


11.1.2 Promieniowanie elektryczne dipolowe



$q(t) = q_0 \cos(\omega t)$ ładunek przepływa

11.1.2 Promieniowanie elektryczne dipolowe



$q(t) = q_0 \cos(\omega t)$ ładunek przepływa

$\boldsymbol{p}(t) = p_0 \cos(\omega t)\hat{\boldsymbol{z}}, \quad p_0 \equiv q_0 d$ drgający dipol

Potencjał opóźniony

$$V(\boldsymbol{r}, t) = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_0\cos[\omega(t-\mathcal{R}_+/c)]}{\mathcal{R}_+} - \frac{q_0\cos[\omega(t-\mathcal{R}_-/c)]}{\mathcal{R}_-}\right\}$$

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$\mathcal{R}_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2}$ z twierdzenia cosinusów przybliżenie 1: $d \ll r$ dipol doskonały

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$$\frac{1}{\mathcal{R}_{\pm}} \cong \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$$

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$$\cos\left[\omega\left(t-\frac{\mathcal{R}_\pm}{c}\right)\right] \cong \cos\left[\omega\left(t-\frac{r}{c}\right) \pm \frac{\omega d}{2c}\cos\theta\right]$$

$$= \cos\left[\omega\left(t-\frac{r}{c}\right)\right]\cos\left(\frac{\omega d}{2c}\cos\theta\right) \mp \sin\left[\omega\left(t-\frac{r}{c}\right)\right]\sin\left(\frac{\omega d}{2c}\cos\theta\right)$$

przybliżenie 2: $d \ll \frac{c}{\omega}$ $(d \ll \lambda)$ dipol doskonały przybliżenie 2: $d \ll \frac{c}{\omega}$ $(d \ll \lambda)$ dipol doskonały

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$$\boxed{V(r,\theta,t)=\frac{p_0\cos\theta}{4\pi\epsilon_0 r}\left\{-\frac{\omega}{c}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]+\frac{1}{r}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right\}}$$

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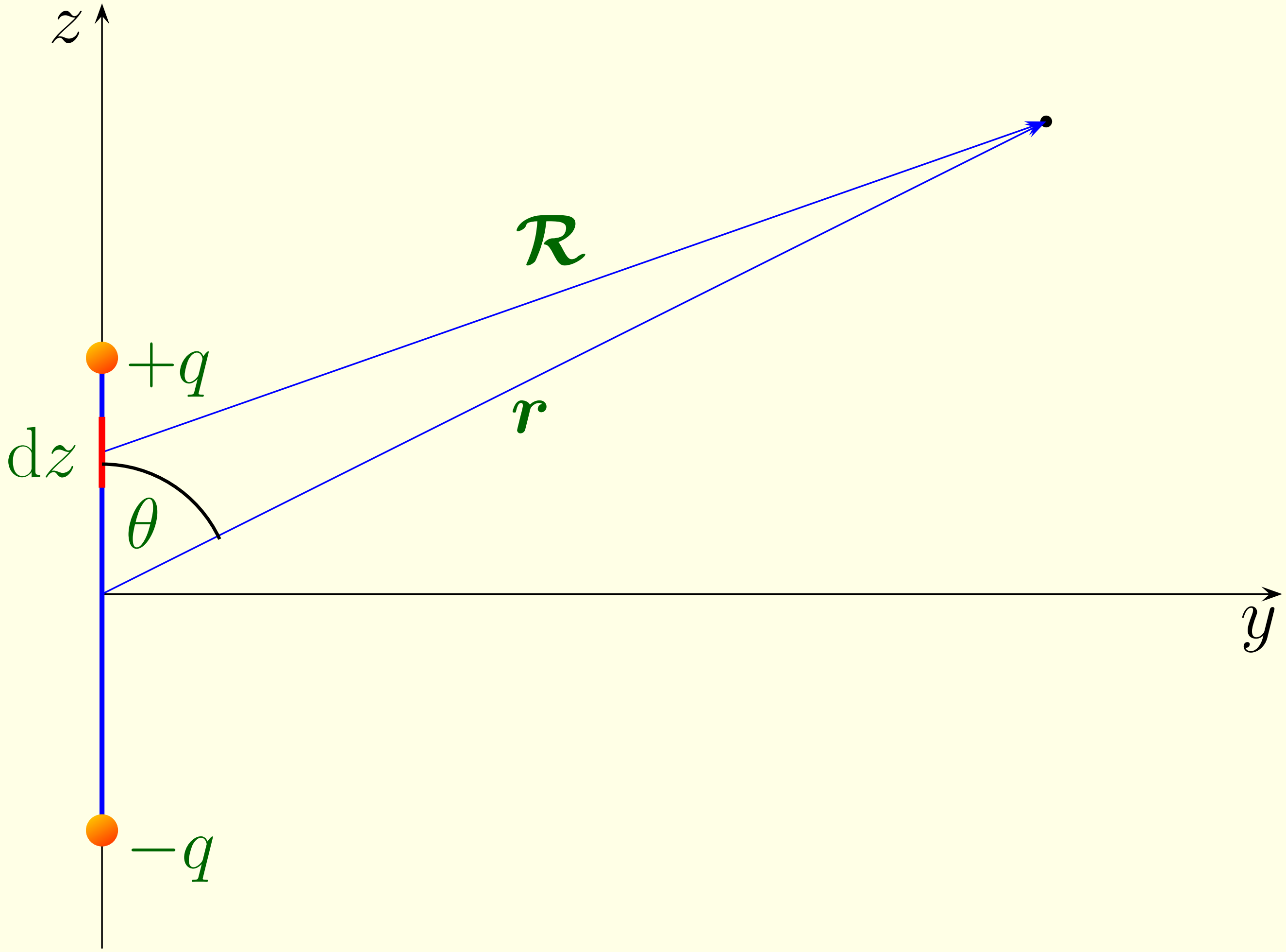
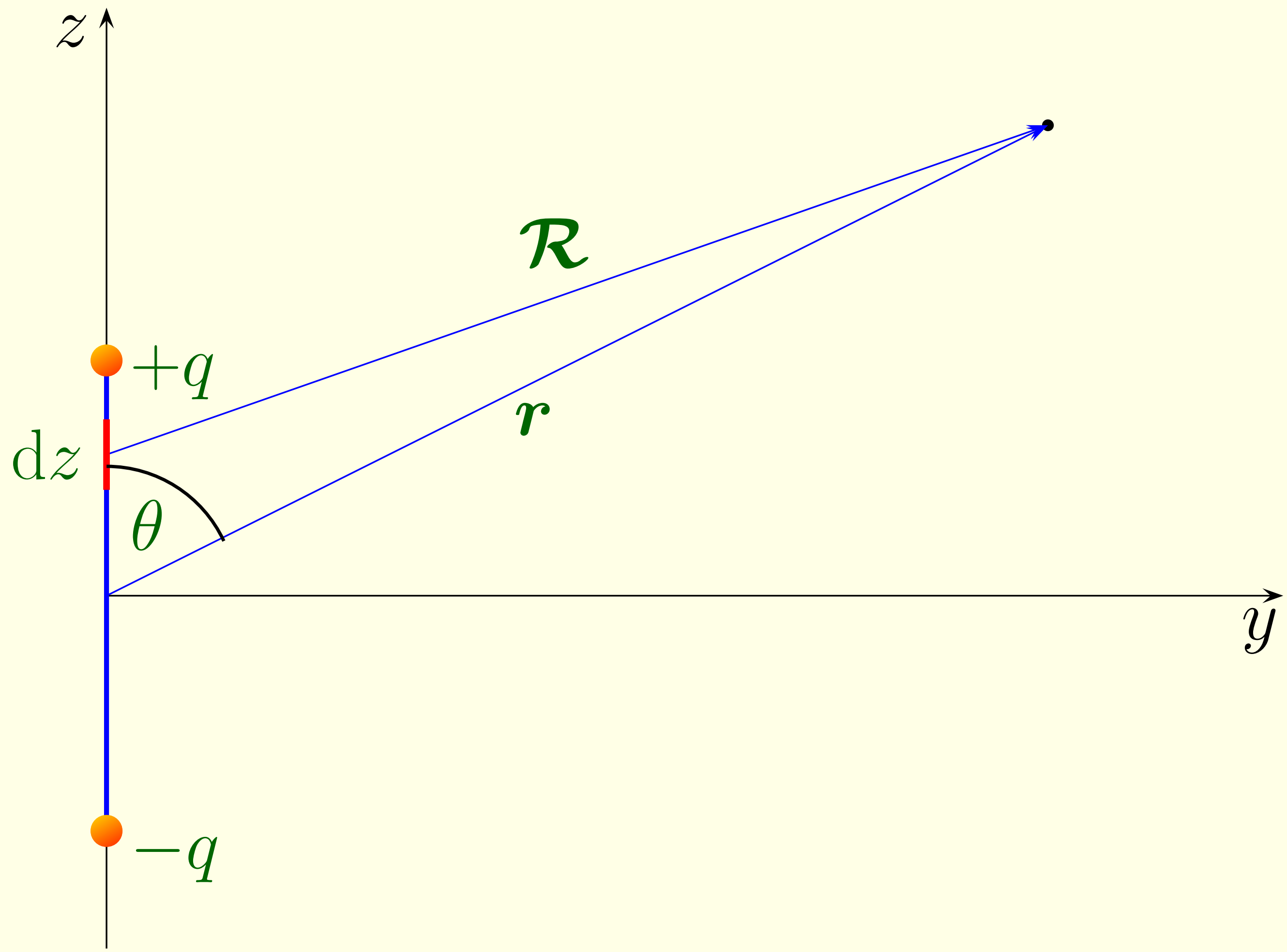
$V = \dfrac{p_0 \cos\theta}{4\pi\epsilon_0 r^2}$ dla $\omega \to 0,$ granica statyczna przybliżenie 3: $r \gg \frac{c}{\omega}$ $(r \gg \lambda)$ strefa promieniowania przybliżenie 3: $r \gg \frac{c}{\omega}$ $(r \gg \lambda)$ strefa promieniowania

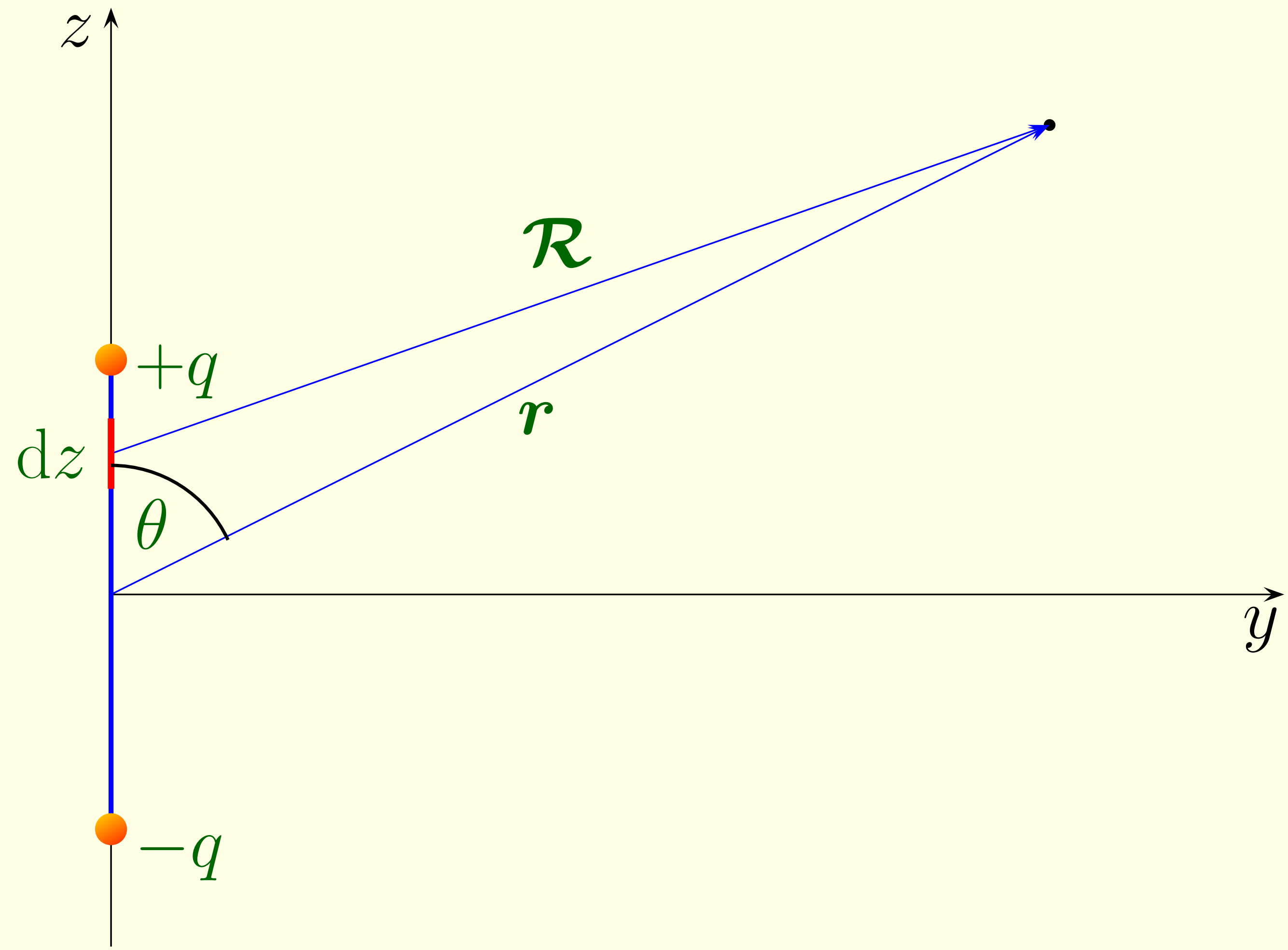
$$V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right]$$



z
$\boldsymbol{\mathcal{R}}$
$+q$
$\boldsymbol{r}$
$\mathrm{d}z$
θ
y
$-q$



$$\boldsymbol{I}(t) = \frac{\mathrm{d}q}{\mathrm{d}t}\hat{\boldsymbol{z}} = -q_0\omega\sin(\omega t)\hat{\boldsymbol{z}}$$ prąd płynący w drucie



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$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2}\frac{-q_0\omega\sin[\omega(t-\mathcal{R}/c)]\hat{\boldsymbol{z}}}{\mathcal{R}}\,\mathrm{d}z$$ potencjał wektorowy

$$\boxed{\boldsymbol{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{z}}}$$

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Obliczamy pola:

$$\nabla V = \frac{\partial V}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}}$$

$$= -\frac{p_0\omega}{4\pi\epsilon_0 c}\left\{\cos\theta\left(-\frac{1}{r^2}\sin\left[\omega\left(t-\frac{r}{c}\right)\right] - \frac{\omega}{rc}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)\hat{\boldsymbol{r}}\right.$$

$$\left. -\frac{\sin\theta}{r^2}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\theta}}\right\}$$

$$\cong \frac{p_0\omega^2}{4\pi\epsilon_0 c^2}\left(\frac{\cos\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{r}}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi r} \cos\left[\omega\left(t - \frac{r}{c}\right)\right] \underbrace{(\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})}_{\hat{\boldsymbol{z}}}$$

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$$\boxed{\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t - \frac{r}{c}\right)\right]\,\hat{\boldsymbol{\theta}}}$$

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$$\nabla \times \boldsymbol{A} = \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_z}{\partial \theta}\right]\,\hat{\boldsymbol{\phi}}$$

$$= -\frac{\mu_0 p_0 \omega^2}{4\pi r}\left\{\frac{\omega}{c}\sin\theta\cos\left[\omega\left(t - \frac{r}{c}\right)\right] + \frac{\sin\theta}{r}\sin\left[\omega\left(t - \frac{r}{c}\right)\right]\right\}\hat{\boldsymbol{\phi}}$$

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$$\boxed{\boldsymbol{B} = \nabla \times \boldsymbol{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \left(\frac{\sin\theta}{r}\right) \cos\left[\omega\left(t - \frac{r}{c}\right)\right] \hat{\boldsymbol{\phi}}}$$

$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{\mu_0}{c} \left\{ \frac{p_0 \omega^2}{4\pi} \left(\frac{\sin\theta}{r} \right) \cos\left[\omega \left(t - \frac{r}{c} \right) \right] \right\}^2 \hat{\boldsymbol{r}}$$

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$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2\theta}{r^2} \hat{\boldsymbol{r}}$$ natężenie promieniowania

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$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \mathrm{d}\boldsymbol{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \,\mathrm{d}\theta\,\mathrm{d}\phi = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$

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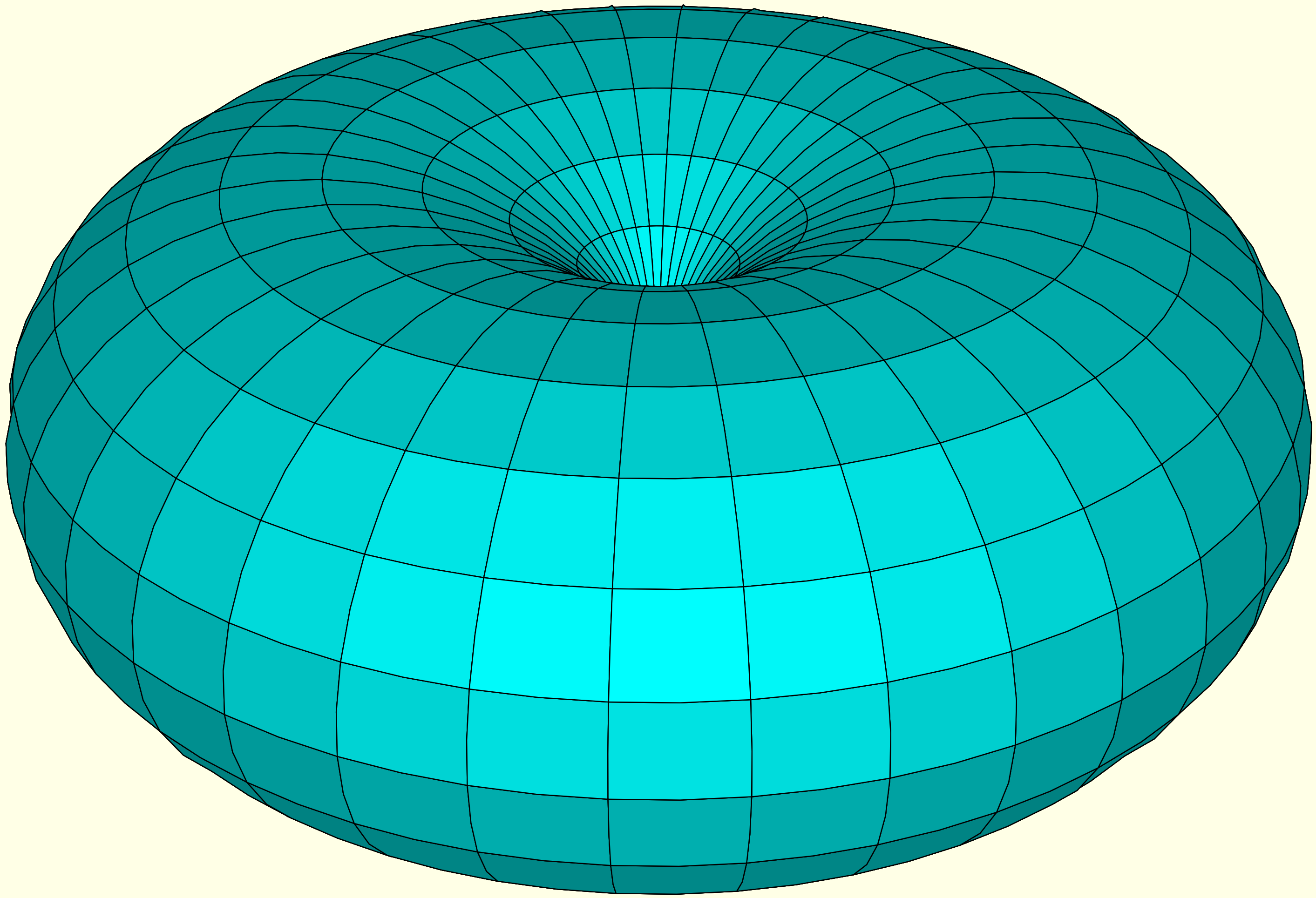
natężenie promieniowania

$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \mathrm{d}\boldsymbol{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$

Wynik nie zależy od promienia sfery

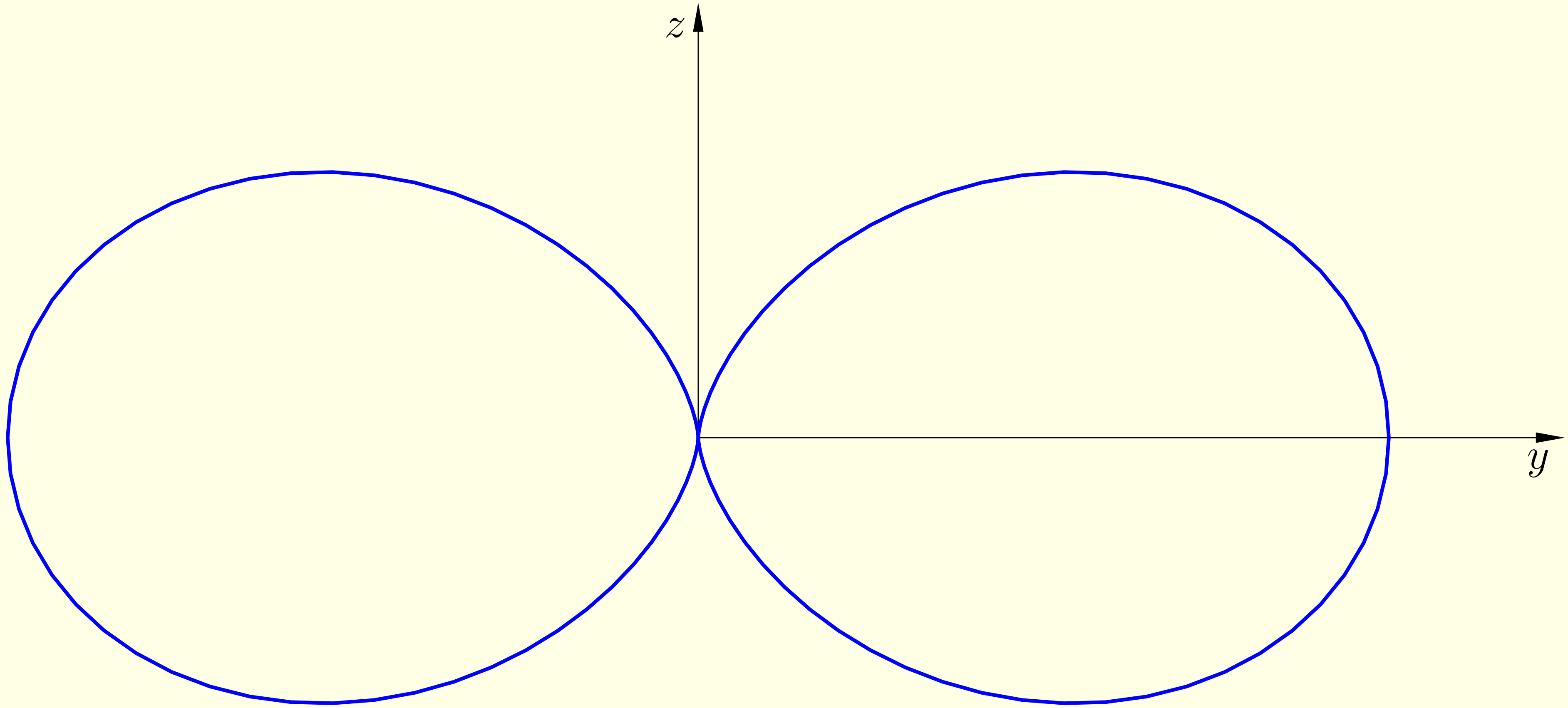
$$\frac{\mathrm{d}\langle P\rangle}{\mathrm{d}\Omega} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\sin^2\theta$$ moc wypromieniowana w kąt bryłowy $\mathrm{d}\Omega$

$$\frac{\mathrm{d}\langle P\rangle}{\mathrm{d}\Omega} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \sin^2\theta$$ moc wypromieniowana w kąt bryłowy $\mathrm{d}\Omega$



promieniowanie elektryczne dipolowe

charakterystyka kierunkowa



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