Elektrodynamika

Część 10

Promieniowanie

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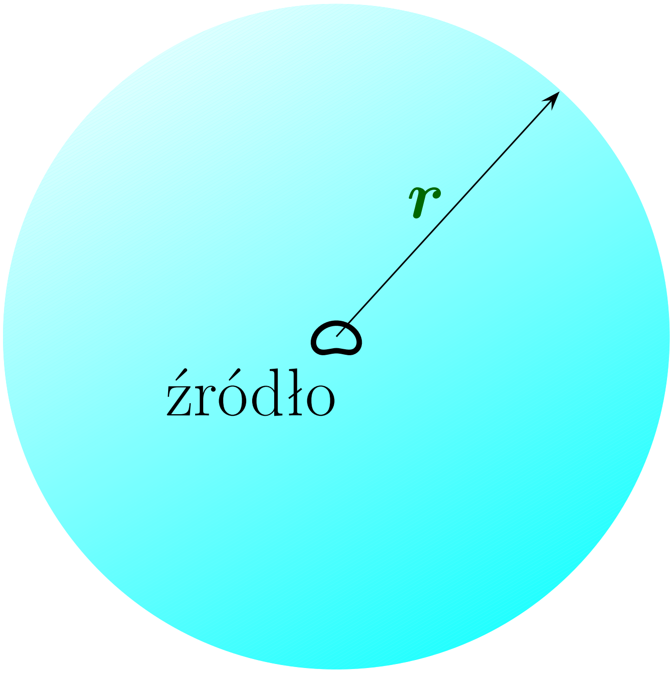
Spis treści

11 Promieniowanie

11.1 Promieniowanie dipolowe

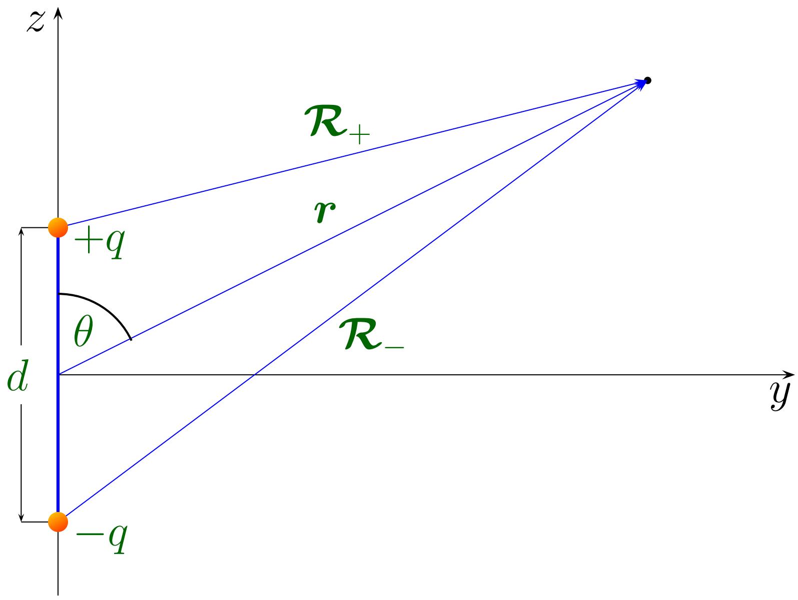
11.1.1 Czym jest promieniowanie?



$$P(r) = \oint \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = \frac{1}{\mu_0} \oint (\boldsymbol{E} \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{a}$$ moc przechodząca przez powierzchnię

$$P_{\mathrm{rad}} \equiv \lim_{r \to \infty} P(r)$$ **moc wypromieniowana**

11.1.2 Promieniowanie elektryczne dipolowe



$q(t) = q_0 \cos(\omega t)$ ładunek przepływa

$\boldsymbol{p}(t) = p_0 \cos(\omega t)\hat{\boldsymbol{z}}, \quad p_0 \equiv q_0 d$ drgający dipol

Potencjał opóźniony

$$V(\boldsymbol{r}, t) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{q_0 \cos[\omega(t - \mathcal{R}_+/c)]}{\mathcal{R}_+} - \frac{q_0 \cos[\omega(t - \mathcal{R}_-/c)]}{\mathcal{R}_-} \right\}$$

$\mathcal{R}_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2}$ z twierdzenia cosinusów

$\boxed{\textbf{przybliżenie 1:} \quad d \ll r \quad \text{dipol doskonały}}$

$$\mathcal{R}_\pm \cong r\left(1 \mp \boxed{\frac{d}{2r}}\cos\theta\right) \quad \text{przybliżenie liniowe ze względu na } d$$

$$\frac{1}{\mathcal{R}_\pm} \cong \frac{1}{r}\left(1 \pm \boxed{\frac{d}{2r}}\cos\theta\right)$$

$$\cos\left[\omega\left(t - \frac{\mathcal{R}_\pm}{c}\right)\right] \cong \cos\left[\omega\left(t - \frac{r}{c}\right) \pm \boxed{\frac{\omega d}{2c}}\cos\theta\right]$$

$$= \cos\left[\omega\left(t - \frac{r}{c}\right)\right] \cos\left(\boxed{\frac{\omega d}{2c}}\cos\theta\right) \mp \sin\left[\omega\left(t - \frac{r}{c}\right)\right] \sin\left(\boxed{\frac{\omega d}{2c}}\cos\theta\right)$$

$\boxed{\textbf{przybliżenie 2:} \quad d \ll \frac{c}{\omega} \ (d \ll \lambda) \quad \text{dipol doskonały}}$

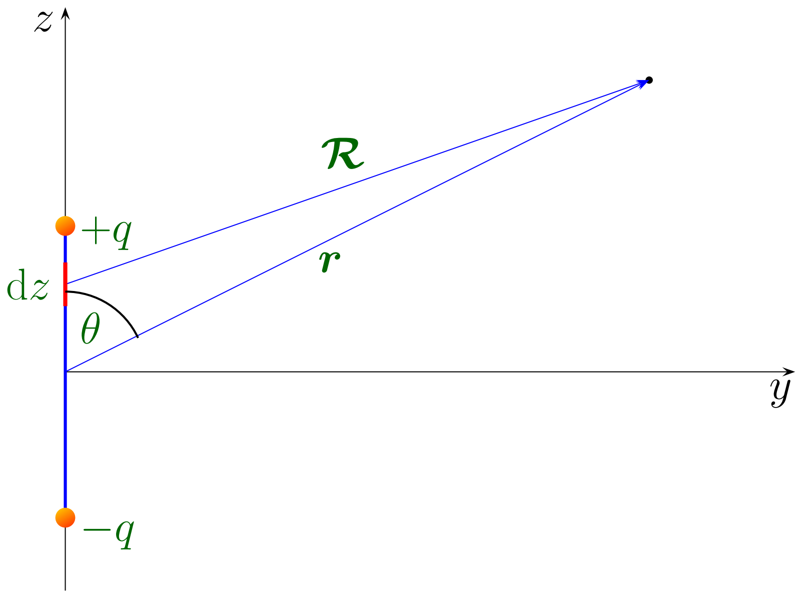
$$\cos\left[\omega\left(t - \frac{\mathcal{R}_\pm}{c}\right)\right] \cong \cos\left[\omega\left(t - \frac{r}{c}\right)\right] \mp \boxed{\frac{\omega d}{2c}}\cos\theta \sin\left[\omega\left(t - \frac{r}{c}\right)\right]$$

$$\boxed{V(r,\theta,t) = \frac{p_0\cos\theta}{4\pi\epsilon_0 r}\left\{-\frac{\omega}{c}\sin\left[\omega\left(t - \frac{r}{c}\right)\right] + \frac{1}{r}\cos\left[\omega\left(t - \frac{r}{c}\right)\right]\right\}}$$

$$V = \frac{p_0\cos\theta}{4\pi\epsilon_0 r^2} \quad \text{dla } \omega \to 0, \quad \text{granica statyczna}$$

przybliżenie 3: $r \gg \frac{c}{\omega}$ $(r \gg \lambda)$ strefa promieniowania

$$\boxed{V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right]}$$



$\boldsymbol{I}(t) = \frac{\mathrm{d}q}{\mathrm{d}t}\hat{\boldsymbol{z}} = -q_0\omega\sin(\omega t)\hat{\boldsymbol{z}}$ prąd płynący w drucie

$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int\limits_{-d/2}^{d/2}\frac{-q_0\omega\sin[\omega(t-\mathcal{R}/c)]\hat{\boldsymbol{z}}}{\mathcal{R}}\,\mathrm{d}z$ potencjał wektorowy

$$\boxed{\boldsymbol{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{z}}}$$

Obliczamy pola:

$$\nabla V = \frac{\partial V}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}}$$

$$= -\frac{p_0\omega}{4\pi\epsilon_0 c}\left\{\cos\theta\left(-\boxed{\frac{1}{r^2}}\sin\left[\omega\left(t-\frac{r}{c}\right)\right] - \frac{\omega}{rc}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)\hat{\boldsymbol{r}}\right.$$

$$\left.-\boxed{\frac{\sin\theta}{r^2}}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\theta}}\right\}$$

$$\cong \frac{p_0\omega^2}{4\pi\epsilon_0 c^2}\left(\frac{\cos\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{r}}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi r}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\underbrace{(\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})}_{\hat{\boldsymbol{z}}}$$

$$\boxed{\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\theta}}}$$

$$\nabla\times\boldsymbol{A} = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_z}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

$$= -\frac{\mu_0 p_0 \omega^2}{4\pi r}\left\{\frac{\omega}{c}\sin\theta\cos\left[\omega\left(t-\frac{r}{c}\right)\right] + \boxed{\frac{\sin\theta}{r}}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\right\}\hat{\boldsymbol{\phi}}$$

$$\boxed{\boldsymbol{B} = \nabla\times\boldsymbol{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\phi}}}$$

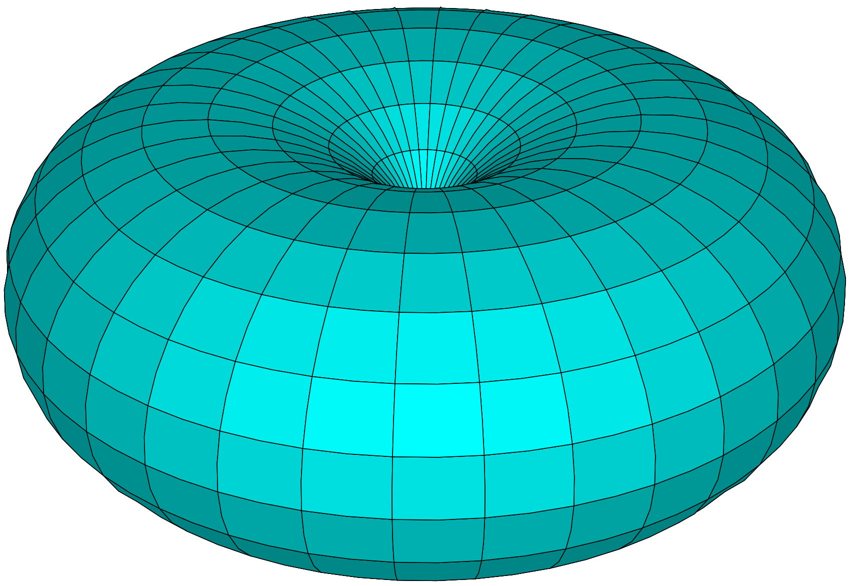
$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{\mu_0}{c} \left\{ \frac{p_0 \omega^2}{4\pi} \left(\frac{\sin\theta}{r} \right) \cos\left[\omega \left(t - \frac{r}{c} \right) \right] \right\}^2 \hat{\boldsymbol{r}}$$

$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2\theta}{r^2} \hat{\boldsymbol{r}}$$ natężenie promieniowania

$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \mathrm{d}\boldsymbol{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$

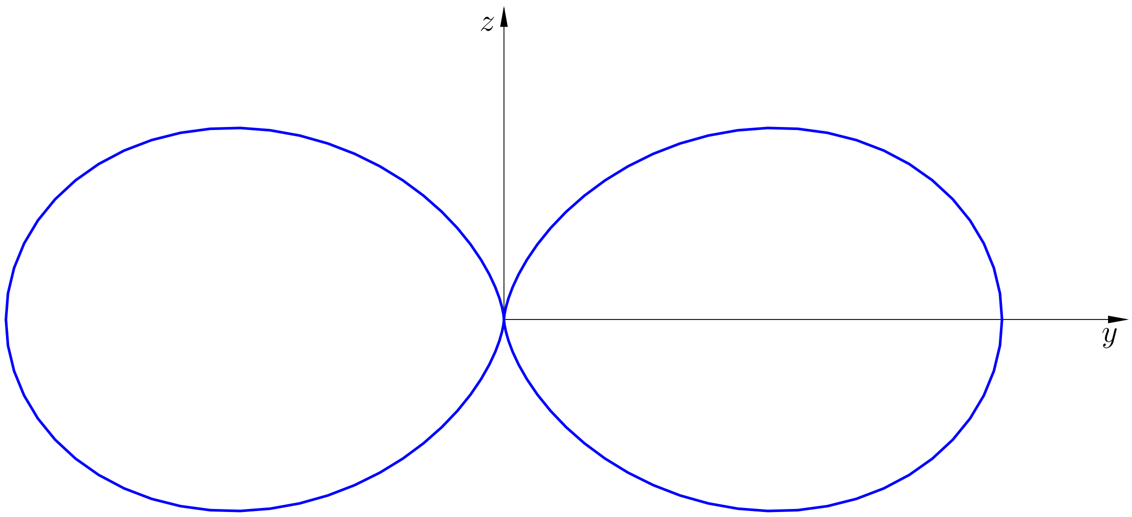
Wynik nie zależy od promienia sfery

$$\frac{\mathrm{d}\langle P \rangle}{\mathrm{d}\Omega} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \boxed{\sin^2\theta}$$ moc wypromieniowana w kąt bryłowy $\mathrm{d}\Omega$



promieniowanie elektryczne dipolowe

charakterystyka kierunkowa



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