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Discrete superpositions of coherent states and phase properties of the m-photon anharmonic oscillator

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Abstract. The formation of discrete superpositions of coherent states via the unitary evolution of the m-photon anharmonic oscillator is studied. Exact analytical formulae for the superposition coefficients are obtained. It is shown that, in contrast to the two-photon anharmonic oscillator, for $m > 2$ the superposition components enter the superposition with different amplitudes. The Pegg–Barnett phase formalism is used to calculate the phase distributions for the resulting states and to visualize their symmetry.

1. Introduction

It is now a well known fact that the quantum evolution of the anharmonic oscillator leads to the quantum states being discrete superpositions of coherent states if the evolution time is properly chosen.This fact was indicated by Yurke and Stoler [1], who considered the m-photon anharmonic oscillator and obtained some special examples of the superposition states. Tombesi and Mecozzi [2] have discussed a possibility of generating quantum mechanical superposition of macroscopically distinguishable states via the non-linear interaction of two modes with orthogonal polarizations in a Kerr medium. The two-photon anharmonic oscillator model was earlier used by Tanaś [3] to show a high degree of squeezing for a large number of photons. The two-mode version of the model was used by Tanaś and Kielich [4] to describe non-linear propagation of light in a Kerr medium, predicting a high degree of what was called 'self-squeezing' of strong light. The comparison of quantum and classical Liouville dynamics of the anharmonic oscillator was made by Milburn [5] and Milburn and Holmes [6]. Kitagawa and Yamamoto [7] have used the model in their discussion of the number-phase minimum uncertainty state that can be obtained in a non-linear Mach–Zehnder interferometer with a Kerr medium. The anharmonic oscillator model has also been discussed by Peřinova and Lukš [8] from the point of view of photon statistics and squeezing. Quantum field superpositions have been discussed by Kennedy and Drummond [9] and by Sanders [10]. Miranowicz *et al* [11] have shown that, in the two-photon anharmonic oscillator model, superpositions of coherent states can be obtained with not only even but also odd numbers of components. They have used the Q function to illustrate the formation of such superpositions and have also shown that the maximum number of well distinguished states is proportional to the field amplitude. Recently, Gantsog and Tanaś [12] have applied the new Hermitian phase formalism of Pegg and Barnett [13–15] to study the formation of discrete superpositions of coherent states and phase properties of elliptically polarized light propagating in a Kerr medium. Tanaś *et al* [16] have compared two descriptions of the superpositions generated in the anharmonic oscillator model: one using the Q function, and the other one using the phase distribution function. Gerry [17] has considered the m-photon anharmonic oscillator model, with the interaction Hamiltonian $\sim \kappa a^{\dagger m} a^m$, showing that squeezing can also be obtained for higher values of m ($m = 3, 4$). The role of the higher optical Kerr non-linearities in self-squeezing of light has recently been discussed by Tanaś and Kielich [18].

In this paper we study the problem of formation of discrete superpositions of coherent states via unitary evolution of the m-photon anharmonic oscillator. Exact formulae for the superposition coefficients are obtained for the superpositions with an arbitrary number of components. Our results are generalizations of the results obtained earlier for the two-photon anharmonic oscillator [11, 12, 19] to the m-photon case with arbitrary $m > 2$. It is shown that for $m > 2$, in contrast to the $m = 2$ case, the superposition components usually enter the superposition with different probabilities. We use the phase distribution function $P(\theta)$ obtained within the Hermitian phase formalism of Pegg and Barnett [13–15] to visualize the superpositions. Polar plots of this function indicate the superpositions of well distinguished coherent states in a very spectacular way showing the number of components, their probabilities and phases. The phase distributions confirm clearly the properties predicted from analytical formulae.

2. The state evolution

The m-photon anharmonic oscillator is described by the following model Hamiltonian

$$H = H_0 + H_I = \hbar\omega a^\dagger a + \hbar\frac{\kappa}{m} a^{\dagger m} a^m \tag{1}$$

and κ is the anharmonicity parameter that can be associated with corresponding non-linearities of the medium when the model is to be used to describe the propagation of optical field in a non-linear medium. We use here the normal ordering of the operators in the interaction Hamiltonian. For $m = 2$ the model becomes that of the well known two-photon anharmonic oscillator. The evolution of the system with the Hamiltonian (1) is described by the Schrödinger equation (in the interaction picture)

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}U(t) = H_I U(t) \tag{2}$$

where the evolution operator $U(t)$ has the form

$$U(t) = \exp\left(-\mathrm{i}\frac{\kappa t}{m}\hat{n}(\hat{n}-1)\ldots(\hat{n}-m+1)\right) \tag{3}$$

with $\hat{n} = \hat{a}^\dagger\hat{a}$ being the photon number operator. Replacing the time t by $-z/v$, and introducing the dimensionless propagation length

$$\tau = \kappa z/v \tag{4}$$

we can write the evolution operator as

$$U(\tau) = \exp\left(\mathrm{i}\frac{\tau}{m}\hat{n}(\hat{n}-1)...(\hat{n}-m+1)\right). \tag{5}$$

The resulting state of the field is thus given by

$$|\psi(\tau)\rangle = U(\tau)|\psi(0)\rangle \tag{6}$$

where $|\psi(0)\rangle$ is the initial state of the field. If the initial state of the field is a coherent state $|\alpha_0\rangle$, the resulting state of the field is

$$\begin{aligned}|\psi(\tau)\rangle &= U(\tau)|\alpha_0\rangle \\ &= \exp(-|\alpha_0|^2/2)\sum_n \frac{\alpha_0{}^n}{\sqrt{n!}}\exp\left(\mathrm{i}\frac{\tau}{m}\hat{n}(\hat{n}-1)...(\hat{n}-m+1)\right)|n\rangle \\ &= \sum_n b_n \exp\left[\mathrm{i}\left(n\varphi_0 + \frac{\tau}{m}A_n^m\right)\right]|n\rangle \end{aligned} \tag{7}$$

where we have used the notation

$$A_n^m = n(n-1)...(n-m+1) \tag{8}$$

$$b_n = \exp(-|\alpha_0|^2/2)\frac{|\alpha_0|^n}{\sqrt{n!}} \tag{9}$$

$$\alpha_0 = |\alpha_0|\exp(\mathrm{i}\varphi_0). \tag{10}$$

The state (7) differs from the initial coherent state $|\alpha_0\rangle$ by the additional (non-linear in n) phase proportional to the numbers A_n^m, that is, the resulting state belongs to a class of generalized coherent states [20, 21]. Such states under certain conditions can become discrete superpositions of coherent states [22].

3. Discrete superpositions of coherent states

The problem of generating discrete superpositions of coherent states for the two-photon ($m = 2$) anharmonic oscillator has been discussed in detail by Miranowicz *et al* [11], who have shown that superpositions with both even and odd numbers of components are possible, but all the components enter a superposition with the same probabilities. Here, we shall give exact analytical formulae for the superpositions with any numbers, N, of components for the generalized, m-photon anharmonic oscillator model. We shall study some new features of the superpositions that appear for $m > 2$.

It was shown by Białynicka-Birula [22] that under periodic conditions generalized coherent states, to a class of which the states (7) belong, become a discrete superposition of N coherent states, and that the superposition coefficients can be found by solving a system of N algebraic equations. Such a system of equations has been solved for several N values by Miranowicz *et al* [11] for the two-photon model. Averbukh and Perelman [23] have considered the problem of the evolution of wavepackets formed by highly excited states of quantum systems showing a possibility of 'fractional revivals' of the initial wavepacket. Their calculations effectively lead to the anharmonic oscillator model. They have shown that because of the periodicity the superposition coefficients can be obtained analytically for arbitrary N. We adopt their approach in our calculations here.

First of all, it is easy to note that the state (7) is periodic in τ

$$|\psi(\tau+T)\rangle = |\psi(\tau)\rangle \tag{11}$$

with the period $T = 2\pi$. This is true because A_n^m/m is an integer number. Moreover, we have

$$A_{n+mN}^m = A_n^m + mN \sum_{s=0}^{m-1} A_n^s A_{n-1+mN-s}^{m-1-s} \tag{12}$$

and

$$\exp\left(\mathrm{i}\frac{\tau}{m}A_{n+mN}^m\right) = \exp\left(\mathrm{i}\frac{\tau}{m}A_n^m\right)\exp\left(\mathrm{i}\tau N\sum_{s=0}^{m-1} A_n^s A_{n-s+mN-1}^{m-s-1}\right) \tag{13}$$

which means that for

$$\tau = \frac{M}{N}2\pi = \frac{M}{N}T \tag{14}$$

the exponential becomes periodic with the period mN. We assume that M and N are mutually prime integers. This means that for τ being a fraction of the period, as in (14), the state (7) becomes a superposition of coherent states [22]

$$|\psi(\tau = MN^{-1}T)\rangle = \sum_{k=0}^{mN-1} c_k|\exp(\mathrm{i}\varphi_k)\alpha_0\rangle \tag{15}$$

where $|\alpha_0\rangle$ is the initial coherent state.

The phases φ_k are given by

$$\varphi_k = \frac{2\pi}{mN}k \qquad k = 0, 1, \ldots, mN-1 \tag{16}$$

and the coefficients c_k are given by the set of mN equations

$$\sum_{k=0}^{mN-1} c_k \exp(\mathrm{i}n\varphi_k) = \exp\left(\mathrm{i}2\pi\frac{M}{mN}A_n^m\right) \tag{17}$$

where $n = 0, 1, \ldots, mN-1$. Equation (17) can be rewritten as

$$\sum_{k=0}^{mN-1} c_k \exp\left(\mathrm{i}\frac{2\pi}{mN}(nk - MA_n^m)\right) = 1 \tag{18}$$

which after summing over n and a minor rearrangement gives

$$\sum_{k=0}^{mN-1} c_k \frac{1}{mN}\sum_{n=0}^{mN-1}\exp\left(\mathrm{i}\frac{2\pi}{mN}(nk - MA_n^m)\right) = 1. \tag{19}$$

In view of the normalization condition

$$\sum_k c_k c_k^* = 1 \tag{20}$$

we immediately obtain the expression for the coefficients c_k in the form

$$c_k = \frac{1}{mN} \sum_{n=0}^{mN-1} \exp\left(-\mathrm{i}\frac{2\pi}{mN}(nk - MA_n^m)\right). \tag{21}$$

Formula (21) gives the coefficients c_k of the superposition (15) for any M and N. Because of the symmetry of the system formula (21) can be rewritten in the form

$$c_k = \frac{1}{m} \sum_{s=0}^{m-1} \exp\left(-\mathrm{i}\frac{2\pi s}{m}(k - MA_{N-1}^{m-1})\right) \frac{1}{N} \sum_{n=0}^{N-1} \exp\left(-\mathrm{i}\frac{2\pi n}{mN}(k - MA_{n-1}^{m-1})\right). \tag{22}$$

The first sum in formula (22) is simply the Kronecker delta, δ_{1r}, where

$$r = \begin{cases} 1 & \text{if } k - MA_{N-1}^{m-1} \text{ is a multiple of } m \\ 0 & \text{otherwise} . \end{cases} \tag{23}$$

This means that only the coefficients c_k with

$$k = k_0 + sm \qquad s = 0, 1, \ldots, N-1 \tag{24}$$

where

$$k_0 = MA_{N-1}^{m-1} \pmod m \tag{25}$$

can be different from zero. So, the maximum number of components in the superposition is equal to N. Anticipating this we have extended the summations in formulae (15)–(21) to mN terms in order to preserve N for the maximum number of components. Thus, the denominator in (14), which is N, defines the maximum number of components in the superposition, and the second summation in (22) has only N terms. Having defined the k for the non-zero coefficients c_k by the equations (24) and (25), we can rename the coefficients using the numbers s, instead of k, omitting in this way the coefficients that are zeros. After such change of the indices the superposition coefficients are given by

$$c_s = \frac{1}{N} \sum_{n=0}^{N-1} \exp\left(-\mathrm{i}\frac{2\pi n}{mN}(k_0 + sm - MA_{n-1}^{m-1})\right) \tag{26}$$

and the superposition (15) can be rewritten in the form

$$|\psi(\tau = MN^{-1}T)\rangle = \sum_{s=0}^{N-1} c_s |\exp(\mathrm{i}\varphi_s)\alpha_0\rangle \tag{27}$$

with

$$\varphi_s = \frac{2\pi}{mN}(k_0 + sm) \quad s = 0, 1, ..., N-1. \tag{28}$$

In fact, the actual number of components in the superposition can in some cases be smaller than N, because for given m and N the sum (26) can still become zero for a particular value of s.

In some special cases expression (26) can be summed up analytically. For instance, it is easy to note that for $N < m$, $nA^{m-1}_{n-1} = A^m_n = 0$, $k_0 = 0$, and the sum (26) is equal to zero unless $s = 0$. That is, for $N < m$, the only non-zero coefficient is $c_0 = 1$, and the state remains unchanged. Thus, the lowest N for which the state (27) becomes a non-trivial superposition of coherent states is $N = m$. In this case $nA^{m-1}_{n-1} = A^m_n$ is still zero, but $k_0 = M(m-1)! \pmod m$ is different from zero if $M \pmod m$ is different from zero, and we have

$$c_s = \frac{1}{m}\sum_{n=0}^{m-1} \exp\left(-\mathrm{i}\frac{2\pi n}{m^2}(k_0 + sm)\right) \tag{29}$$

where $s = 0, 1, ..., m-1$. The geometrical series in (29) can be summed up giving the result

$$\begin{aligned} c_s &= \frac{1}{m}\frac{1 - \exp[-\mathrm{i}(2\pi/m)k_0]}{1 - \exp[-\mathrm{i}(2\pi/m^2)(k_0 + sm)]} \\ &= \frac{1}{m}\exp\left(-\mathrm{i}\frac{\pi}{m^2}[k_0(m-1) - sm]\right)\frac{\sin[(\pi/m)k_0]}{\sin[(\pi/m^2)(k_0 + sm)]}. \end{aligned} \tag{30}$$

In particular, for $M = 1$ and $N = m = 2$, $k_0 = 1$, we have

$$c_0 = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\pi/4} \quad c_1 = \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\pi/4}. \tag{31}$$

The resulting state in this case is

$$|\psi(\tau = 2\pi/2)\rangle_{m=2} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\pi/4}|\mathrm{e}^{\mathrm{i}\pi/2}\alpha_0\rangle + \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\pi/4}|\mathrm{e}^{-\mathrm{i}\pi/2}\alpha_0\rangle. \tag{32}$$

Apart from the rotation by $\pi/2$ in the phase space, the state (32) is the superposition of two coherent states indicated by Yurke and Stoler [1]. The rotation is related to the fact that we use normal ordering of the interaction Hamiltonian. The difference between two possible orderings has been discussed by Miranowicz *et al* [11].

For $M = 1$ and $N = m = 3$, we have $k_0 = 2$ (according to equation (25)), and the non-zero coefficients are

$$c_0 = \frac{\exp(-\frac{4}{9}\mathrm{i}\pi)}{2\sqrt{3}\sin(\frac{2}{9}\pi)} \qquad c_1 = \frac{\exp(-\frac{1}{9}\mathrm{i}\pi)}{2\sqrt{3}\sin(\frac{5}{9}\pi)} \qquad c_2 = \frac{\exp(\frac{2}{9}\mathrm{i}\pi)}{2\sqrt{3}\sin(\frac{8}{9}\pi)} \tag{33}$$

and we get the superposition

$$|\psi(\tau = 2\pi/3)\rangle_{m=3} = c_0|\mathrm{e}^{\mathrm{i}4\pi/9}\alpha_0\rangle + c_1|\mathrm{e}^{\mathrm{i}10\pi/9}\alpha_0\rangle + c_2|\mathrm{e}^{\mathrm{i}16\pi/9}\alpha_0\rangle. \tag{34}$$

The probabilities $|c_0|^2$, $|c_1|^2$ and $|c_2|^2$ with which the component states enter the superposition (34) are different, and the superposition is not symmetrical.

In the case $M = 1$, $N = m = 4$, we have $k_0 = 2$, and the non-zero coefficients are given by

$$c_0 = c_3^* = \tfrac{1}{4}[1 - i(\sqrt{2} + 1)] \qquad c_1 = c_2^* = \tfrac{1}{4}[1 - i(\sqrt{2} - 1)]. \tag{35}$$

The resulting superposition in this case is the following

$$|\psi(\tau = 2\pi/4)\rangle_{m=4} = c_0|e^{i\pi/4}\alpha_0\rangle + c_1|e^{i3\pi/4}\alpha_0\rangle + c_2|e^{-i3\pi/4}\alpha_0\rangle + c_3|e^{-i\pi/4}\alpha_0\rangle. \tag{36}$$

There are two pairs of states with mutually complex conjugate amplitudes and the phases disposed symmetrically with respect to the phase φ_0 of the initial coherent state $|\alpha_0\rangle$. The two examples of the superpositions considered above already show that for $m > 2$ the resulting superpositions are generally less symmetrical than in the case of a two-photon anharmonic oscillator ($m = 2$) for which all the components have the same probability [11, 19].

It turns out also that the expression (26) for the superposition coefficients c_s, which in the case $m = 2$ and $M = 1$ is an example of the Gaussian sum [24], can also be summed up and the result is given by the following simple formula

$$c_s = \frac{1}{\sqrt{N}} \exp\left(i\frac{\pi}{2N} l_s\right) \tag{37}$$

where

$$l_s = \frac{N - k_0 - 1}{2} - 2(s + 1)(s + k_0). \tag{38}$$

These are important results that allow us to write down the superposition states generated in the most important case of the two-photon anharmonic oscillator immediately in a compact and very simple form.

For $m > 2$ we were unable to find formulae, similar to (37) and (38), describing the superposition coefficients c_s, and we use the formula (26) to calculate them. We should emphasize, however, that formula (26) is exact and can be easily evaluated numerically for any finite number of components N.

Knowing the coefficients c_s we can write down the superposition (27), in which $|c_s|^2$ give the probabilities with which particular components enter the superposition. Both amplitudes and phases of the components entering the superposition are clearly indicated by the phase distribution for the superposition if the components are well separated. Examples of such distributions are given in the next section.

4. Phase distributions

We use the Pegg–Barnett [13–15] Hermitian phase formalism to illustrate some properties of the resulting superposition states. The idea of Pegg and Barnett is based on introducing, for one mode of the field, a finite $(s + 1)$-dimensional space Ψ spanned by the number states $|0\rangle, |1\rangle, \ldots, |s\rangle$. The Hermitian phase operator operates on this finite space, and after all necessary expectation values have been calculated in Ψ, the value of s is allowed to tend to infinity. A complete orthonormal basis of $(s+1)$ states is defined on Ψ as

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle \tag{39}$$

where

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1} \qquad (m = 0, 1, ..., s). \tag{40}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states.

The Hermitian phase operator is defined as

$$\hat{\Phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m| \tag{41}$$

where the subscript θ indicates the dependence on the choice of θ_0. The phase states (39) are eigenstates of the phase operator (41) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0 + 2\pi$.

If the state of the field is $\psi(\tau)\rangle$, as given by equation (7), we have

$$\langle\theta_m|\psi(\tau)\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} b_n \exp\left[-i\left(n(\theta_m - \varphi_0) - \frac{\tau}{m} A_n^m\right)\right]. \tag{42}$$

Symmetrizing the phase window with respect to the phase φ_0, i.e. assuming

$$\theta_0 = \varphi_0 - \frac{\pi s}{s+1} \tag{43}$$

and introducing a new phase label $\mu = m - s/2$, which goes in integer steps from $-s/2$ to $s/2$, we have

$$\langle\theta_\mu|\psi(\tau)\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} b_n \exp\left[-i\left(n\theta_\mu - \frac{\tau}{m} A_n^m\right)\right] \tag{44}$$

where $\theta_\mu = \mu 2\pi/(s+1)$. So, the probability of being in the phase state $|\theta_\mu\rangle$ is given by

$$|\langle\theta_\mu|\psi(\tau)\rangle|^2 = \frac{1}{s+1} + \frac{2}{s+1} \sum_{n>k} b_n b_k \cos\left((n-k)\theta_\mu - \frac{\tau}{m}(A_n^m - A_k^m)\right). \tag{45}$$

In the limit as s tends to infinity, the continuous phase variable can be introduced replacing $\mu 2\pi/(s+1)$ by θ and $2\pi/(s+1)$ by $d\theta$. This leads to the continuous phase probability distribution given by the formula

$$P(\theta) = \frac{1}{2\pi}\left[1 + 2\sum_{n>k} b_n b_k \cos\left((n-k)\theta - \frac{\tau}{m}(A_n^m - A_k^m)\right)\right] \tag{46}$$

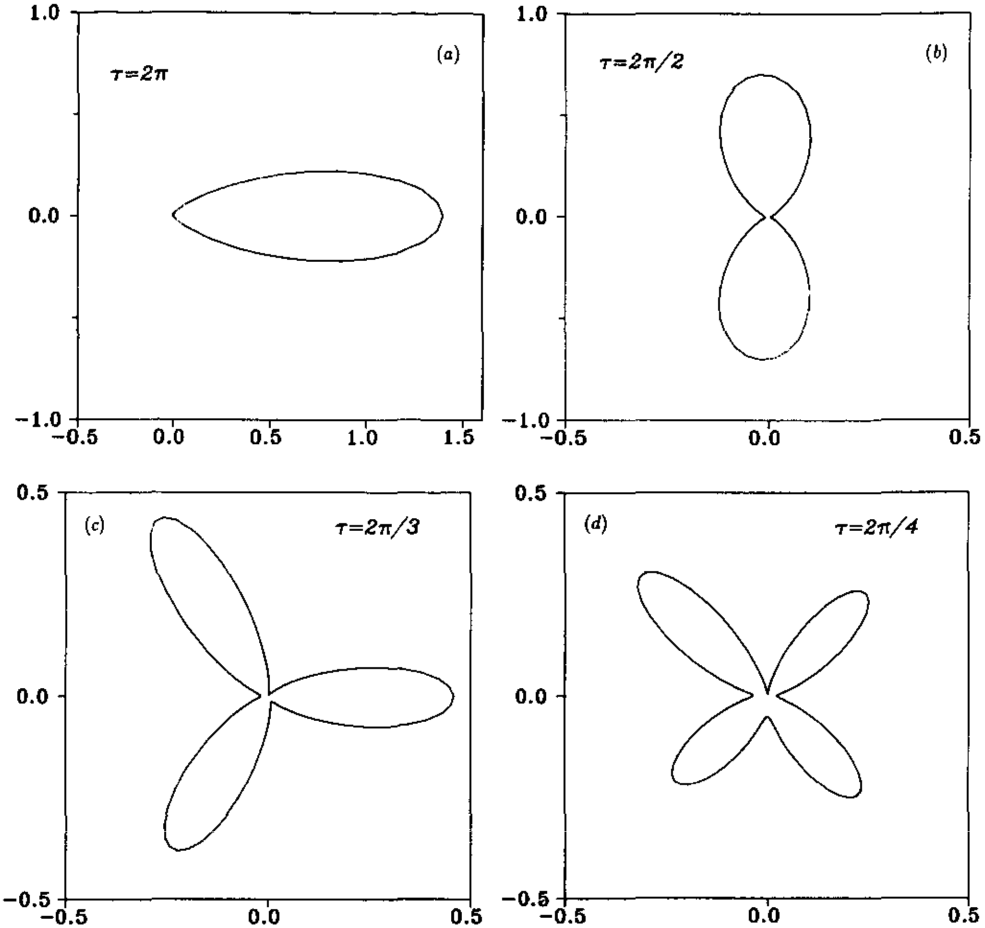

Figure 1. Phase probability distribution $P(\theta)$ plotted against θ in the polar coordinate system for $m = 2$ and $\tau = 2\pi/N$: (*a*) $N = 1$, (*b*) $N = 2$, (*c*) $N = 3$, (*d*) $N = 4$.

with the normalization

$$\int_{-\pi}^{\pi} P(\theta)\,\mathrm{d}\theta = 1. \tag{47}$$

For $m = 2$, i.e. the two-photon anharmonic oscillator, the phase properties have been studied by Gerry [25] and Gantsog and Tanaś [19]. It has been shown [19] that the phase distribution function $P(\theta)$ when plotted in polar coordinates clearly indicates the superposition of coherent states that appear during the evolution of the anharmonic oscillator. Here, we use the same idea, to apply the phase distribution $P(\theta)$ for studying the superposition of coherent states appearing in the course of evolution of the m-photon anharmonic oscillator.

In figure 1 we present the polar plots of the phase distribution $P(\theta)$ for the two-photon anharmonic oscillator. Such plots have already been shown in [19], and we adduce them here just for reference. The distributions were obtained according to formula (46) for the mean number of photons $|\alpha_0|^2 = 4$. We have assumed $M = 1$ everywhere. The symmetry of the states is clearly visible, although for $\tau = 2\pi/4$

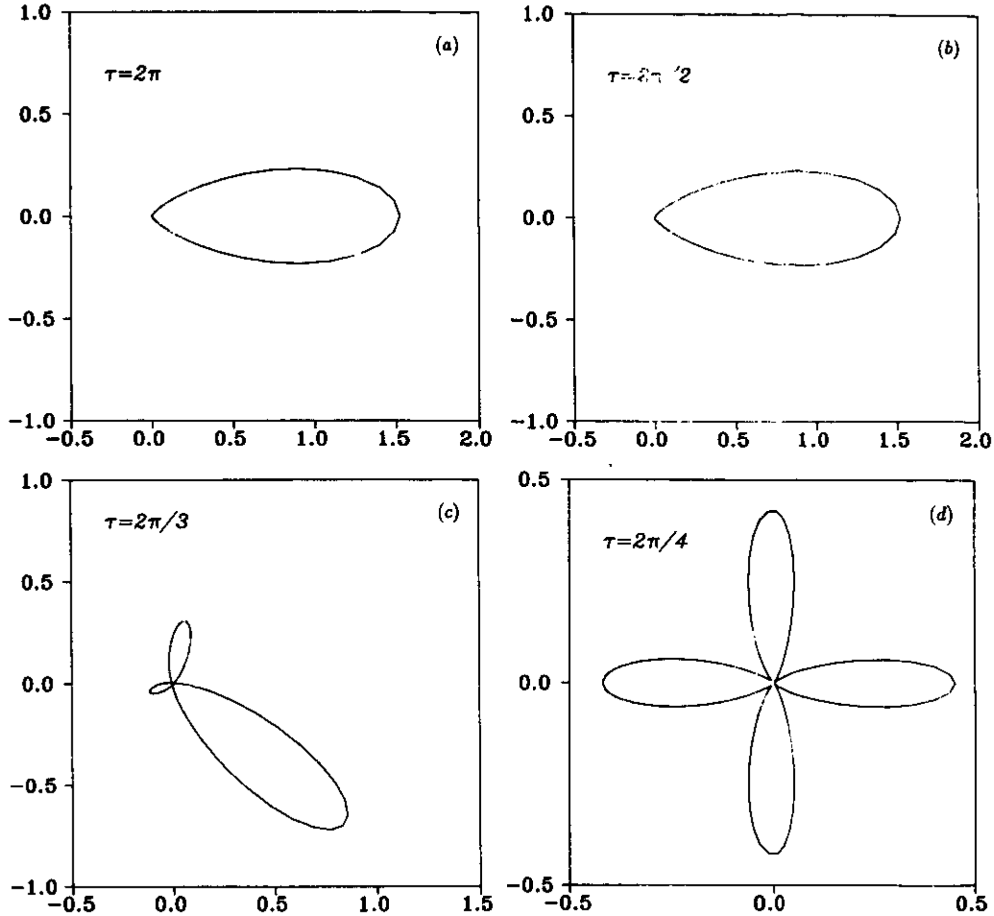

Figure 2. The polar coordinate pictures of $P(\theta)$ for $m = 3$ and $\tau = 2\pi/N$: (*a*) $N = 1$, (*b*) $N = 2$, (*c*) $N = 3$, (*d*) $N = 4$.

the fourfold rotational symmetry is already distorted. This is caused by the influence of the interference terms, which play an increasing role and the components start to overlap. The maximum number of well separated states is proportional to $|\alpha_0|^2$ and was estimated earlier [11].

In figures 2 and 3 we show a few examples of the superposition states for the three- and four-photon processes. The differences are quite clear and confirm the results obtained analytically for the superposition coefficients c_s. Whenever the probability $|c_s|^2$ for a given peak is smaller than for other peaks, the lobe in the phase distribution is also smaller. It is also seen that some components disappeared, as is evident for example for $m = 3$ and $\tau = \pi = 2\pi/2$, where instead of two peaks only one peak occurs. Generally, for $m > 2$, the phase peaks have different heights, which is in sharp contrast with the two-photon case. This means that the phase properties of the superposition with the same number of components are quite different. The Pegg–Barnett phase formalism allows us to calculate any phase characteristics of the field (such as, for example, phase variance), but we are not going to do it here. We have treated the phase distribution rather as a useful tool to visualize our analytical results and not as a main subject of the paper.

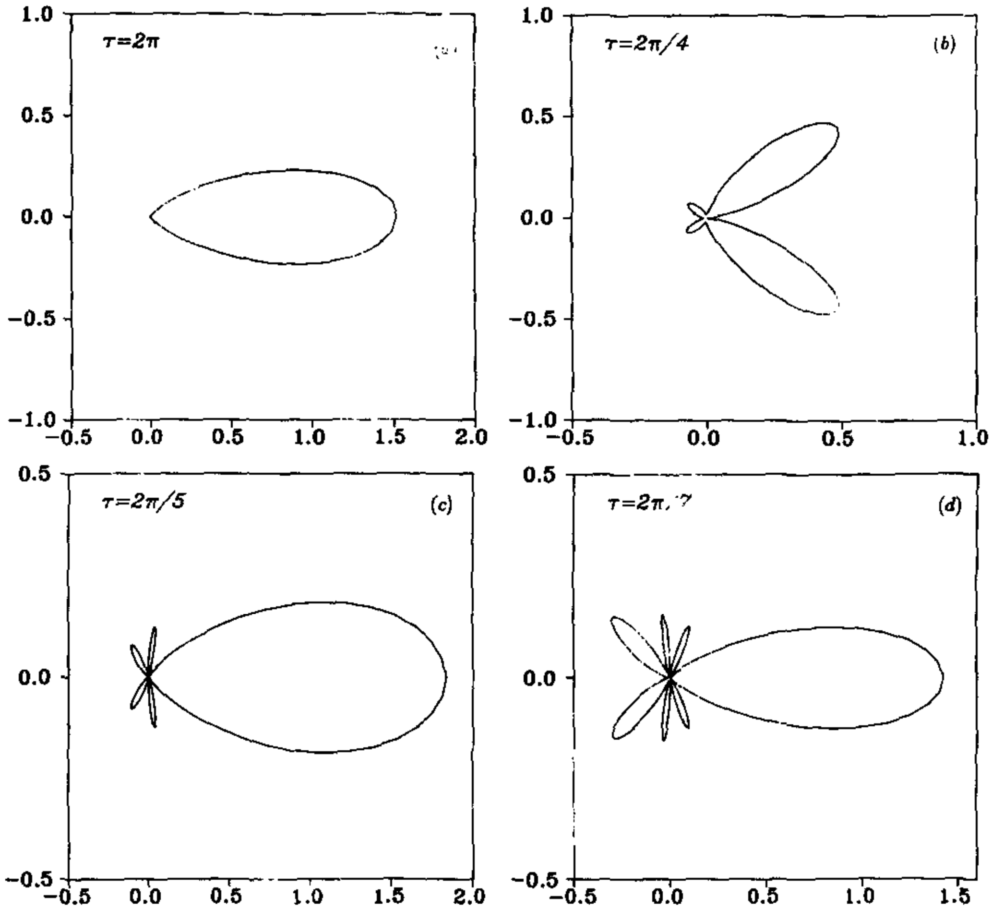

Figure 3. The polar coordinate pictures of $P(\theta)$ for $m = 4$ and $\tau = 2\pi/N$: (*a*) $N = 1$, (*b*) $N = 4$, (*c*) $N = 5$, (*d*) $N = 7$.

5. Conclusion

In this paper we have studied the problem of generation of discrete superpositions of coherent states in the course of evolution of the m-photon anharmonic oscillator. We have obtained exact analytical formulae for the superposition coefficients with an arbitrary number of components. We have shown that, in contrast to the two-photon process, the superposition components for $m > 2$ enter the superposition with different probabilities making the superposition less symmetrical. We have applied the phase distribution function $P(\theta)$ obtained from the Pegg–Barnett Hermitian phase formalism to show explicitly the symmetry of the superpositions. The polar plots of this function clearly reflect such symmetry showing the number of components (if the states are well separated), their probabilities and phases. The phase distributions calculated numerically from the exact quantum state of the field, for the evolution times $\tau = 2\pi M/N$, supported our predictions based on the analytical formulae for the superposition coefficients.

References

[1] Yurke B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13
[2] Tombesi P and Mecozzi A 1987 *J. Opt. Soc. Am.* A **4** 1700
[3] Tanaś R 1984 *Coherence and Quantum Optics V* ed L Mandel and E Wolf (New York: Plenum) p 645
[4] Tanaś R and Kielich S 1983 *Opt. Commun.* **45** 351; 1984 *Opt. Acta* **31** 81
[5] Milburn G J 1986 *Phys. Rev.* A **33** 674
[6] Milburn G J and Holmes C A 1986 *Phys. Rev. Lett.* **56** 2237
[7] Kitagawa M and Yamamoto Y 1986 *Phys. Rev.* A **34** 3974
[8] Peřinowa V and Lukš A 1988 *J. Mod. Opt.* **35** 1513
[9] Kennedy T A B and Drummond P D 1988 *Phys. Rev.* A **38** 1319
[10] Sanders B C 1989 *Phys. Rev.* A **39** 4284
[11] Miranowicz A, Tanaś R and Kielich S 1990 *Quantum Opt.* **2** 253
[12] Gantsog Ts and Tanaś R 1991 *Quantum Opt.* **3** 33
[13] Pegg D T and Barnett S M 1988 *Europhys. Lett.* **6** 483
[14] Barnett S M and Pegg D T 1989 *J. Mod. Opt.* **36** 7
[15] Pegg D T and Barnett S M 1989 *Phys. Rev.* A **39** 1665
[16] Tanaś R, Gantsog Ts, Miranowicz A and Kielich S 1991 *J. Opt. Soc. Am.* B **8** 1576
[17] Gerry C C 1987 *Phys. Lett.* **124A** 237
[18] Tanaś R and Kielich S 1990 *Quantum Opt.* **2** 23
[19] Gantsog Ts and Tanaś R 1991 *J. Mod. Opt.* **38** 1021
[20] Titulaer U and Glauber R J 1965 *Phys. Rev.* **145** 1041
[21] Stoler D 1971 *Phys. Rev.* D **4** 2309
[22] Białynicka-Birula Z 1968 *Phys. Rev.* **173** 1207
[23] Averbukh I Sh and Perelman N F 1989 *Phys. Lett.* **139A** 449
[24] Korobov N M 1989 *Trigonometric Sums and their Applications* (Moscow: Nauka) (in Russian)
[25] Gerry C C 1990 *Opt. Commun.* **75** 168