We study state-dependent quantum cloning which can outperform universal cloning. This is possible by using some a priori information on a given quantum state to be cloned. Specifically, we propose a generalization and optical implementation of quantum optimal mirror phase-covariant cloning, which refers to optimal cloning of sets of qubits of known modulus of expectation value of Pauli’s $Z$ operator. Our results can be applied for cloning of an arbitrary mirror-symmetric distribution of qubits on Bloch sphere including in special cases the universal cloning and phase-covariant cloning. We show that the cloning is optimal by adapting our former optimality proof for axisymmetric cloning [Phys. Rev. 82, 042330 (2010)]. Moreover, we propose an optical realization of the optimal mirror phase-covariant $1 \rightarrow 2$ cloning of a qubit, for which the mean probability of successful cloning varies from 1/6 to 1/3 depending on prior information on the set of qubits to be cloned. The qubits are represented by polarization states of photons generated by the type-I spontaneous parametric down-conversion. The scheme is based on the interference of two photons on an unbalanced polarization-dependent beam splitter with different splitting ratios for vertical and horizontal polarization components and the additional application of feedforward by means of Pockels cells. The experimental feasibility of the proposed setup is carefully studied including various kinds of imperfections and losses including: (i) finite efficiency of generating a pair of entangled photons in the type-I spontaneous parametric down conversion, (ii) the influence of choosing various splitting ratios of the unbalanced beam splitter, (iii) the application of conventional and single-photon discriminating detectors, (iv) dark counts and finite efficiency of the detectors.

I. INTRODUCTION

The no-cloning theorem [1, 2] tells that unknown quantum states cannot be copied perfectly, which is implied by the linearity of quantum mechanics. The no-cloning theorem guaranties, e.g., the security (or privacy) of quantum communication protocols including quantum key distribution and excludes naïve protocols of superluminal communication with entangled particles.

As perfect quantum cloning is impossible, much attention has been devoted to approximate and probabilistic quantum cloning. Such studies are especially important for quantum cryptography [3], but also for quantum communication [4] and computation [5].

It is worth noting that quantum cloning is not only of theoretical interest. In fact, a few experimental realizations of quantum cloning have been reported [6]. In particular, quantum cloning with prior partial information, which is the main topic of our paper, was experimentally demonstrated using nuclear magnetic resonance [5] and optical systems [10, 11]. Also quantum-dot implementations of cloning machines were considered [12].

The first $1 \rightarrow 2$ optimal cloning machine was designed by Bužek and Hillery [3]. This cloning machine, referred to as the universal cloning machine (UC), prepares two approximate copies of an unknown pure qubit state with the same fidelity $F = 5/6$. This means that the UC is state independent (i.e., the cloning is equally good for any pure qubit state) and symmetric (i.e., the copies are identical). The case of the UC producing the infinite number of copies [13] allowed to establish the classical limit of $F = 5/6$ for copying quantum information, which corresponds to the best copying operation achieved by classical operations.

Next, the concept of optimal cloning was extended to include cloning of qubits, cloning of continuous-variable systems, and state-dependent cloning (non-universal cloning).

The state-dependent cloning machines can produce clones of a specific set of qubits with much higher fidelity than $F = 5/6$ [14–24] (see also reviews [25, 26] and references therein). The study of state-dependent cloning is well motivated since we often have some a priori information about a given quantum state that we want to clone and by employing the available information, we can construct a cloning machine which surpasses the UC for some a priori specified set of qubits.

For example, if the qubits are taken from the equator of the Bloch sphere then by using the so-called optimal phase-covariant cloners (PCCs) [15, 21], one achieves higher fidelity than that for the UC. The phase-covariant or phase-independent cloning was further generalized by Fiurášek [19] who studied the PCCs of qubits of known expectation value of Pauli’s $Z$ operator and provided two optimal symmetric cloners: one for the states in the northern and the other for those in the southern hemisphere of the Bloch sphere.

Further works on phase-independent cloning included cloning of qubits uniformly distributed on a belt of the Bloch sphere.
and is larger than the fidelity in the case of the UC, of mode mismatch on the fidelity of the MPCC. Analysis version (SPDC), the influence of choosing various parameters as follows:

\[ \langle \sigma_z \rangle = \cos \theta \]

(3)

and \( \langle \sigma_z \rangle = \cos \theta \). The average fidelity \( F \) over the input states of the cloning machine is equal to \( F = 0.8594 \) and is larger than the fidelity in the case of the UC, which is equal to \( F = 0.8333 \). Moreover, from Ref. 24 follows that any optimal cloning machine that copies a phase-covariant set of qubits and exhibits mirror \( xy \)-plane symmetry is described by such general transformation. Therefore, the proposed experimental setup can be used for cloning an arbitrary set of qubits of the described symmetry. It is worth noting that former proposals of realizations of the MPCC in linear-optical systems, 23, 24 and quantum dots, 23, 27 were discussed formally without referring to experimental setups.

In this paper we propose an optical implementation of the MPCC 23 based on a generalized version of the setup described by Černoch et al. 10 (see also Ref. 11). The experimental setup can equally well perform operations of the UC, PCC, and MPCC in special cases, i.e., for the proper choice of \( \Lambda \) (the explicit formulas can be found in Ref. 24).

In the following sections we analyze the performance of our setup accounting for various losses and imperfections such as finite efficiency of generating a pair of entangled photons in the type-I spontaneous parametric down conversion (SPDC), the influence of choosing various parameters of an unbalanced beam splitter (splitting ratios for vertical and horizontal polarization components), finite detector efficiency, dark counts, and finite resolution of applied detectors. For simplicity, we neglect the effects of mode mismatch on the fidelity of the MPCC. Analysis of such losses would require application of a pulse-mode formalism (see, e.g., Ref. 28).

The paper is organized as follows. In Sec. II, we present a setup implementing the optimal symmetric \( 1 \rightarrow 2 \) mirror phase-covariant cloning of a qubit and study the influence of imperfections of the beam splitter on the performance of cloning. In Sec. III, we study the performance of the setup assuming imperfect photon detectors by means of the positive operator valued measure (POVM) formalism. We conclude in Sec. IV.

FIG. 1. Scheme of the experimental setup used to implement the MPCC. We use the following acronyms for the standard optical elements: QWP – quarter-wave plate, HWP – half-wave plate, PBS – balanced polarizing beam splitter, PDBS – polarization-dependent beam splitter for different splitting ratios for H and V polarizations, D – detector, PC – Pockels cell, PL – pulsed laser, SHG – second-harmonic generation and M – mirror. Double solid lines denote transfer of frequency and M – mirror. Double solid lines denote transfer of frequency and spatial degrees of freedom. The input state \( |\Psi\rangle_{0012} \) is prepared by means of spontaneous parametric down conversion of type-I (see, e.g., Ref. 24) using a stack of two \( \beta \)-barium borate (BBO) crystals. Next, \( |\Psi\rangle_{0012} \) is transformed into \( |\Psi\rangle_{00122'} \) by first setting the input state \( |\psi\rangle_2 \) to be cloned with the QWP, and then mixing the modes 1 and 2 on the PDBS (this can be considered as the first step of the actual cloning). Finally, \( |\Psi\rangle_{00122'} \) is subject to classical feedforward, driven by the measurement outcomes of D1 and D2. As a result, we obtain state \( \hat{\rho}_{out} \) which is the output of the cloning machine as long as there is one photon in mode 1’ and 2’. Detector D3 is used as a trigger for the experiment, which practically eliminates the probability of having vacuum in mode 2. This is due to low dark-count rate of modern photon detectors.

II. A PROPOSAL FOR PRACTICAL PHOTONIC IMPLEMENTATION OF THE MPCC

A. Initialization

The initial entangled state is prepared by using parametric down conversion of the first type (see Refs. 29–32). The output of a pulsed laser (PL), with angular frequency \( \omega_0 \), is frequency doubled in a nonlinear crystal to produce pulses of ultraviolet (UV) light of angular frequency 2\( \omega_0 \). The UV pulses are then used to pump twice (in forward and backward directions) a pair of nonlinear
crystals which are stacked together such that their optical axes are orthogonal to each other \(29, 33\). The crystals are for the type-I SPDC to produce photon pairs in two modes (idler and signal) of the same polarization and of half the frequency of the PL. In the forward pumping direction, the polarization of the UV beam is set to vertical so that an H-polarized photon pair in modes 2 and 0' are generated. The remaining (non-down-converted) portion of the UV beam first passes through a quarter wave plate (QWP) which changes its polarization into an ellipsoidal polarization. A mirror \(M_1\) placed after the QWP reflects this beam and sends it through the QWP again which further changes the polarization of the beam into diagonal polarization. This diagonally polarized beam pumps the crystals in the backward direction creating the entangled photon pair \(|\psi\rangle = (|1_H\rangle_0|1_V\rangle_1 + |1_V\rangle_0|1_H\rangle_1)/\sqrt{2}\). However, the total state of the system in modes 0, 0', 1, and 2 is more complex than

\[
|\Psi\rangle = |\psi_{+}\rangle_{0,1}|1_H\rangle_{0'}|1_H\rangle_2,
\]

which we use in our further analytical considerations. On the one hand, the SPDC is probabilistic and the state \(|\Psi\rangle\) consists also of the vacuum and higher-order SPDC terms. On the other hand, for the circuit to work we require fourfold coincidence count in all modes and a very low dark-count rate of modern photon detectors (dark count probability of the order of \(10^{-6}\)) allows us to effectively eliminate the vacuum state from the further considerations. Moreover, the measurement of a photon in mode 0 is polarization dependent, which is further used in the feedforward processing. So, finally the system is prepared in the state

\[
|\Psi\rangle = \mathcal{N} \left[ \gamma^2 e^{2i\phi} |\psi_{+}\rangle_{01}|1_H\rangle_{0'}|1_H\rangle_2 \\
+ \gamma^3 e^{3i\phi} (|\psi_{+}\rangle_{01}|2_H\rangle_{0'}|2_H\rangle_2 \\
+ |\psi_{+}\rangle_{01}|1_H\rangle_{0'}|1_H\rangle_2 + \mathcal{O}(\gamma^5) \right],
\]

(5)

where

\[
|\psi_{+}\rangle_{01} = \frac{1}{2}(|1_{H1V}\rangle_0 + |1_{H1V}\rangle_1 + |2_{H0}\rangle_0|2_{V1}\rangle_1 + |2_{V0}\rangle_0|2_{H1}\rangle_1).
\]

Moreover, \(\gamma\) describes the efficiency of the SPDC and depends on the amplitude of the incident field and properties of the nonlinear crystal, \(\phi\) is the phase shift caused by the SPDC, and \(\mathcal{N}\) is the normalization constant. Typically \(\gamma^2 = 0.01\ \[33\] for simplicity we can neglect the terms of amplitudes of order higher than \(\gamma^3\) since probability of the occurrence of such event is very low as \(|\mathcal{O}(\gamma^4)|^2 = \mathcal{O}(\gamma^8)\). We will consider a more complete form of \(|\Psi\rangle\) only in Sect. III.

Next, we prepare the arbitrary state to be cloned in mode 2 by a combination of a half-wave plate (HWP) and QWP. The input state is passed into mode 2. It is given in the following form:

\[
|\psi\rangle_2 = (\alpha a_{2H}^\dagger + \beta a_{2V}^\dagger)|0\rangle_2,
\]

(6)

where \(\alpha = \cos(\theta/2)\) and \(\beta = e^{i\delta}\sin(\theta/2)\). Later, modes 1 and 2 are mixed on an unbalanced polarization-dependent beam splitter (PDBS). The PDBS transforms the input in the following way

\[
\begin{align*}
\hat{a}_{1H}^\dagger &\rightarrow \sqrt{1 - \mu^2} \hat{a}_{1H}^\dagger - \sqrt{\mu^2} \hat{a}_{2H}^\dagger, \\
\hat{a}_{1V}^\dagger &\rightarrow \sqrt{1 - \nu^2} \hat{a}_{1V}^\dagger + \sqrt{\nu^2} \hat{a}_{2V}^\dagger, \\
\hat{a}_{2H}^\dagger &\rightarrow \sqrt{1 - \mu^2} \hat{a}_{2H}^\dagger + \sqrt{\mu^2} \hat{a}_{1H}^\dagger, \\
\hat{a}_{2V}^\dagger &\rightarrow \sqrt{1 - \nu^2} \hat{a}_{2V}^\dagger - \sqrt{\nu^2} \hat{a}_{1V}^\dagger.
\end{align*}
\]

(7)

The MPCC can be implemented when

\[
\mu + \nu = 1.
\]

(8)

The most convenient situation is when \(\mu = \mu_0 = (1 - 1/\sqrt{3})/2\) and \(\nu = \nu_0 = (1 + 1/\sqrt{3})/2\), i.e., \(1 - 2\mu = 2\nu - 1 = \sqrt{2}\mu\nu = 1/\sqrt{3}\). Analogical conditions for the PCC were given by Fiurášek \[19\]. Finally, the state of the system after the action of the PDBS (for \(\mu = \nu\)) is given by the following expression:

\[
|\Psi'\rangle = \mathcal{N}' \left[ \alpha a_{0V}^\dagger \left( \hat{\gamma} \hat{a}_{1V}^\dagger \hat{a}_{1H}^\dagger - \hat{a}_{2H}^\dagger \hat{a}_{2V}^\dagger \right) \\
+ (1 - 2\mu) \hat{a}_{1H}^\dagger \hat{a}_{2H}^\dagger \\
+ \beta a_{0V}^\dagger \left( \nu \hat{a}_{1V}^\dagger \hat{a}_{1H}^\dagger + \mu \hat{a}_{2H}^\dagger \hat{a}_{2V}^\dagger \\
- \sqrt{\mu^2} \hat{a}_{1V}^\dagger \hat{a}_{2V}^\dagger + \hat{a}_{1H}^\dagger \hat{a}_{2V}^\dagger \right) \\
+ \alpha a_{0H}^\dagger \left( \mu \hat{a}_{1H}^\dagger \hat{a}_{1V}^\dagger + \nu \hat{a}_{2V}^\dagger \hat{a}_{2H}^\dagger \\
+ \sqrt{\mu^2} \hat{a}_{1V}^\dagger \hat{a}_{2V}^\dagger + \hat{a}_{1H}^\dagger \hat{a}_{2V}^\dagger \right) \\
+ \beta a_{0H}^\dagger \left( \nu \hat{a}_{1V}^\dagger \hat{a}_{1V}^\dagger - \hat{a}_{2V}^\dagger \hat{a}_{2V}^\dagger \right) \\
+ (1 - 2\mu) \hat{a}_{1V}^\dagger \hat{a}_{2V}^\dagger \right] \hat{a}_{2H}^\dagger |0\rangle_{00'}|1'\rangle_2,
\]

(9)

where \(\mathcal{N}'\) is a normalization constant.

### B. Feedforward

In order to implement the MPCC we also apply a feedforward technique (see Refs. \[34, 37\]), i.e., photons of the same polarization as detected in mode 0 are dumped in modes 1' and 2'. The element implementing the dumping is based on a Pockels cell and two PBSs and is presented in Fig. 1. As it was shown in Ref. \[34\] that such operation can be performed with high fidelity of more than 99%. The final density matrix of the system is given as

\[
\hat{\rho}_{\text{out}} = \text{Tr}_{00'} \left[ (\hat{\Pi}^0_{1H} \hat{\Pi}^0_{1V} \hat{D}_H \hat{\rho} \hat{D}_H^\dagger + \hat{\Pi}^0_{1V} \hat{\Pi}^0_{0H} \hat{D}_V \hat{\rho} \hat{D}_V^\dagger) \hat{\Pi}^0_{1V} \right],
\]

(10)

where \(\hat{\rho}' = |\psi'\rangle \langle \psi'|\) is the output state after the action of the unbalanced PDBS, \(\hat{D}_H = \hat{\Gamma}_{1H}^0 \hat{\Gamma}_{2H}^0\), \(\hat{D}_V = \hat{\Gamma}_{1V}^0 \hat{\Gamma}_{2V}^0\), where \(\hat{\Gamma}_{iV}^0 (\hat{\Gamma}_{iH}^0)\) is the operation acting on photons in the \(i\)th spatial mode, which corresponds to the conditional application of a Pockels cell (see Fig. 1). \(\hat{\Pi}^i_{1H} (\hat{\Pi}^i_{1V})\) are the POVM operators describing the probability of detection of the \(j\) H(V)-polarized photons in the
only one photon in every outgoing mode. Probability of
when two of the detectors click, one of the pair (D_4,D_5) and
one of the pair (D_6,D_7). Four detectors are used in order
to evaluate the fidelity of the cloning operations as given in
Eq. (22), where the pairs of detectors (D_4,D_5) and (D_6,D_7)
correspond to the POVMs (Π_1,Π_1) and (Π_1,Π_1), respectively.

D. Fidelity of the proposed experimental setup

In order to describe the quality of the cloning we use
single-copy fidelity
\[ F_i = \frac{\langle \psi | \text{Tr}_{\text{out}} \hat{\rho}_{\text{out}} | \psi \rangle}{\text{Tr} \hat{\rho}_{\text{out}}}. \] (13)

However, to describe the overall performance of a
cloning machine it is more convenient to use the average
single-copy fidelity
\[ F = \frac{1}{2} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \ g(\theta, \phi) [F_1(\theta, \phi) + F_2(\theta, \phi)] \] (14)

which is an average over all possible input qubits defined
by the distribution function \( g(\theta, \phi) \). In case of the
MPCC, \( g \) distribution form Eq. (14) is given by
\[ g_0(\theta, \phi) = \frac{1}{4\pi} [\delta(\theta - \theta') + \delta(\theta + \theta' - \pi)]. \] (15)

In terms of Dirac’s δ-function. Moreover, we added sub-
script \( \theta \) to indicate \textit{a priori} knowledge about the input state.

One can easily check that the resulting expression for
the average single-copy fidelity is the same as for the
MPCC (22) and given by
\[ F = F_1 = F_2 = \frac{1 + \Lambda^2}{2} - \frac{1}{2} \Lambda \left( \Lambda - \Lambda \sqrt{2} \right) \sin^2 \theta. \] (16)

From Eq. (16) follows that the average cloning fidelity over all possible input states of the MPCC (over all \( \theta \) -
the average is over all possible circles and their mirror-
symmetric counterparts) is \( F = 0.8594 \).

Note that for simplicity of exposition, we focus here on
the MPCC, which is the simplest nontrivial example of
cloning of mirror-symmetric distributions \( g(\theta) \) on Bloch
sphere, where \( g(\theta) \) is a sum of two Dirac’s δ-functions.
However, in the case of other mirror-symmetric phase-
covariant qubit distributions we obtain different values
the average cloning fidelity and success rate of the prop-
osed experimental setup. For example, in the case of the
UC we obtain \( F = 0.8333 \) (the average is over the
whole Bloch sphere), and \( F = 0.8536 \) (the average over
the equator of Bloch sphere) in the case of the PCC. For
the UC and PCC, we have \( \Lambda = \sqrt{2} / 3 \) and \( \Lambda = 1 / \sqrt{2} \),
respectively.

III. PRACTICAL CONSIDERATIONS FOR
EXPERIMENTAL IMPLEMENTATION

A. Choosing the parameters of unbalanced
polarization-dependent beam splitter

In order to perform the required quantum transfor-
mation in some cases one needs to use a polarization-
dependent beam splitter of some strictly chosen values
two clones reaches its maximum for the MPCC. The average fidelity over the first clone (a), the second clone (b) and its average over the two clones (c) for the MPCC. The average fidelity drops and cloning is no longer symmetric (see Fig. 3). This happens when \( \nu = 1 - \mu \) and the inequality given in Eq. (19) is satisfied.

![Graphs showing the average success probability of the proposed setup in the case of the MPCC.](image)

Hence, imperfections of the PDBS (see Refs. [10, 33]) result only in decreasing the success rate of the setup (see Fig. 3). Therefore, the probability of successful cloning is given by the following expression:

\[
P_{\text{success}} = \frac{(1 - 2\mu)^2}{2\bar{\Lambda}^2},
\]

where

\[
\frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) \leq \mu \leq \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right).
\]
B. Influence of detector imperfections

Detectors play an important part in the proposed experiment. As one can see in Eq. (10), the density matrix $\tilde{\rho}_{\text{out}}$ depends explicitly on the measurements performed on the ancillary qubits. Also in practical realizations of the cloning machine, the fidelity of the cloning process can be evaluated by measuring polarization of photons in modes $1^0$ and $2^0$ in the basis of $|\psi\rangle$ and $|\bar{\psi}\rangle$ as described in Ref. 10. This gives seven detectors in total, however we analyze only the cases when four detectors (one photon per detector) click at the same time. For simplicity, we assume that all the detectors are characterized by the same parameters.

There are two basic types of photon detectors that can be used in the experiment: single-photon counters and ON/OFF detectors. Since we cannot exclude completely the possibility of higher-order SPDC events [see Eq. (3)] we investigate the implications of using both types of the detectors.

1. Single-photon counters

First we analyze single-photon counters, which can discriminate between vacuum, detection one photon, and detection of many photons. We describe imperfections of these detectors by the following POVM operators [38]:

\[
\hat{\Pi}_0 = \sum_{m=0}^{\infty} e^{-\zeta} (1 - \eta)^m |m\rangle \langle m|,
\]

\[
\hat{\Pi}_1 = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} e^{-\zeta} \eta^n m^n (1 - \eta)^{m-n} |m\rangle \langle m|,
\]

\[
\hat{\Pi}_{N \geq 2} = \mathbb{I} - \hat{\Pi}_0 - \hat{\Pi}_1.
\]

where $\eta$ is quantum efficiency of the detectors and $\zeta$ stands for the dark-count rate (typically of the order of $10^{-6}$).

2. ON/OFF detectors

We also analyze the ON/OFF detectors (also referred to as conventional or bucket detectors), which can discriminate only between vacuum and any other number of photons. The difference between the single-photon counters and ON/OFF detectors is negligible in the case of low dark-count rate. Since we are interested only in such events where the number detector “clicks” is equal to the assumed number of photons in the system. We use the following POVM operators [38]:

\[
\hat{\Pi}_0 = \sum_{m=0}^{\infty} e^{-\zeta} (1 - \eta)^m |m\rangle \langle m|,
\]

\[
\hat{\Pi}_{N \geq 1} = \mathbb{I} - \hat{\Pi}_0.
\]

3. Expected fidelity and probability of cloning

In our proposed experimental setup we use post-selection, thus the average fidelities of the clones can be expressed via coincidences as [10]:

\[
F_1 = \frac{C_{11} + C_{10}}{P_{\text{success}}}, \quad F_2 = \frac{C_{11} + C_{01}}{P_{\text{success}}},
\]

where

\[
P_{\text{success}} = C_{00} + C_{01} + C_{10} + C_{11}
\]

is the probability of success (successful post-selection) and $C_{ij}$ ($i,j \in \{0,1\}$) are the following coincidences

\[
C_{11} = \text{Tr} (\hat{\rho}_{\text{out}} \hat{\Pi}_1 \otimes \hat{\Pi}_1), \quad C_{10} = \text{Tr} (\hat{\rho}_{\text{out}} \hat{\Pi}_1 \otimes \hat{\Pi}_0),
\]

\[
C_{01} = \text{Tr} (\hat{\rho}_{\text{out}} \hat{\Pi}_0 \otimes \hat{\Pi}_1), \quad C_{00} = \text{Tr} (\hat{\rho}_{\text{out}} \hat{\Pi}_0 \otimes \hat{\Pi}_0).
\]

Here, the POVMs $\hat{\Pi}_1$ and $\hat{\Pi}_0$ correspond to the detection of a photon in the state $|\psi\rangle$ and $|\bar{\psi}\rangle$, respectively. As one can see in Eq. (10), $\hat{\rho}_{\text{out}}$ depends on the quality and type of the photon detectors. Moreover, it also depends on the efficiency of generation of the entangled photon pairs [see Eq. (3)]. In the case of perfect detectors (both single-photon counters and ON/OFF detectors) we have $\hat{\Pi}_1 = |1\psi\rangle\langle 1\psi|$. The influence of imperfections of measurements on the fidelity of cloning and success rate for single-photon counters (ON/OFF detectors) is summarized in Tables I and II.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$P_{\text{success}}$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$P_{\text{success}}$</th>
<th>$F_1$</th>
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</tr>
<tr>
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<td>0.1387</td>
<td>0.8562</td>
<td>0.8564</td>
</tr>
<tr>
<td>0.80</td>
<td>0.0671</td>
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<td>0.8589</td>
<td>0.0688</td>
<td>0.8555</td>
<td>0.8558</td>
</tr>
<tr>
<td>0.70</td>
<td>0.0302</td>
<td>0.8583</td>
<td>0.8584</td>
<td>0.0311</td>
<td>0.8548</td>
<td>0.8551</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.8543</td>
<td>0.0002</td>
<td>0.8510</td>
<td>0.8513</td>
</tr>
</tbody>
</table>

\[a\] Single-photon counters.

\[b\] ON/OFF detectors.
Heisenberg group) distribution we obtain a symmetric (invariant to the action of discrete Weyl expansion. Moreover, for a normalized \( \zeta \) of the order of 0.001, it is seen that probability of coincidence count increases with \( \zeta \) for the ON/OFF detectors and drops in the case of the photon-number discriminating detectors (single-photon counters). The ON/OFF detectors register false successful events as true coincidences. The single-photon counters are better in the case of low-dark-count rates (most of practical situations), but for \( \zeta > 0.0001 \) we observe that the performance of the machine is better when the ON/OFF detectors are applied.

## IV. Applicability to Arbitrary Mirror-Symmetric Phase-Covariant Cloning

For simplicity, so far we analyzed the setup for the MPCC \( \Lambda \) only. However, the general cloning transformation given in Eq. (4) is optimal for cloning of arbitrary mirror-symmetric distributions on Bloch sphere.

Recently we showed \( \Lambda \) that the optimal symmetric 1 \( \rightarrow \) 2 of an arbitrary axisymmetric distribution of qubits \( g(\theta) \) (distribution of expectation values \( \langle \hat{\sigma}_z \rangle = \cos \theta \) for a set of qubits). Any \( g(\theta) \) can be expanded in the basis of Legendre polynomials \( P_n(\cos \theta) \) as

\[
g(\theta) = \frac{1}{4\pi} \sum_{n=0}^{\infty} (2n+1) a_n P_n(\cos \theta),
\]

\[
a_n = \int_{0}^{2\pi} \int_{-1}^{1} g(\theta) P_n(\cos \theta) \, d\cos \theta \, d\phi.
\]

In Ref. \( \Lambda \) we showed that the optimal cloning transformation depends only on first three terms of this expansion. Moreover, for a normalized \( (a_0 = 1) \) mirror symmetric (invariant to the action of discrete Weyl-Heisenberg group) distribution we obtain \( a_1 = 0 \). Such case includes as special cases the PCC for \( \theta = \pi/2 \), the MPCC \( \Lambda \), and the UC of Bužek and Hillery \( \Lambda \).

By comparing the results from Ref. \( \Lambda \) with those from \( \Lambda \) we find that \( \Lambda \) in general depends on a single parameter as follows:

\[
\Lambda = \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8(1-a_2)^2}{3(4a_2^2 - 4a_2)}}}.
\]

Thus, by using appropriate functional form of \( \Lambda \) we can implement various optimal cloning machines such as the PCC, MPCC and UC with the same experimental setup. Note that for the UC \( a_2 = 0 \) and for the PCC \( a_2 = -1/2 \), i.e., \( \Lambda = \sqrt{2/3} \) and \( \Lambda = 1/\sqrt{2} \), respectively.

## V. Conclusions

W investigated experimentally-feasible optimal mirror phase-covariant cloning, i.e., optimal cloning of arbitrary sets of qubits of known modulus of expectation value of Pauli’s \( \hat{\sigma}_z \) operator. Our definition of the mirror phase-covariant cloning includes in special cases the universal cloning (corresponding to cloning of a uniform distribution of qubits on Bloch sphere) and the phase-covariant cloning (cloning of equatorial qubits). By identifying the class of mirror-symmetric phase-covariant distributions of qubits as subclass of axisymmetric distributions, for which optimal cloning transformations were obtained in Ref. \( \Lambda \), we showed that the cloning transformation we implemented is optimal.

We proposed an optical realization of optimal quantum mirror phase-covariant 1 \( \rightarrow \) 2 cloning of a qubit, for which the mean probability of successful cloning varies from 1/6 to 1/3 depending on the prior information on the set of qubits to be cloned. The qubits are represented by polarization states of photons generated by spontaneous parametric down-conversion of the first type. The scheme is based on the interference of two photons on a beam splitter with different splitting ratios for vertical and horizontal polarization components and additional application of feedforward by means of Pockels cells.

The phase-covariant cloning machine implemented by Černoch et al. \( \Lambda \) is less general as it does not include feedforward that allows to use the setup in cases other than implementation of the PCC. Moreover, we showed that the feedforward also allows using a wider range of splitting ratios of the polarization-dependent beam splitter than in the schemes without feedforward.

The experimental feasibility of the proposed setup was studied including various kinds of losses: (i) finite efficiency of generating a pair of entangled photons in the type-I spontaneous parametric down conversion, (ii) the influence of choosing various splitting ratios of an unbalanced beam splitter, (iii) the use of conventional (ON/OFF detectors) and single-photon discriminating detectors, (iv) finite detector efficiency, and (v) dark counts.

For simplicity, we studied the experimental feasibility of our setup implementing only the standard MPCC, i.e., which corresponds to cloning distribution \( g(\theta) \) described...
by two Dirac’s $\delta$-functions. Such analysis can be easily extended to show the feasibility of our setup for the optimal cloning of arbitrary distributions $g(\theta)$ that are mirror-symmetric on Bloch sphere.

We showed that the cloning machine is robust, its fidelity is expected to be very close to the theoretical limit and is expected to stay unaffected by the imperfections of the particular elements other than the PDBS. Robustness of the proposed experimental setup was confirmed by investigation of influence of the mentioned imperfections on the average fidelity of clones and success probability of the MPCC.

Both the success rate and average cloning fidelity were estimated by means of simplified qubit tomography setup [12]. In our case, similarly as Černoch et al. [10], we do not need to use the complete tomography to determine the fidelity of the clones (since we a priori know the input state to some extent).

The probability of successful cloning is high if compared the logical circuit described in Ref. [23] with all the CNOT operations replaced with the best optical gates. The setup proposed in this paper is not only suitable for the MPCC, but also for any optimal cloning of an arbitrary set of qubits of the axial and mirror $xy$ symmetry [24] including the universal, phase-covariant and mirror-phase covariant cloning.

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