

Photon-counting statistics of squeezed states in collective resonance fluorescence

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Abstract. The photon-counting statistics of squeezed states in collective time-dependent resonance fluorescence is calculated for the homodyne detection technique. The secular approximation is applied to obtain analytical expressions for the photon-counting distribution produced by homodyning the fluorescent field and coherent radiation. Squeezing is shown to appear only in the transient regime of resonance fluorescence. The time location of squeezing is independent of the number of atoms, N , but squeezing increases with increasing N . Moreover, squeezing occurs only for a judicious choice of the phase of the local oscillator and occurs over long counting times T . The time interval T is strongly dependent on the initial atomic population. If initially all atoms are in their excited states, squeezing appears with a delay twice as long as that for initially non-excited atoms.

1. Introduction

Squeezed states of light have attracted considerable attention in the last few years. They reduce the fluctuations in one of the two quadrature components of the electric field below the vacuum limit, or below that achievable in a completely coherent field, while the other quadrature exhibits enhanced fluctuations. This presents obvious advantages for optical communication and gravitational wave detection, for example, where signals even below the vacuum limit are expected. Various theoretical methods for the generation of squeezed states have recently been proposed[†] and the first successful experiments have been reported [1–3]. The usual way to identify squeezed states is to homodyne measured light against the beam from a local oscillator and to measure the fluctuations of the superposed field with one or more photodetectors. Squeezing is manifest in the appearance of sub-Poissonian photon statistics in the homodyne experiment. The theory of homodyne detection of squeezed states as applied to resonance fluorescence has been developed by Mandel [4] and Loudon [5]. In particular, they have shown that in the one-port homodyne scheme the quantity $S(T) = \langle n^{(2)} \rangle - \langle n \rangle^2$, which is related to the second factorial moment $\langle n^{(2)} \rangle = \langle n^2 \rangle - \langle n \rangle^2$ of the photon-counting distri-

bution for the superposition of the coherent radiation (local oscillator) and the radiation scattered by a two-level atom (resonance fluorescence), can serve to monitor squeezing in the scattered radiation. However, they dealt with the case of counting times T much shorter than the correlation time of resonance fluorescence. Lakshmi and Agarwal [6] considered the case of large T and showed that, for a particular choice of the parameters, squeezing can appear for long counting time intervals T . The single-atom resonance fluorescence situation, while relatively straightforward in principle, necessarily involves an extremely weak scattered field and generates relatively low levels of squeezing.

One way to obtain greater squeezing in resonance fluorescence is to use a larger number of atoms interacting with the external coherent field. Vogel and Welsch [7] have shown that in resonance fluorescence from N independent atoms, distributed regularly in a linear chain, squeezing is N times stronger than for a single atom. Heidmann and Reynaud [8] considered resonance fluorescence radiated in the forward direction by an ensemble of N independent atoms and showed that squeezing is not modified by multi-atom effects. Lakshmi and Agarwal [9] and Wahiddin *et al* [10] have shown that cooperative effects can eliminate or magnify (depending on the parameters used) the squeezing in steady state resonance fluorescence compared with that obtained for the single atom. However, squeezing appears only if the Rabi frequency of the exciting field is sufficiently low. For strong exciting fields, as has been

[†] The literature on the subject is quite extensive. For a survey of recent publications and some new results, see the special issues of *J. Mod. Opt.* **34** (1987), issues 6/7 and *J. Opt. Soc. Am. B* **4** (1987), issue 10.

shown in [11, 12], squeezing can occur in the transient regime of resonance fluorescence and shifts towards shorter times as the intensity of the exciting field increases.

Here we consider the possibility of detecting squeezed states in the transient regime of cooperative N -atom resonance fluorescence by the photon-counting technique. We show that the properties of the photon-counting distribution produced by a mixture of coherent radiation (local oscillator) and the radiation scattered by a strongly pumped N -atom system depend on the number of atoms, the initial atomic populations, the phase of the local oscillator and the counting intervals T .

2. Photon-counting distribution

To identify squeezed states in cooperative resonance fluorescence, we apply the theory of homodyne detection [13], in which the light is homodyned against the beam from a local oscillator and the fluctuations of the superposed optical field are measured with one or more photodetectors. The quantity of interest is $S(T) = \langle n^{(2)} \rangle - \langle n \rangle^2$, which is related to the second factorial moment $\langle n^{(2)} \rangle = \langle n^2 \rangle - \langle n \rangle^2$ of the photon-counting distribution of the superposed optical field. In homodyne detection squeezing is manifest in the appearance of sub-Poissonian photon statistics ($S(T) < 0$).

The second factorial moment $\langle n^{(2)} \rangle$ of the photoelectron distribution is related to the intensity fluctuations of the light incident on the detector [14]:

$$\langle n^{(2)} \rangle = 2\beta^2 \int_t^{t+T} dt_1 \int_t^{t_1} dt_2 \times \langle E^{(-)}(t_1)E^{(-)}(t_2)E^{(+)}(t_2)E^{(+)}(t_1) \rangle \quad (1)$$

where T is the counting interval and β is the quantum efficiency parameter of the detector. In homodyne experiments the field amplitude $E^{(+)}(t)$ consists of two field amplitudes:

$$E^{(+)}(t) = E_s^{(+)}(t) + E_c^{(+)}(t) \quad (2)$$

where $E_s^{(+)}(t)$ is the amplitude of the field scattered by N two-level atoms (resonance fluorescence) and $E_c^{(+)}(t)$ is that of the local oscillator, assumed to be in a coherent state. In the far-field limit $E_s^{(+)}(t)$ can be expressed in terms of the atomic operators $s^\pm(t)$ as follows [15]:

$$E_s^{(+)}(t) = E_0^{(+)}(t) + \frac{1}{2}\psi(r)s^-(t - r/c) \quad (3)$$

where $\psi^2(r) = (3\hbar\omega_0\gamma/2cr^2) \sin^2 \varphi$ (with φ the angle between the observation direction \mathbf{r} and the atomic transition dipole moment $\boldsymbol{\mu}$) and $s^-(t)$ is the collective atomic operator which, together with the operator $s^+(t)$ and the atomic energy operator $s^z(t)$, obeys the commutation relations $2s^z(t) = [s^+(t), s^-(t)]$, $[s^z(t), s^\pm(t)] = \pm s^\pm(t)$.

Assuming that the coherent field E_c is made sufficiently intense and much stronger than the fluorescent

field, we find from equations (1)–(3), to second order in $|E_c|$,

$$S(T) = \beta^2 |E_c|^2 \psi^2(r) \operatorname{Re} \int_t^{t+T} dt_1 \int_t^{t_1} dt_2 \times [\langle s^+(t_1)s^-(t_2) \rangle - \langle s^+(t_1) \rangle \langle s^-(t_2) \rangle] + e^{2i\theta} (\langle s^+(t_1)s^+(t_2) \rangle - \langle s^+(t_1) \rangle \langle s^+(t_2) \rangle) \quad (4)$$

where θ is the phase of the local oscillator. The time correlations of the atomic operators $s^\pm(t)$ of equation (4) are obtained from the master equation [15]

$$\partial\rho/\partial t = \frac{1}{2}i\Omega[s^+ + s^-, \rho] - \gamma(s^+s^- \rho + \rho s^+s^- - 2s^- \rho s^+) \quad (5)$$

where Ω is the Rabi frequency and 2γ is the Einstein A coefficient. For a strongly driven collective N -atom system, $\Omega \gg N\gamma$, we can make the secular approximation [16] on the master equation (5) and obtain exact solutions for the atomic correlation functions [17]. Inserting these solutions into (4) and integrating, we obtain

$$S(T) = 2\beta^2 |E_c|^2 \psi^2(r) \left[\left(\frac{N(N+2)}{6} \times (2\gamma T - 2 + 2e^{-\gamma T}) + \frac{N(N-1)}{18} \times e^{-3\gamma T} (3e^{-\gamma T} - e^{-3\gamma T} - 2) \right) \cos^2 \theta + \frac{N\gamma^2 \operatorname{sgn}\langle s^z(0) \rangle}{\Omega} T e^{-3\gamma(t+T)/2} \sin \Omega(t+T) \sin^2 \theta \right] \quad (6)$$

where $\operatorname{sgn}\langle s^z(0) \rangle = -1$ if initially the atoms are in their ground state and $\operatorname{sgn}\langle s^z(0) \rangle = 1$ if they start in their excited state.

Our analytical solution (6) describes the time dependence of the photon-counting distribution of the cooperative N -atom fluorescent field superposed with the coherent radiation. Equation (6) holds for arbitrary initial conditions, numbers of atoms and times t and T . For steady state resonance fluorescence ($t \rightarrow \infty$) we have by (6)

$$S(T) = 2\beta^2 |E_c|^2 \psi^2(r) \left(\frac{N(N+2)}{6} \times [2\gamma T - 2(1 - e^{-\gamma T})] \cos^2 \theta \right). \quad (7)$$

In this case $S(T)$ is always positive, irrespective of N and T . Squeezing, i.e. negative values of $S(T)$, appears only in the transient regime of resonance fluorescence. Our solution (6) is illustrated graphically in figures 1 and 2, with $S(T)$ plotted against the normalised counting time $2\gamma T$ for different parameters Ω , N and θ . Figure 1 shows the influence of the phase θ of the local oscillator and the number of atoms, N , on $S(T)$. Obviously, negative values of $S(T)$ appear only for a judicious choice of θ . Squeezing is present only for θ near $\pi/2$ and is greatest for $\theta = \pi/2$, meaning that only the last term in (6) (i.e.

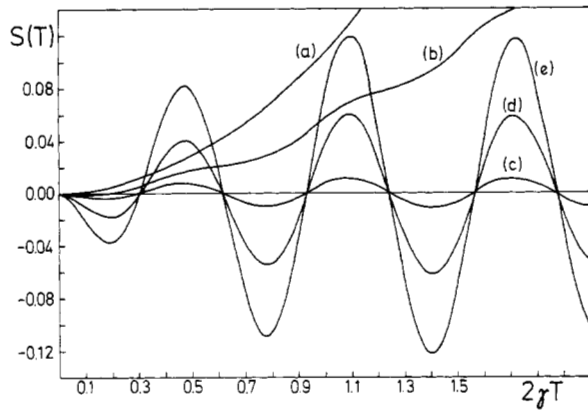


Figure 1. $S(T)$ in units of $2\beta^2|E_c|^2\psi^2(r)$ versus the counting time $2\gamma T$ for Rabi frequency $\Omega = 20\gamma$, $\text{sgn}\langle s^z(0) \rangle = -1$ and $t = 0.02\gamma$: curve (a), $\theta = 0$, $N = 1$; curve (b), $\theta = \pi/4$, $N = 1$; curve (c), $\theta = \pi/2$, $N = 1$; curve (d), $\theta = \pi/2$, $N = 5$; curve (e) $\theta = \pi/2$, $N = 10$.

that multiplied by $\sin^2 \theta$ gives negative values of $S(T)$. Initially $S(T)$ is negative, then dips to a minimum (i.e. reaches a local maximum of squeezing), rises to positive values and then becomes negative again (squeezing). Its behaviour is pronouncedly oscillatory, reflecting the Rabi oscillations. Moreover, as the number of atoms, N , increases, the amount of squeezing increases proportionally to N . It is interesting to note from figure 1 that the positions of the minima of $S(T)$ are independent of the number of atoms. The positions of the minima of $S(T)$ depend on the time t and the initial atomic populations. This is shown in figure 2, where $S(T)$ is plotted against T for $N = 100$, $\Omega = 80\gamma$, different times t and different initial atomic populations. If initially all atoms are in their excited state, then $\text{sgn}\langle s^z(0) \rangle = 1$ and, for the same time t , squeezing appears with a delay twice as long as that for initially non-excited atoms. Moreover, a comparison of the full and broken curves in figure 2 shows that squeezing can appear at the same T for different initial atomic populations provided that the times t are different. This suggests that it is possible to choose t and the initial population $\langle s^z(0) \rangle$ so that squeezing ($S(T) < 0$) can appear for long counting times

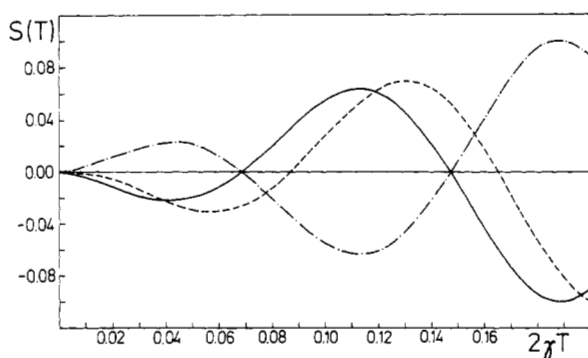


Figure 2. $S(T)$ in units of $2\beta^2|E_c|^2\psi^2(r)$ versus $2\gamma T$ for $\Omega = 80\gamma$, $N = 100$, $\theta = \pi/2$ and different initial atomic populations and times t : —, $\text{sgn}\langle s^z(0) \rangle = -1$, $t = 0.02\gamma$; ---, $\text{sgn}\langle s^z(0) \rangle = 1$, $t = 0.02\gamma$; -.-.-, $\text{sgn}\langle s^z(0) \rangle = 1$, $t = 0.14\gamma$.

T . Thus we show that a judicious choice of the parameters will reveal the squeezing properties of the fluorescent field.

3. Conclusions

We have studied the photon-counting statistics of squeezed states in collective time-dependent resonance fluorescence from N two-level atoms coherently driven by a strong resonant laser field. The secular approximation proposed by Agarwal *et al* [16] has been used to obtain analytical solutions for the first, $\langle n \rangle$, and second, $\langle n^{(2)} \rangle$, factorial moments.

Our results show that, for strong driving fields, squeezing, i.e. negative values of $S(T) = \langle n^{(2)} \rangle - \langle n \rangle^2$, appears only in the transient regime of resonance fluorescence. The location of squeezing in time is independent of the number of atoms but is dependent on the initial atomic populations and the counting time T . If initially all the atoms are in their excited state, squeezing appears with a delay twice as long as that for initially non-excited atoms. We also show that squeezing occurs only for a judicious choice of the phase of the local oscillator. Moreover, it is also feasible to adjust the parameters so that negative values of $S(T)$ will persist and recur over long counting intervals T .

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