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Role of the higher optical Kerr nonlinearities in self-squeezing of light

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Abstract. The part played by higher-order optical nonlinearities in self-squeezing of intense light propagating in a nonlinear Kerr medium is discussed. An analytical formula for the normally ordered variances of the field is derived under the assumption that the nonlinearity parameters of the medium are small and the number of photons is very great. The saddle-point approach is used to evaluate the sums over large numbers of photons. The analytical formula is illustrated graphically for several sets of nonlinearity coefficients. It is shown that, for large numbers of photons, the higher-order contributions are substantial and can modify the magnitude of squeezing as well as the range of the parameters over which squeezing can be observed.

1. Introduction

The problem of squeezing of quantum fluctuations in optical fields is a subject of intensive research in recent years and the number of publications on the subject is growing at a high rate (see, for example, two special issues [1] of optical journals). Several experiments have successfully verified the existence of squeezed states of light [2–8]. Some time ago, we have shown [9] that intense strong light propagating through a nonlinear Kerr medium can squeeze itself, and we referred to this effect as self-squeezing. We have proved the possibility of as much as 98% of squeezing in the above process. The one-mode version of the self-squeezing effect has been dealt with by Tanaś [10] in terms of the anharmonic oscillator model with interaction Hamiltonian $\sim\kappa a^{+2}a^2$. This very simple and strictly solvable model is in fact highly instructive and many aspects of squeezing obtainable from this model have been discussed recently [11–17]. The squeezed states to which it leads are not minimum-uncertainty and differ essentially from the two-photon coherent states [18] that are most often used as models for squeezed states. Kitagawa and Yamamoto [19], who considered the quasiprobability distribution Q for such states, refer to the squeezing obtained in such a model as 'crescent' squeezing (in contrast to elliptic squeezing) because of the crescent shape of the quasiprobability distribution. Our 'self-squeezing' and their 'crescent squeezing' are but different terms for what is virtually the same mechanism of squeezing. Some aspects of third- [20] and second- [21] harmonic generation by self-squeezed light have also been discussed. The feasibility of controlling the self-squeezing process by means of an external magnetic field has been predicted [22]. Recently, Gerry [23] has generalised the model to the k-photon anharmonic oscillator, with the interaction Hamiltonian $\sim\kappa a^{+k}a^k$, showing that squeezing can also be obtained for higher values of k (k=3,4). In this case, however, closed form expressions for the field variances cannot be found and the results are expressed in the form of infinite sums difficult to evaluate numerically for the most interesting case (from the experimental point of view) of a small nonlinearity parameter and a large number of photons.

The k-photon anharmonic oscillator has been earlier considered by Yurke and Stoler [24] in their discussion of the generation of macroscopically distinguishable quantum states.

In this paper we derive approximate analytical formulae describing the variances of the field for the case of small nonlinearities and large number of photons using the saddle-point technique to evaluate the integrals that are used to replace the sums. Moreover, we consider the higher-order nonlinearities not as separate terms but add their contributions to the lowest optical Kerr nonlinearity contribution. This approach is more closely related to the real physical situation and allows us to discuss the role of higher nonlinearities in self-squeezing.

2. The model

We consider a system the dynamics of which is governed by the following Hamiltonian

$$H = \hbar\omega a^+a + \frac{\hbar}{2}\kappa_1 a^{+2}a^2 + \frac{\hbar}{3}\kappa_2 a^{+3}a^3 + \frac{\hbar}{4}\kappa_3 a^{+4}a^4 \tag{1}$$

where a and a^+ are the annihilation and creation operators and κ_1, κ_2, κ_3 are the coupling constants describing the successive nonlinearities of the system. Without the higher-order nonlinearities κ_2 and κ_3 we have the anharmonic oscillator model considered by Tanaś [10], whereas taking separately one of the higher-order terms together with the free-field term we have the k-photon (k=3,4) anharmonic oscillator considered by Gerry [23]. In practice, the model described by (1) can be realised when intense circularly polarised laser light propagates through a nonlinear, isotropic Kerr medium. The coupling constants κ_1, κ_2 and κ_3 are then related to the nonlinear susceptibilities of the medium $\chi^{(3)}$, $\chi^{(5)}$ and $\chi^{(7)}$. The lowest-order nonlinearity κ_1 is that related to the optical Kerr effect [25, 26] and it always differs from zero, although its numerical value is dependent on the medium and is usually very small. The values of the successive nonlinearity parameters decrease by many orders of magnitude with growing order of the nonlinearity. Thus, the physical effects related to these nonlinearities can manifest themselves only if the light intensity is sufficiently high. The higher-order nonlinearities, however, are known to play a crucial role, e.g., in the laser beam autocollimation process [27].

Here, we are interested in the role played by the higher-order terms of (1) in the self-squeezing process. According to the Hamiltonian (1), the equation of motion for the annihilation operator is given by

$$\frac{da}{dt} = -\frac{i}{\hbar}[a, H] = -i(\omega + \kappa_1 a^+a + \kappa_2 a^{+2}a^2 + \kappa_3 a^{+3}a^3)a \tag{2}$$

and, since the terms $a^{+k}\,a^k$ are all constants of motion, the solution is

$$a(t) = \exp[-\mathrm{i}t(\omega + \kappa_1 a^+(0)a(0) + \kappa_2 a^{+2}(0)a^2(0) + \kappa_3 a^{+3}(0)a^3(0))]a(0)\,. \quad (3)$$

In the case of light propagating through a nonlinear medium with refractive index n (instead of a field in a cavity), we can replace t by $-(n/c)z$ (on neglecting the dispersion of the medium) and, on discarding the free evolution, we arrive at the equation

$$a(z) = \exp[\mathrm{i}(\tau_1 a^+(0)a(0) + \tau_2 a^{+2}(0)a^2(0) + \tau_3 a^{+3}(0)a^3(0))]a(0) \quad (4)$$

where we have used the notation

$$\tau_i = -\frac{n}{c} z\kappa_i\,. \quad (5)$$

Equation (4) is the exact operator equation describing the evolution of the field propagating through the nonlinear medium. With the solution (4) and knowing the initial state of the field, one can calculate all characteristics of the field after its traversal of the path z in the medium.

Since we are interested in squeezing, we define the hermitian operator

$$Q_\varphi = a\mathrm{e}^{-\mathrm{i}\varphi} + a^+\mathrm{e}^{\mathrm{i}\varphi} \quad (6)$$

which for $\varphi = 0$ corresponds to the in-phase quadrature component of the field and for $\varphi = \pi/2$ to the out-of-phase component.

The variance of such an operator is given by

$$\begin{aligned}\mathrm{Var}\,[Q_\varphi] &= \langle Q_\varphi^2\rangle - \langle Q_\varphi\rangle^2 \\ &= 2\mathrm{Re}\,\{\langle a^2\rangle \mathrm{e}^{-2\mathrm{i}\varphi} - \langle a\rangle^2\mathrm{e}^{-2\mathrm{i}\varphi}\} + 2\{\langle a^+a\rangle - \langle a^+\rangle\langle a\rangle\} + 1\,. \quad (7)\end{aligned}$$

For the vacuum state as well as coherent states this variance is equal to unity. If it becomes smaller than unity the state of the field for which this occurs is referred to as squeezed [28]. Perfect squeezing is obtained if $\mathrm{Var}\,[Q_\varphi] = 0$. Later on, we will use the normally ordered variance

$$\begin{aligned}V_\varphi(z) &= \langle :Q_\varphi^2(z): \rangle - \langle Q_\varphi(z)\rangle^2 \\ &= 2\,\mathrm{Re}\,\{\langle a^2(z)\rangle \mathrm{e}^{-2\mathrm{i}\varphi} - \langle a(z)\rangle^2 \mathrm{e}^{-2\mathrm{i}\varphi}\} + 2\{\langle a^+(z)a(z)\rangle - \langle a^+(z)\rangle\langle a(z)\rangle\}\,. \quad (8)\end{aligned}$$

Negative values of this variance mean squeezing and its value equal to -1 means perfect squeezing.

From equation (4) we immediately see that $\langle a^+(z)a(z)\rangle = \langle a^+(0)a(0)\rangle = N$ is the mean number of photons in the beam. Their number is constant. What we thus need in order to calculate the variance (8) are the quantities $\langle a(z)\rangle$ and $\langle a^2(z)\rangle$. Assuming that the initial state of the field is a coherent state

$$|\alpha\rangle = \mathrm{e}^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle \quad (9)$$

with the mean number of photons $N = |\alpha|^2$, we obtain from (4)

$$\langle\alpha|a(z)|\alpha\rangle = \alpha e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} e^{i\vartheta_n} \tag{10}$$

$$\langle\alpha|a^2(z)|\alpha\rangle = \alpha^2 e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} e^{i(\vartheta_n+\vartheta_{n+1})} \tag{11}$$

where

$$\vartheta_n = \tau_1 n + \tau_2 n(n-1) + \tau_3 n(n-1)(n-2) . \tag{12}$$

The formulae (10)–(12) are generalisations of those obtained by Gerry [23] for k-photon anharmonic oscillators. The generalisation is somewhat trivial because we study the joint effect of all the nonlinearities rather than the effect of each term separately. However, this approach is closer to the real physical situation when strong light propagates in a Kerr nonlinear medium. The lowest-order Kerr nonlinearity is always different from zero and certainly plays the most important role in the process. The influence of the higher-order terms can be essential, however, if the field is very strong. We shall discuss this situation here. Unfortunately, for very great numbers of photons, the numerical calculation of the sums in (10) and (11) is by no means feasible. For numerical reasons, Gerry [23] assumed for τ_2 and τ_3 values of 10^{-2}. Albeit estimates based on real values of the nonlinear susceptibilities [25, 26] give for τ_1 values $\sim 10^{-6}$ and the corresponding values for τ_2 and τ_3 at $\sim 10^{-12}$ and $\sim 10^{-18}$. This means that the number of photons $N = |\alpha|^2$ must be of the order of $\sim 10^6$ if considerable effects are to be expected. This, however, makes direct numerical evaluation of the sums (10) and (11) very difficult. Only in the case $\tau_2 = \tau_3 = 0$ can the sums be performed giving closed analytical formulae for $\langle a(z)\rangle$, $\langle a^2(z)$ and the variance $V_\varphi(z)$, leading to the result obtained by Tanaś [10]. But even in this case numerical calculations are not easy to perform for the parameter values under discussion. To overcome the difficulties some approximations are needed. Since the sums involve the Poisson weight factor, which is peaked at $n = N$, the sums for $N >> 1$ can be replaced by integrals accessible to evaluation using the saddle-point technique. This is the subject of the next section.

3. The saddle-point approach in calculations of self-squeezing

To calculate the sums (10) and (11) we use the technique applied in calculations of the collapses and revivals [29, 30] in the Jaynes–Cummings model. The sums to be calculated have the form

$$S_i = \sum_{n=0}^{\infty} p_n \cos \psi_i(y) , \qquad (i = 1, 2) \tag{13}$$

where

$$p_n = \mathrm{e}^{-N} \frac{N^n}{n!} \tag{14}$$

is the Poisson weight factor, and

$$\psi_1(y) = -\phi + \tau_1 N y^2 + \tau_2 N y^2 (N y^2 - 1) + \tau_3 N y^2 (N y^2 - 1)(N y^2 - 2) \tag{15}$$

$$\psi_2(y) = -2\phi + \tau_1 + 2\tau_1 N y^2 + 2\tau_2 (N y^2)^2 + \tau_3 [2(N y^2)^3 - 3(N y^2)^2 + N y^2] \tag{16}$$

where we have introduced the notation $y^2 = n/N$. To evaluate the sums (13) we rewrite them as the integrals

$$S_i \approx \left(\frac{2N}{\pi}\right)^{1/2} \mathrm{Re} \int_0^\infty \exp[N f_i(y)]\, \mathrm{d}y \tag{17}$$

where

$$f_i(y) = y^2(1 - 2 \ln y) - 1 + \mathrm{i}\, \frac{\psi_i(y)}{N} \,. \tag{18}$$

In obtaining (18) we have used Stirling's formula for $n!$.

The saddle points y_i of $f_i(y)$ are given by

$$\left. \frac{\partial f_i(y)}{\partial y} \right|_{y=y_i} = 0 \tag{19}$$

which leads us to the following set of equations:

$$-4 \ln y_1 + 2\mathrm{i}(\tau_1 - \tau_2) + 4\mathrm{i}\tau_2 N y_1^2 + 6\mathrm{i}\tau_3 N^2 y_1^4 - 12\mathrm{i}\tau_3 N y_1^2 + 4\mathrm{i}\tau_3 = 0$$

$$-4 \ln y_2 + 4\mathrm{i}\tau_1 + 8\mathrm{i}\tau_2 N y_2^2 + 12\mathrm{i}\tau_3 N^2 y_2^4 - 12\mathrm{i}\tau_3 N y_2^2 + 2\mathrm{i}\tau_3 = 0 \,. \tag{20}$$

On introducing $y_i = \rho_i \mathrm{e}^{\mathrm{i}\varphi_i}$ $(i=1,2)$ we have

$$\ln \rho_1 + \tau_2 N \rho_1^2 \sin 2\varphi_1 + \frac{3}{2} \tau_3 N^2 \rho_1^4 \sin 4\varphi_1 - 3\tau_3 N \rho_2^2 \sin 2\psi_2 = 0 \,,$$

$$-\varphi_1 + \frac{1}{2}(\tau_1 - \tau_2) + \tau_3 + \tau_2 N \rho_1^2 \cos 2\varphi_1 + \frac{3}{2} \tau_3 N^2 \rho_1^4 \cos 4\varphi_1$$

$$- 3\tau_3 N \rho_1^2 \cos 2\varphi_1 = 0 \tag{21}$$

$$\ln \rho_2 + 2\tau_2 N\rho_2^2 \sin 2\varphi_2 + 3\tau_3 N^2\rho_2^4 \sin 4\varphi_2 - 3\tau_3 N\rho_2^2 \sin 2\varphi_2 = 0$$

$$-\varphi_2 + \tau_1 + \frac{1}{2}\tau_3 + 2\tau_2 N\rho_2^2 \cos 2\varphi_2 + 3\tau_3 N^2\rho_2^2 \cos 4\psi_2 - 3\tau_3 N\rho_2^2 \cos 2\psi_2 = 0 \,. \quad (22)$$

In the case $\tau_2 = \tau_3 = 0$ the solutions of (21) and (22) are $\rho_1 = \rho_2 = 1$ and $\psi_2 = 2\psi_1 = \tau_1$. They are solutions periodic in τ_1, which suggest the possibility of revivals with growing τ_1. In practice, as already mentioned, τ_1 is very small and will never approach the successive saddle points except for the first one. The smallness of the nonlinearity parameters τ_1 allows us to find approximate solutions for (21) and (22). Assuming $\tau_i << 1$, $N >> 1$, we get in a first approximation

$$\rho_1 = 1 - \tau_2 N \sin \tau_1 - \frac{3}{2}\tau_3 N^2 \sin 2\tau_1$$
$$\varphi_1 = \frac{1}{2}(\tau_1 - \tau_2) + \tau_2 N \cos \tau_1 + \frac{3}{2}\tau_3 N^2 \cos 2\tau_1 \quad (23)$$

$$\rho_2 = 1 - 2\tau_2 N \sin 2\tau_1 - 3\tau_3 N^2 \sin 4\tau_1$$
$$\varphi_2 = \tau_1 + 2\tau_2 N \cos 2\tau_1 + 3\tau_3 N^2 \cos 4\tau_1 \,. \quad (24)$$

In the same approximation the values of the second derivatives at the saddle point are given by

$$f_1^{(2)}(y_1) = -4[1 - 2i\tau_2 N e^{i\tau_1} - 6i\tau_3 N^2 e^{2i\tau_1}]$$
$$f_2^{(2)}(y_2) = -4[1 - 4i\tau_2 N e^{2i\tau_1} - 12i\tau_3 N^2 e^{4i\tau_1}] \,. \quad (25)$$

The integrals (17) can thus be evaluated according to the formula

$$S_i = 2|f_i^{(2)}(y_i)|^{-1/2}\exp[Nf_i(y_i) - \frac{i}{2}\arg f_i^{(2)}(y_i)] \quad (26)$$

where $-\arg f_i^{(2)}(y_i)$ is the angle of steepest descent.

The results are

$$S_1 = \exp[Nf_1(y_1) + i\tau_2 N + 3i\tau_3 N^2]$$
$$S_2 = \exp[Nf_2(y_2) + 2i\tau_2 N + 6i\tau_3 N^2] \,. \quad (27)$$

Again, taking advantage of the inequalities $N >> 1$, $\tau_i << 1$ to evaluate $Nf_i(y_1)$, we finally arrive at the following approximate expressions for S_1 and S_2:

$$S_1 = \exp\left\{-\left[\frac{1}{2}\tau_1^2 N + 2\tau_1\tau_2 N^2 + 2\tau_2^2 N^3 + 3\tau_1\tau_3 N^3 + 6\tau_2\tau_3 N^4 + \frac{9}{2}\tau_3^2 N^5\right] + i\left[-\varphi + \tau_1 N + \tau_2 N^2 + \tau_3 N^3\right]\right\} \tag{28}$$

$$S_2 = \exp\{-[2\tau_1^2 N + 8\tau_1\tau_2 N^2 + 8\tau_2^2 N^3 + 12\tau_1\tau_3 N^3 + 24\tau_2\tau_3 N^4 + 18\tau_3^2 N^5] + i[-2\psi + \tau_1 + 2\tau_1 N + 2\tau_2 N + 2\tau_2 N^2 + 3\tau_3 N^2 + 2\tau_3 N^3]\}\ . \tag{29}$$

Equations (28) and (29) are the approximate expressions for the sums (13), allowing us to calculate the expectation values (10) and (11) and subsequently to calculate the normally ordered variance defined by (8). The approximation is the better, the smaller the nonlinearity parameters $\tau_i << 1$ and the greater the number of photons $N >> 1$. The results based on this approximation should thus satisfactorily reproduce the real physical situation in the nonlinear propagation process.

4. Results

To make our discussion of the results more transparent, we introduce a new variable

$$x = \tau_1 N = -\frac{n}{c} z\,\kappa_1 N \tag{30}$$

which, in the case $\tau_1 << 1$ and $N >> 1$, is in fact the variable that properly describes the scale on which the essential changes in the expectation values take place, in contradistinction to the path z in the medium or the number of photons N in the beam taken separately. We have used the above variable in our earlier papers [9, 10]. Moreover, we introduce the coefficients

$$c_1 = \frac{\kappa_1}{|\kappa_1|}\,, \qquad c_2 = \frac{\kappa_2}{|\kappa_1|^2}\,, \qquad c_3 = \frac{\kappa_3}{|\kappa_1|^3}\,, \tag{31}$$

which properly scale the nonlinearity parameters. Their absolute values are numbers of the order of unity.

On assuming

$$\alpha = \sqrt{N}\, e^{i\varphi_0}\,, \tag{32}$$

the normally ordered variance (8) for the strong field on traversing a path z in the Kerr medium can be written as

$$V_\varphi(x) = 2N\{\mathrm{Re}(S_2 e^{2i\varphi_0}) - \mathrm{Re}(S_1^2 e^{2i\varphi_0}) + 1 - S_1^* S_1\} \tag{33}$$

and on insertion of (28) and (29) into (33) we obtain after retaining leading terms only, the following simple formula for the variance (33):

$$V_\varphi(x) = -2[A \sin 2\theta - A^2(1 - \cos 2\theta)] \tag{34}$$

where

$$A = c_1x + 2c_2x^2 + 3c_3x^3 \tag{35}$$

$$\theta = \varphi_0 - \varphi + c_1x + c_2x^2 + c_3x^3 \,. \tag{36}$$

On choosing in (36) $\varphi_0 - \varphi = 0$, we obtain the variance for the in-phase component of the field — and for $\varphi 20 - \varphi = \pi/2$, that for the out-of-phase component. Here, we apply a notion of the in-phase and the out-of-phase components slightly different from that used in (6). Both definitions are identical if the initial phase of the field $\varphi_0 = 0$; however, the words in-phase and out-of-phase are now better understood. The local oscillator field is in-phase or out-of-phase with the input field.

The expressions (35) and (36) for A and θ have a strikingly simple structure, and one is tempted to write down their next higher terms. This is, however, but a conjecture still unproven in the general case. Nevertheless, the expression (34) for the normally ordered variance in the self-squeezing effect appears to us to possess a quite universal structure, and higher-order terms would only modify the form (and values) of A and θ leaving the form of (34) unaffected.

To illustrate our formula (34), we give several graphs in which the variance $V_0(x)$ for the in-phase component of the field is plotted against x for various sets of the coefficients c_1, c_2 and c_3. Depending on the medium, the coefficients c_i can be positive or negative. It is obvious from (34) that the variance exhibits oscillatory behaviour and a sequence of minima will appear in $V_\varphi(x)$ with values of squeezing approaching closer

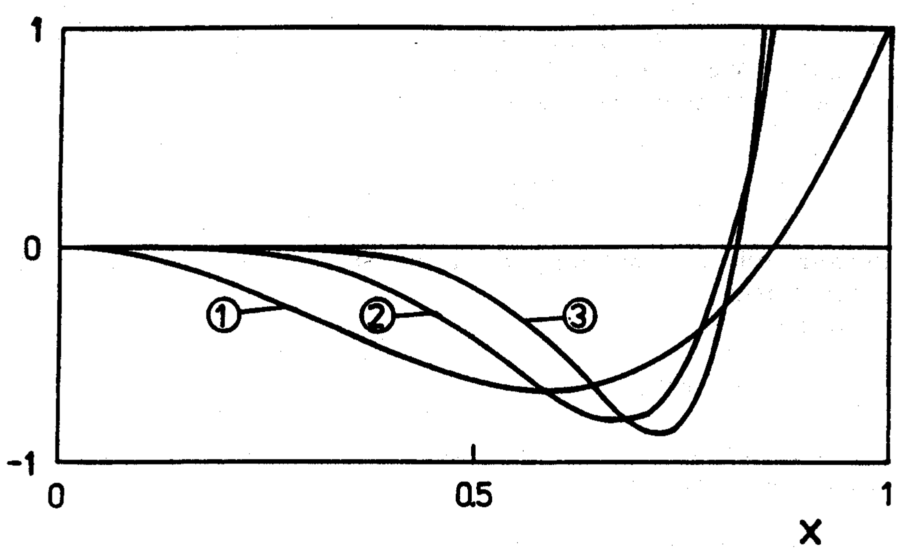

Figure 1. The normally ordered variance for the in-phase component of the field versus x for the parameters: $c_1 = 1$, $c_2 = c_3 = 0$ (curve 1); $c_2 = 1$, $c_1 = c_3 = 0$ (curve 2); $c_3 = 1$, $c_1 = c_2 = 0$ (curve 3).

and closer the perfect squeezing for which the variance $V_\varphi(x) = -1$. However, the subsequent minima become narrower and narrower. In the case of $c_1 = 1, c_2 = c_3 = 0$, the self-squeezing for the second minimum approaches 98 per cent, and corresponding graphs are to be found in [10]. Here, we restrict our discussion to the first minimum only and show how it is affected by the presence of the higher nonlinear terms.

First, to compare our results to those of Gerry [23], we plot in figure 1 the variances for each nonlinear term separately. To make comparisons, however, one has to keep in mind that we use our variable x defined in (30) instead of the number of photons N, so that the minimum for the higher-order terms does not shift towards lower N. After the proper change of the variables our results reproduce the positions of the minima obtained by Gerry [23]. Our results, however, show that the higher the nonlinearity, the greater is the squeezing obtained. This means that the value 10^{-2} used, for numerical reasons, by Gerry is not small enough to obtain quantitative agreement between our formula (34) and his direct numerical calculations.

In figure 2, we show the influence of the c_2 nonlinearity on the self-squeezing effect (assuming $c_3 = 0$). The change in the sign of c_2 has a dramatic effect on the values of squeezing.

In figure 3, we illustrate the effect of the c_3 non-linearity, assuming $c_1 = c_2 = 1$. Especially interesting is the curve 3, which shows that due to counteraction of the individual nonlinear terms it is posible to get considerable squeezing over a wide range of x.

Our graphs are but examples illustrating our analytical formula (34), and many more combinations of the parameters c_1, c_2 and c_3 could be considered. Whatever the combination, our formula gives us the immediate answer to the question of squeezing in such a system.

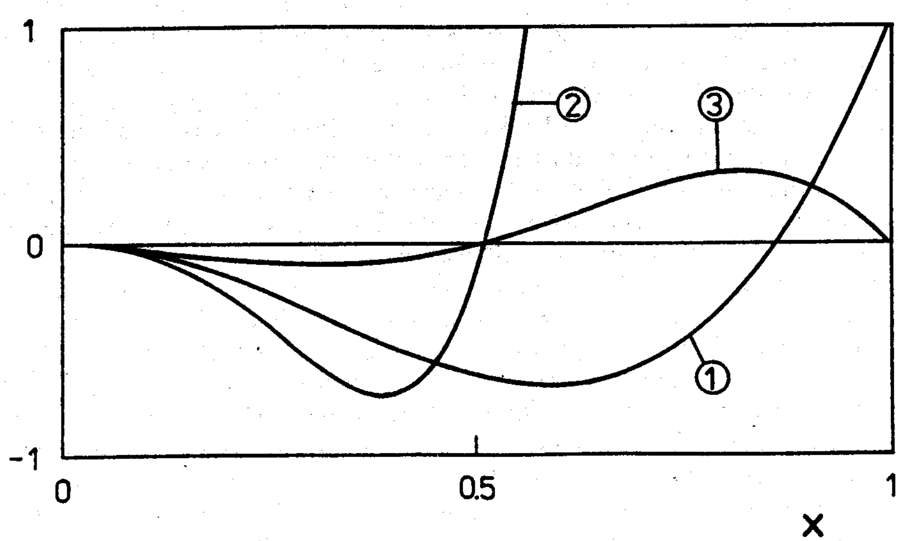

Figure 2. The same as in figure 1, but for the choice of parameters: $c_1 = 1, c_2 = c_3 = 0$ (curve 1); $c_1 = 1, c_2 = 1, c_3 = 0$ (curve 2); $c_1 = 1, c_2 = -1, c_3 = 0$ (curve 3).

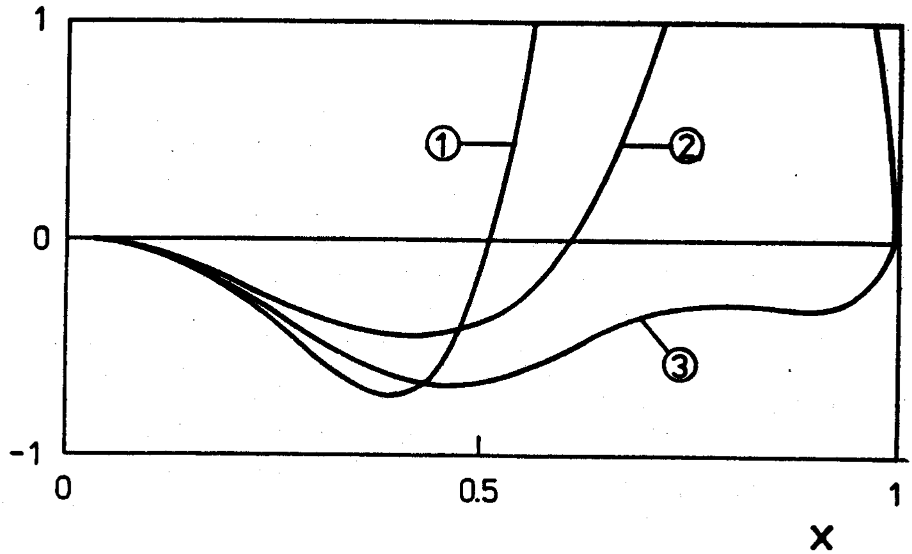

Figure 3. The same as in figure 1, but for the parameters: $c_1 = c_2 = 1$, $c_3 = 0$ (curve 1); $c_1 = c_2 = 1$, $c_3 = -2$ (curve 2 – this curve is broken into two pieces by the frame of the picture); $c_1 = c_2 = 1$, $c_3 = -1$ (curve 3).

5. Conclusions

In this paper we have considered the role played by higher-order nonlinearities in the self-squeezing of light propagating through a Kerr medium. The self-squeezing effect can be remarkable if the light intensity is sufficiently high [9]. So, this is a quantum effect which becomes more and more pronounced for greater numbers of photons. We have calculated the normally ordered variances of strong light propagating in a Kerr medium including higher-order Kerr nonlinearities. We have used the saddle-point technique to get rid of the difficulties besetting numerical evaluations of summations over a large number of photons. Our calculations are based on the assumption of small values for the nonlinear parameters $\tau_i << 1$ and large values for the number of photons $N >> 1$. This assumption allowed us to obtain a very simple analytical formula describing the variances of the field. Our formula appears to be quite universal in form and indicates a way of including arbitrary higher-order Kerr nonlinearities. Some of the consequences of the higher-order terms are illustrated in figures 1–3. Our results are briefly compared to the recent results of Gerry [23]; taking into account the differences in parameter values, agreement is quite satisfactory. There is a gap between the parameter values that can be used in direct numerical calculations so as to make them reliable and the parameter values for which our formula works well. Estimates based on realistic values of the nonlinear susceptibilities give values for which our formula yields precise answers.

Since the interest in Kerr media is growing in the context of so-called quantum nondemolition measurements [31, 32] we believe our results can be useful in this context also.

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References

[1] Special issues: 1987 *J. Modern Optics* **34** No. 6/7; 1987 *J. Opt. Soc. Am.* **B4** No. 10
[2] Slusher R E, Hollberg L W, Yurke B, Mertz J C and Valley J F 1985 *Phys. Rev. Lett.* **55** 2409
[3] Shelby R M, Levenson M D, Perlmutter S H, DeVoe R G and Walls D F 1986 *Phys. Rev. Lett.* **57** 691
[4] Wu L A, Kimble H J, Hall J L and Wu H 1986 *Phys. Rev. Lett.* **57** 2520
[5] Raizen M G, Orozco L A, Xiao M, Boyd T L and Kimble H J 1987 *Phys. Rev. Lett.* **59** 198
[6] Maeda M W, Kumar P and Shapiro J H 1987 *Opt. Lett.* **12** 161
[7] Machida S, Yamamoto Y and Itaya Y 1987 *Phys. Rev. Lett.* **58** 1000
[8] Heidmann A, Horowicz R J, Reynaud S, Giacobino E and Fabre C 1987 *Phys. Rev. Lett.* **59** 2555
[9] Tanaś R and Kielich S 1983 *Opt. Commun.* **45** 351; 1984 *Optica Acta* **31** 81
[10] Tanaś R 1984 in *Coherence and Quantum Optics,* Vol. 5 eds L Mandel and E Wolf (New York: Plenum) p 645
[11] Milburn G J 1986 *Phys. Rev.* A **33** 674
[12] Gerry C C 1987 *Phys. Rev.* A **35** 2146
[13] Gerry C C and Rodrigues S 1987 *Phys. Rev.* A **35** 4440; 1987 *Phys. Rev.* A **35** 5444
[14] Agarwal G S 1987 *Opt. Commun.* **62** 190
[15] Perinova V and Luks A 1988 *J. Mod. Optics* **35** 1513
[16] Gerry C C and Vrscay E R 1988 *Phys. Rev.* A **37** 4265
[17] Tanaś R 1988 *Phys. Rev.* A **38** 1091
[18] Yuen H P 1976 *Phys. Rev.* A **13** 2226
[19] Kitagawa M and Yamamoto Y 1986 *Phys. Rev.* A **34** 3974
[20] Kielich S, Tanaś R and Zawodny R 1987 *J. Opt. Soc. Am.* B **4** 1627
[21] Kielich S, Tanaś R and Zawodny R 1988 *Appl. Phys.* B **45** 249
[22] Kielich S, Tanaś R and Zawodny R 1987 *Phys. Rev.* A **36** 5670
[23] Gerry C C 1987 *Phys. Lett.* **124** 237
[24] Yurke B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13
[25] Kielich S 1981 *Nonlinear Molecular Optics* (Moscow: Nauka)
[26] Kielich S 1967 *Acta Phys. Polonica* **31** 689; 1968 *Acta Phys. Polonica* **34** 1093
[27] Piekara A H, Moore J S and Feld M S 1974 *Phys. Rev.* A **9** 1403
[28] Mandel L 1982 *Phys. Rev. Lett.* **49** 136
[29] Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 *Phys. Rev.* A **23** 236
[30] Yoo H J, Sanchez-Mondragon J J and Eberly J H 1981 *J. Phys.* A **14** 1383
[31] Levenson M J, Shelby R M, Reid M and Walls D F 1986 *Phys. Rev. Lett.* **57** 2473.
[32] Bachor H A, Levenson M D, Walls D F, Perlmutter S H and Shelby R M 1988 *Phys. Rev.* A **38** 180