Magnetic analog of the transverse Pockels effect in transparent antiferromagnetic crystals

R. Zawodny and S. Kielich
Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, 60-780 Poznań, Poland
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A novel effect, the magnetic analog of the transverse Pockels effect (MATPE), is calculated theoretically in a phenomenological approach for transparent crystals. The linearly polarized probe beam on traversal of the crystal under the action of a magnetic field $\mathbf{B}^0$ exhibits elliptical polarization, proportional to the first power of $\mathbf{B}^0$ and also to the axial third-rank tensor ${}^{(1)}_{e}\alpha^{(1)}_{e(ij)k}(\omega)$ of linear magnetic variations in the electric permeability antisymmetric with respect to time inversion and symmetric in its first two indices. Applying group-theoretical methods involving time-inversion arguments, we show that the effect occurs only in magnetic-symmetry classes and is most easily observable in the Voigt setup ($\mathbf{B}^0$ at right angles to the propagation direction of the probe beam) in transparent centrosymmetric antiferromagnetic crystals. The most suitable setup for experimental observations of MATPE is that with the linearly polarized beam propagating along the optical axis of antiferromagnetic crystals with the symmetries $\bar{3}m$, $\underline{6}/\underline{m}$, and $\underline{6}/\underline{m}m\underline{m}$.

I. INTRODUCTION

One of the classical methods for the study of the physical properties of matter in the region of optical frequencies is provided by traditional polarization optics (ellipsometry).[1,2] The method permits their determination from measurements of the angle Ψ of rotation of the polarization plane and the ellipticity Φ of the light wave. It moreover permits the study of changes in the optical properties of matter acted on by a static magnetic field;[3,4] in this situation, depending on the conditions of observation, we deal with the following linear magnetooptical effects: the Zeeman, Faraday, Voigt or Cotton-Mouton, and magnetooptical Kerr effects, magnetic circular dichroism, and linear magnetic dichroism. Their theoretical foundations have been expounded in numerous monographs.[5–12]

Here, we draw attention to the possible existence of magnetooptical birefringence dependent on the first power of the static magnetic field in the Voigt configuration ($\mathbf{B}^0$ perpendicular to the light propagation direction).

In a transparent crystal, a magnetic field $\mathbf{B}^0$ applied at right angles to the propagation direction of linearly polarized light modifies the polarization from linear to elliptic, with ellipticity proportional to the first power of $\mathbf{B}^0$. With regard to its geometry, the ellipticity linearly dependent on $\mathbf{B}^0$, and the electric susceptibility tensor symmetric in its first two indices, the effect is analogous to the electrooptic Pockels effect. We thus propose to refer to it as the magnetic analog of the transverse Pockels effect (MATPE). In the phenomenological approach, it is found that MATPE can take place in magnetic crystals, with the best observability in centrosymmetric, transparent crystals.

The MATPE calculated by us differs essentially from Voigt's effect. The Voigt (or Cotton-Mouton) effect arises when a medium is made birefringent with an applied transverse dc magnetic field. The change in refractive index is proportional to the square of the magnetic field, and hence this phenomenon resembles the electrooptic Kerr effect. More strictly, Voigt's effect is characterized by the fact that the magnetic field $\mathbf{B}^0$, acting perpendicularly to the propagation direction of the linearly polarized light wave and the direction of oscillation of its electric vector $\mathbf{E}(\mathbf{r},t)$, changes the state of polarization of the light wave from linear to elliptic, the ellipticity being proportional to the second power of $\mathbf{B}^0$.

Our primary aim is to enumerate those antiferromagnetic symmetry classes where MATPE will not be perturbed by other optical effects like natural birefringence and optical activity,[13] natural gyroscopic rotation and birefringence,[14,15] or their variations dependent on the first and second power of the magnetic field[16–21] (among which the best known are the Faraday, Voigt, and Cotton-Mouton effects). In order to determine those antiferromagnetic symmetry classes for which the above MATPE-perturbing effects cannot occur, we have to calculate the variation in birefringence coefficient due to the field $\mathbf{B}^0$ with accuracy to terms proportional to the square of $\mathbf{B}^0$.

II. CLASSICAL MAGNETOOPTICS IN A PHENOMENOLOGICAL TREATMENT

The optical properties of a transparent antiferromagnet in the region of optical frequencies under the action of a static magnetic field $\mathbf{B}^0$ in the case of weak excitation[22] by a monochromatic light wave with electric field vector

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\omega,\mathbf{k})\exp\left[-i\omega\left[t-\frac{n}{c}\hat{\mathbf{s}}\cdot\mathbf{r}\right]\right]+\text{c.c.}\ , \qquad (1)$$

and the similarly expressed magnetic vector $\mathbf{H}(\mathbf{r},t)$ oscillating with the circular frequency ω, are described by the vectors of electric induction $\mathbf{D}(\mathbf{r},t)$ and magnetic induction $\mathbf{B}(\mathbf{r},t)$. These, in SI, have the well-known form[10,23]

$$\mathbf{D}(\mathbf{r},t)=\epsilon_0\mathbf{E}(\mathbf{r},t)+\mathbf{P}_e(\mathbf{r},t)\ , \tag{2}$$

$$\mathbf{B}(\mathbf{r},t)=\mu_0[\mathbf{H}(\mathbf{r},t)+\mathbf{P}_m(\mathbf{r},t)]\ . \tag{3}$$

The amplitudes $\mathbf{P}_e(\omega,\mathbf{k})$ and $\mathbf{P}_m(\omega,\mathbf{k})$ of the electric polarization vector $\mathbf{P}_e(\mathbf{r},t)$ and magnetic polarization vector $\mathbf{P}_m(\mathbf{r},t)$ are

$$\mathbf{P}_A(\omega,\mathbf{k})={}_A\overleftrightarrow{\chi}_e(\omega,\mathbf{k},\mathbf{B}^0)\cdot\mathbf{E}(\omega,\mathbf{k}) +\mu_0\,{}_A\overleftrightarrow{\chi}_m(\omega,\mathbf{k},\mathbf{B}^0)\cdot\mathbf{H}(\omega,\mathbf{k})\ , \tag{4}$$

with $A=e$ or m. $\hat{\mathbf{s}}$ is the unit vector of propagation of the light wave, n the light refractive index of the medium in the absence of absorption, c the velocity of light in vacuum, $\mathbf{k}$ the wave vector of length $k=\omega n/c$, ϵ_0 and μ_0 the electric and magnetic permittivity of vacuum, and c.c. stands for complex conjugate.

The polar tensors of second rank ${}_e\overleftrightarrow{\chi}_e(\omega,\mathbf{k},\mathbf{B}^0)$ and ${}_m\overleftrightarrow{\chi}_m(\omega,\mathbf{k},\mathbf{B}^0)$ describe the linear electroelectric and magnetomagnetic susceptibilities of the antiferromagnet under the action of the static magnetic field of induction $\mathbf{B}^0$. The axial tensors ${}_e\overleftrightarrow{\chi}_m(\omega,\mathbf{k},\mathbf{B}^0)$ and ${}_m\overleftrightarrow{\chi}_e(\omega,\mathbf{k},\mathbf{B}^0)$ describe its linear electromagnetic and magnetoelectric susceptibilities.

In the case of crystals, where spatial dispersion is not excessively great,[24] and in a moderately strong magnetic field $\mathbf{B}^0$, the linear tensor susceptibilities can be written in expansion form[25] as follows (we apply the Einstein summation convention):

$$\begin{aligned}{}_e\chi_{e\,ij}(\omega,\mathbf{k},\mathbf{B}^0)={}&{}^{(1)}_{e}\chi^{(1)}_{e\,ij}(\omega)+\frac{i\omega n}{3c}[{}^{(1)}_{e}\chi^{(2)}_{e\,i(jl)}(\omega)-{}^{(2)}_{e}\chi^{(1)}_{e\,(il)j}(\omega)]s_1\\&+\left[{}^{(1)}_{e}\chi^{(1)m}_{e\,iju}(\omega)+\frac{i\omega n}{3c}[{}^{(1)}_{e}\chi^{(2)m}_{e\,i(jl)u}(\omega)-{}^{(2)}_{e}\chi^{(1)m}_{e\,(il)ju}(\omega)]s_1\right]B^0_u\\&+\left[{}^{(1)}_{e}\chi^{(1)mm}_{e\,ij(uw)}(\omega)+\frac{i\omega n}{3c}[{}^{(1)}_{e}\chi^{(2)mm}_{e\,i(jl)(uw)}(\omega)-{}^{(2)}_{e}\chi^{(1)mm}_{e\,(il)j(uw)}(\omega)]s_1\right]B^0_uB^0_w+\cdots\ ,\end{aligned} \tag{5}$$

$${}_e\chi_{m\,ij}(\omega,\mathbf{k},\mathbf{B}^0)={}^{(1)}_{e}\chi^{(1)}_{m\,ij}(\omega)+{}^{(1)}_{e}\chi^{(1)m}_{m\,iju}(\omega)B^0_u+{}^{(1)}_{e}\chi^{(1)mm}_{m\,ij(uw)}(\omega)B^0_uB^0_w+\cdots\ , \tag{6}$$

$${}_m\chi_{e\,ij}(\omega,\mathbf{k},\mathbf{B}^0)={}^{(1)}_{m}\chi^{(1)}_{e\,ij}(\omega)+{}^{(1)}_{m}\chi^{(1)m}_{e\,iju}(\omega)B^0_u+{}^{(1)}_{m}\chi^{(1)mm}_{e\,ij(uw)}(\omega)B^0_uB^0_w+\cdots\ , \tag{7}$$

$${}_m\chi_{m\,ij}(\omega,\mathbf{k},\mathbf{B}^0)={}^{(1)}_{m}\chi^{(1)}_{m\,ij}(\omega)+\cdots\ , \tag{8}$$

with ${}^{(1)}_{m}\chi^{(1)}_{m\,ij}(\omega)=0$ in the optical region.[23] The tensor components indices i, j, k, and l refer to the laboratory coordinates and take the values x,y,z. Above, ${}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)}_{Q}(\omega)$ describes the linear electric-multipole $(A=e)$ or magnetic-multipole $(A=m)$ susceptibility of order a related with electric multipole $(Q=e)$ and magnetic multipole $(Q=m)$ transitions of order q.[13,25] The tensors ${}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)m}_{Q}(\omega)$ and ${}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)mm}_{Q}(\omega)$ represent the changes in ${}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)}_{Q}(\omega)$ induced by the static magnetic field in a linear (first-order stationary perturbation calculus) and quadratic approximation (second-order stationary perturbation calculus).

In a transparent loss-less medium, the relation[23,26]

$$\langle \mathrm{div}\mathbf{S}(\mathbf{r},t)\rangle=0\ , \tag{9}$$

has to be fulfilled, with $\mathbf{S}(\mathbf{r},t)$ denoting Poynting's vector in the medium, and $\langle\cdots\rangle$ standing for the time average.

With regard to the condition (9) and the expansions (5)–(8) it can be shown that in loss-less medium the multipole susceptibilities in the absence as well as in the presence of $\mathbf{B}^0$ fulfil the following relations:[14,15]

$$\begin{aligned}{}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)}_{Q}(\omega)&=[{}^{(q)}_{Q}\overleftrightarrow{\chi}^{(a)}_{A}(\omega)]^*\ ,\\{}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)m}_{Q}(\omega)&=[{}^{(q)}_{Q}\overleftrightarrow{\chi}^{(a)m}_{A}(\omega)]^*\ ,\\{}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)mm}_{Q}(\omega)&=[{}^{(q)}_{Q}\overleftrightarrow{\chi}^{(a)mm}_{A}(\omega)]^*\ ,\end{aligned} \tag{10}$$

and, consequently, are Hermitian. Moreover, in magnetic materials, the multipole susceptibilities are conjugate[7,10,14,15] so that each of them can be expressed in the form

$${}^{(a)}_{A}\overleftrightarrow{\chi}^{(q)}_{Q}(\omega)={}^{(a)}_{A}\overleftrightarrow{\delta}^{(q)}_{Q}(\omega)+i\,{}^{(a)}_{A}\overleftrightarrow{\gamma}^{(q)}_{Q}(\omega)\ . \tag{11}$$

Equations (10) and (11) lead to the following transposition relations:

$$\begin{aligned}{}^{(a)}_{A}\overleftrightarrow{\alpha}^{(q)}_{Q}(\omega)&={}^{(q)}_{Q}\overleftrightarrow{\alpha}^{(a)}_{A}(\omega)\ ,\\{}^{(a)}_{A}\overleftrightarrow{\gamma}^{(q)}_{Q}(\omega)&=-{}^{(q)}_{Q}\overleftrightarrow{\gamma}^{(a)}_{A}(\omega)\ ,\\{}^{(a)}_{A}\overleftrightarrow{\alpha}^{(q)m}_{Q}(\omega)&={}^{(q)}_{Q}\overleftrightarrow{\alpha}^{(a)m}_{A}(\omega)\ ,\\{}^{(a)}_{A}\overleftrightarrow{\gamma}^{(q)m}_{Q}(\omega)&=-{}^{(q)}_{Q}\overleftrightarrow{\gamma}^{(a)m}_{A}(\omega)\ ,\\{}^{(a)}_{A}\overleftrightarrow{\alpha}^{(q)mm}_{Q}(\omega)&={}^{(q)}_{Q}\overleftrightarrow{\alpha}^{(a)mm}_{A}(\omega)\ ,\\{}^{(a)}_{A}\overleftrightarrow{\gamma}^{(q)mm}_{Q}(\omega)&=-{}^{(q)}_{Q}\overleftrightarrow{\gamma}^{(a)mm}_{A}(\omega)\ .\end{aligned} \tag{12}$$

Making use of the relations (10)–(12) we can express the electric polarization vector $\mathbf{P}_e(\mathbf{r},t)$ and magnetic polarization vector $\mathbf{P}_m(\mathbf{r},t)$ in a form involving the electric and magnetic field strength vectors $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ as well as their time derivatives $\dot{\mathbf{E}}(\mathbf{r},t)$ and $\dot{\mathbf{H}}(\mathbf{r},t)$. With the respective expressions, and keeping in mind that $\mathbf{E}(\mathbf{r},t)$, $\dot{\mathbf{H}}(\mathbf{r},t)$ and $\mathbf{P}_e(\mathbf{r},t)$ are invariant with respect to time inversion, whereas $\dot{\mathbf{E}}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$, $\mathbf{P}_m(\mathbf{r},t)$, $\mathbf{B}^0$, and $\mathbf{k}$ undergo a change in sign if $t \to -t$, we are immediately in a position to determine how the linear multipole susceptibilities transform on time inversion. In this way we find that

$$\begin{aligned} &{}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)}_{e}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)m}_{m}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)mm}_{e}(\omega)\ ,\\ &{}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)}_{m}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)m}_{e}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)mm}_{m}(\omega)\ , \end{aligned} \tag{13}$$

are invariant under time inversion (after Birss[7] we shall be referring to them as i tensors), whereas

$$\begin{aligned} &{}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)}_{e}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)m}_{m}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\gamma}^{(q)mm}_{e}(\omega)\ ,\\ &{}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)}_{m}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)m}_{e}(\omega),\quad {}^{(a)}_{e}\overleftrightarrow{\alpha}^{(q)mm}_{m}(\omega)\ , \end{aligned} \tag{14}$$

undergo a change in sign (c tensors) for arbitrary a and q.

It follows from Neumann's principle[7,9] that i tensors can exist both in magnetic and nonmagnetic crystals, whereas c tensors can exist in magnetic crystals only. Hence, the expressions (5)–(8) determining the linear electroelectric and, respectively, electro-magnetic susceptibilities of nonmagnetic crystals will involve only i tensors. For this case these susceptibilities will fulfill the relations

$${}_{e}\chi_{e\,ij}(\omega,\mathbf{k},\mathbf{B}^0)={}_{e}\chi_{e\,ji}(\omega,-\mathbf{k},-\mathbf{B}^0)\ , \tag{15}$$

$${}_{e}\chi_{m\,ij}(\omega,\mathbf{k},\mathbf{B}^0)=-{}_{m}\chi_{e\,ji}(\omega,-\mathbf{k},-\mathbf{B}^0)\ , \tag{16}$$

in complete agreement with the result of Onsager's symmetry principle for kinetic coefficients[23] which is considered in the literature as the selection rule for the existence of linear magnetooptical effects in crystals.

Kleiner[27] has shown that the relations (15) and (16) are valid only for nonmagnetic crystals. For the magnetic crystals whose directional symmetry is described by 32 magnetic point groups[7] not comprising time inversion among their elements of symmetry, no Onsager principle can be enunciated, whereas for the remaining 58 magnetic groups a generalized Onsager principle can be formulated. From what has been said we draw the conclusion that with regard to magnetic crystals one has to drop the Onsager principle as a selection rule for the existence of linear magnetooptical effects. Here, the Neumann principle imposes itself as a more adequate selection rule: It is applicable to nonmagnetic as well as magnetic crystals provided that the transformation properties of the tensors under time inversion are known. In other words, the expansions (5)–(8) in the case of magnetic crystals will involve i tensors as well as c tensors, the latter leading to new magnetooptical effects, forbidden by Onsager's principle with regard to nonmagnetic crystals, Eqs. (15) and (16), one of which is MATPE.

III. THE REFRACTIVE INDICES

For magnetic insulators, on replacing $\mathbf{D}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ in the Maxwell equations[24]

$$\mathrm{curl}\mathbf{E}(\mathbf{r},t)=-\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}\ , \tag{17}$$

$$\mathrm{curl}\mathbf{H}(\mathbf{r},t)=\frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}\ , \tag{18}$$

by the expressions (2) and (3), and taking into account (1), (4), and (5)–(8), we get the equation for the light refractive index

$$\begin{aligned}\Bigg[&n^2(\delta_{ij}s_k^2-s_is_j)+n\left[\frac{\mu_0}{\epsilon_0}\right]^{1/2}[\delta_{iuw\,m}\chi_{e\,wj}(\omega,\mathbf{k},\mathbf{B}^0)+{}_{e}\chi_{m\,ip}(\omega,\mathbf{k},\mathbf{B}^0)\delta_{pju}\\ &+i({}_{e}\chi^{\nabla}_{e\,iju}(\omega)+{}_{e}\chi^{\nabla m}_{e\,ijup}(\omega)B_p^0+{}_{e}\chi^{\nabla mm}_{e\,iju(pr)}(\omega)B_p^0B_r^0)]s_u-\delta_{ij}\\ &-\frac{1}{\epsilon_0}[{}^{(1)}_{e}\chi^{(1)}_{e\,ij}(\omega)+{}^{(1)}_{e}\chi^{(1)m}_{e\,ijp}(\omega)B_p^0+{}^{(1)}_{e}\chi^{(1)mm}_{e\,ij(pr)}(\omega)B_p^0B_r^0]+\frac{\mu_0}{\epsilon_0}{}_{e}\chi_{m\,ip}(\omega,\mathbf{k},\mathbf{B}^0)\,{}_{m}\chi_{e\,pj}(\omega,\mathbf{k},\mathbf{B}^0)\Bigg]E_j(\omega,\mathbf{k})=0\ ,\end{aligned} \tag{19}$$

with

$${}_{e}\chi^{\nabla}_{e\,iju}(\omega)=\frac{\omega}{3}[{}^{(1)}_{e}\chi^{(2)}_{e\,i(ju)}(\omega)-{}^{(2)}_{e}\chi^{(1)}_{e\,(iu)j}(\omega)]\ , \tag{20}$$

$${}_{e}\chi^{\nabla m}_{e\,ijup}(\omega)=\frac{\omega}{3}[{}^{(1)}_{e}\chi^{(2)m}_{e\,i(ju)p}(\omega)-{}^{(2)}_{e}\chi^{(1)m}_{e\,(iu)jp}(\omega)]\ , \tag{21}$$

$${}_{e}\chi^{\nabla mm}_{e\,iju(pr)}(\omega)=\frac{\omega}{3}[{}^{(1)}_{e}\chi^{(2)mm}_{e\,i(ju)(pr)}(\omega)-{}^{(2)}_{e}\chi^{(1)mm}_{e\,(iu)j(pr)}(\omega)]\ . \tag{21$'$}$$

Here δ_{ij} and δ_{ijk} denote, respectively, the Kronecker and Levi-Cività unit tensors.

Making use of the tabulated polar and axial i and c tensors of the second, third, fourth and fifth rank[7] one easily checks that for centrosymmetric magnetic crystals the following conditions hold additionally:

$$\begin{aligned}&{}_{e}\chi^{\nabla}_{e\,iju}(\omega)=0,\quad {}_{e}\chi^{\nabla m}_{e\,ijup}(\omega)=0\ ,\\ &{}_{e}\chi^{\nabla mm}_{e\,iju(pr)}(\omega)=0\ ,\\ &{}_{e}\chi_{m\,ij}(\omega,\mathbf{k},\mathbf{B}^0)=0,\quad {}_{m}\chi_{e\,ij}(\omega,\mathbf{k},\mathbf{B}^0)=0\ .\end{aligned} \tag{22}$$

Let the light wave propagate in the crystal along the z axis taken as parallel to the highest of its axes of symmetry. We accordingly have

$$s_x = s_y = 0, \quad s_z = 1 \,, \qquad E_z(\omega, \mathbf{k}) = 0 \,. \tag{23}$$

By (22) and (23), the set of equations (19) reduces to

$$\{(n^2 s_z^2 - 1)\delta_{ij} - \frac{1}{\epsilon_0}[{}_e^{(1)}\chi_{e\,ij}^{(1)}(\omega) + {}_e^{(1)}\chi_{e\,iju}^{(1)m}(\omega)B_u^0 + {}_e^{(1)}\chi_{e\,ij(uw)}^{(1)mm}(\omega)B_u^0 B_w^0 + \cdots]\}E_j(\omega,\mathbf{k}) = 0 \,, \tag{24}$$

for $i,j = x,y$.

On equating to zero the determinant of the coefficients at $E_j(\omega,\mathbf{k})$ in (24) we obtain an equation of the fourth degree in n. On solving it, we arrive at two solutions of the form

$$n_\pm = n_0 + n(B^0) \pm \delta(B^0) \,, \tag{25}$$

where we have introduced the notations

$$n_0 = [(n_x^2 + n_y^2)/2]^{1/2} \,, \tag{26}$$

$$n(B^0) = \frac{1}{4\epsilon_0 n_0}\{[{}_e^{(1)}\alpha_{e\,xxu}^{(1)m}(\omega) + {}_e^{(1)}\alpha_{e\,yyu}^{(1)m}(\omega)]B_u^0 + [{}_e^{(1)}\alpha_{e\,xx(uw)}^{(1)mm}(\omega) + {}_e^{(1)}\alpha_{e\,yy(uw)}^{(1)mm}(\omega)]B_u^0 B_w^0 + \cdots\} \,, \tag{27}$$

$$\delta(B^0) = \frac{1}{4n_0}\{[n_y^2 - n_x^2 + h(B^0)]^2 + 4[g(B^0)^2 + f(B^0)^2]\}^{1/2} \,, \tag{28}$$

with

$$n_x^2 = 1 + \frac{1}{\epsilon_0}{}_e^{(1)}\alpha_{e\,xx}^{(1)}(\omega) \,, \qquad n_y^2 = 1 + \frac{1}{\epsilon_0}{}_e^{(1)}\alpha_{e\,yy}^{(1)}(\omega) \,, \tag{29}$$

$$h(B^0) = \frac{1}{\epsilon_0}\{[{}_e^{(1)}\alpha_{e\,yyu}^{(1)m}(\omega) - {}_e^{(1)}\alpha_{e\,xxu}^{(1)m}(\omega)]B_u^0 + [{}_e^{(1)}\alpha_{e\,yy(uw)}^{(1)mm}(\omega) - {}_e^{(1)}\alpha_{e\,xx(uw)}^{(1)mm}(\omega)]B_u^0 B_w^0 + \cdots\} \,, \tag{30}$$

$$g(B^0) = \frac{1}{\epsilon_0}\{{}_e^{(1)}\alpha_{e\,(xy)}^{(1)}(\omega) + {}_e^{(1)}\alpha_{e\,(xy)u}^{(1)m}(\omega)B_u^0 + {}_e^{(1)}\alpha_{e\,(xy)(uw)}^{(1)mm}(\omega)B_u^0 B_w^0 + \cdots\} \,, \tag{31}$$

$$f(B^0) = \frac{1}{\epsilon_0}[{}_e^{(1)}\gamma_{e\,[xy]}^{(1)}(\omega) + {}_e^{(1)}\gamma_{e\,[xy]u}^{(1)m}(\omega)B_u^0 + {}_e^{(1)}\gamma_{e\,[xy](uw)}^{(1)mm}(\omega)B_u^0 B_w^0 + \cdots] \,. \tag{32}$$

On insertion of n_+ and n_-, respectively, into the first and second equation of the set (24) we get the following relations:

$$E_x(\omega,k_+) - a_+ E_y(\omega,k_+) = 0 \,, \tag{33}$$

$$a_- E_x(\omega,k_-) + E_y(\omega,k_-) = 0 \,, \tag{34}$$

where

$$a_\pm = \frac{2[g(B^0) \pm i f(B^0)]}{n_y^2 - n_x^2 + h(B^0) + 4n_0\delta(B^0)} \,, \tag{35}$$

where obviously $a_- = (a_+)^*$.

Thus, two waves propagate in the medium along the z axis with velocities, respectively, equal to $V_+ = c/n_+$ and $V_- = c/n_-$.

By (33) and (34), the electric field strength vectors $\mathbf{E}_1(z,t)$ and $\mathbf{E}_2(z,t)$ in a point z of the light waves propagating in the crystal with the velocities V_+ and V_- take the form

$$\mathbf{E}_1(z,t) = E(\omega,k_+)\left[\hat{\mathbf{e}}_x + \frac{1}{a_+}\hat{\mathbf{e}}_y\right] \times \exp\left[-i\omega\left[t - \frac{n_+ z}{c}\right]\right] + \text{c.c.} \,, \tag{36}$$

$$\mathbf{E}_2(z,t) = E(\omega,k_-)(\hat{\mathbf{e}}_x - a_-\hat{\mathbf{e}}_y) \times \exp\left[-i\omega\left[t - \frac{n_- z}{c}\right]\right] + \text{c.c.} \tag{36'}$$

It is more convenient to have recourse to the resultant field $[\mathbf{E}(z,t) = \mathbf{E}_1(z,t) + \mathbf{E}_2(z,t)]$ which can be written in the form

$$\mathbf{E}(z,t) = \hat{\mathbf{e}}_x E_x(z,t) + \hat{\mathbf{e}}_y E_y(z,t) + \text{c.c.} \,, \tag{37}$$

where

$$\begin{pmatrix} E_x(z,t) \\ E_y(z,t) \end{pmatrix} = \begin{pmatrix} P_- & Q_+ \\ Q_- & P_+ \end{pmatrix} \begin{pmatrix} E_x(0,t) \\ E_y(0,t) \end{pmatrix} \times \exp\left[\frac{i\omega[n_0+n(B^0)]z}{c}\right] , \quad (38)$$

with

$$P_\mp = \cos\left[\frac{\omega z \delta(B^0)}{c}\right] \mp i \frac{1-a_+a_-}{1+a_+a_-} \sin\left[\frac{\omega z \delta(B^0)}{c}\right] ,$$

$$Q_\pm = \frac{2ia_\pm}{1+a_+a_-} \sin\left[\frac{\omega z \delta(B^0)}{c}\right] , \quad (39)$$

whereas $E_x(0,t)$ and $E_y(0,t)$ are the components of the electric field vector oscillating along the x and y axis in the point $z=0$, i.e., at the input to the crystal

$$E_x(0,t) = E_x(\omega)\exp(-i\omega t) ,$$
$$E_y(0,t) = E_y(\omega)\exp(-i\omega t) . \quad (40)$$

IV. THE STOKES PARAMETERS

Complete information regarding the state of polarization of the field $\mathbf{E}(z,t)$ is best gained by determining the Stokes' parameters[2] S_0, S_1, S_2, and S_3. With regard to our formula (38), S_0, S_1, S_2, and S_3 particularize to the form

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ A_1 & -A_1 & A_2 & (A_2)^* \\ A_3 & -A_3 & A_{4-} & (A_{4-})^* \\ A_5 & -A_5 & iA_{4+} & -i(A_{4+})^* \end{pmatrix} \begin{pmatrix} |E_x(0,t)|^2 \\ |E_y(0,t)|^2 \\ E_x(0,t)E_y(0,t)^* \\ E_x(0,t)^*E_y(0,t) \end{pmatrix} , \quad (41)$$

where we have introduced the parameters

$$A_1 = 1 - \frac{8|a_+|^2}{(1+|a_+|^2)^2} \sin^2\left[\frac{\omega z \delta(B^0)}{c}\right] , \quad (42)$$

$$A_2 = -\frac{2i(a_+)^*}{1+|a_+|^2} \sin\left[2\left[\frac{\omega z \delta(B^0)}{c}\right]\right] \left[1 - i\frac{1-|a_+|^2}{1+|a_+|^2} \tan\left[\frac{\omega z \delta(B^0)}{c}\right]\right] , \quad (43)$$

$$A_3 = -\frac{2[a_+ + (a_+)^*](1-|a_+|^2)}{(1+|a_+|^2)^2} \sin^2\left[\frac{\omega z \delta(B^0)}{c}\right] - i\frac{a_+ - (a_+)^*}{1+|a_+|^2} \sin\left[2\left[\frac{\omega z \delta(B^0)}{c}\right]\right] , \quad (44)$$

$$A_4 = \cos^2\left[\frac{\omega z \delta(B^0)}{c}\right] - \frac{(1-|a_+|^2)^2}{(1+|a_+|^2)^2} \sin^2\left[\frac{\omega z \delta(B^0)}{c}\right]$$
$$- \frac{i(1-|a_+|^4)\sin\left[2\left[\frac{\omega z \delta(B^0)}{c}\right]\right] \mp 4[(a_+)^*]^2 \sin^2\left[\frac{\omega z \delta(B^0)}{c}\right]}{(1+|a_+|^2)^2} , \quad (45)$$

$$A_5 = \frac{a_+ + (a_+)^*}{1+|a_+|^2} \sin\left[2\left[\frac{\omega z \delta(B^0)}{c}\right]\right] - \frac{2i[a_+ - (a_+)^*](1-|a_+|^2)}{(1+|a_+|^2)^2} \sin^2\left[\frac{\omega z \delta(B^0)}{c}\right] . \quad (46)$$

Obviously, $(S_1^2+S_2^2+S_3^2)^{1/2}=S_0$, stating that the light wave $\mathbf{E}(z,t)$ within the medium is totally polarized. For a totally polarized light wave, the azimuth Ψ and the ellipticity Φ are equal to[2]

$$\Psi = \tfrac{1}{2}\arctan\left[\frac{S_2}{S_1}\right] , \quad (47)$$

$$\Phi = \tfrac{1}{2}\arcsin\left[\frac{S_3}{S_0}\right] , \quad (48)$$

where Ψ is the angle between the major axis of the ellipse and the x axis, and Φ is the ratio of the minor and major axes.

V. APPLICATION AND DISCUSSION

Assume the light wave to be linearly polarized along the x axis at the input to the crystal ($z=0$). Putting $E_y(0,t)=0$ in (41) we get

$$\Psi=\tfrac{1}{2}\arctan\left[\frac{A_3}{A_1}\right], \qquad \Phi=\tfrac{1}{2}\arcsin A_5 . \tag{49}$$

From (35), the coefficients a_+ and a_- are in general complex quantities. Let us assume that, moreover, the following two extreme cases are possible:

$$a_+=ib, \quad (a_+)^*=-ib , \tag{50}$$

$$a_+=(a_+)^*=a , \tag{51}$$

where

$$a=\frac{2g(B^0)}{n_y^2-n_x^2+h(B^0)+4n_0\delta(B^0)} , \tag{52}$$

$$b=\frac{2f(B^0)}{n_y^2-n_x^2+h(B^0)+4n_0\delta(B^0)} , \tag{53}$$

a and b being real.

A. Circular birefringence

The first case, given by the relation (50), takes place if

$$g(B^0)=0 . \tag{54}$$

Two elliptically polarized waves propagate in the medium with polarization versors $\hat{e}_+$ and $\hat{e}_-$, respectively, equal to $\hat{e}_+=\hat{e}_x-i/b\hat{e}_y$ and $\hat{e}_-=\hat{e}_x+ib\hat{e}_y$. Their superposition is also an elliptically polarized wave since the parameters A_5 and A_3/A_1 are nonzero, amounting to

$$A_5=f(B^0)[n_y^2-n_x^2+h(B^0)]\left[\frac{\omega z}{2n_0c}\right]^2 \times\left[1+\sum_{k=1}^{\infty}\frac{(-1)^kU(h,f)^{2k}}{(2k+1)!}\right]^2 , \tag{55}$$

$$\frac{A_3}{A_1}=\frac{\dfrac{\omega z}{n_0c}f(B^0)\left[1+\sum_{k=1}^{\infty}\dfrac{(-1)^k[2U(h,f)]^{2k}}{(2k+1)!}\right]}{1-2\left[\dfrac{\omega zf(B^0)}{2n_0c}\right]^2\left[1+\sum_{k=1}^{\infty}\dfrac{(-1)^kU(h,f)^{2k}}{(2k+1)!}\right]^2} , \tag{56}$$

where

$$U(h,f)=\frac{\omega z}{4n_0c}\{[n_y^2-n_x^2+h(B^0)]^2+[2f(B^0)]^2\}^{1/2} , \tag{57}$$

as a result of which Ψ and Φ too are nonzero.

The last two expressions are obtained from (42), (44), and (46) with the condition (54) and on expanding the functions $\sin\{2[\omega z\delta(B^0)/c]\}$ and $\sin^2[\omega z\delta(B^0)/c]$.

In particular, if $b=1$—as it is the case if, additionally, the two following conditions are fulfilled: $n_x=n_y$ and $h(B^0)=0$—two circularly polarized waves with opposite senses will be propagating in the medium, their superposition giving a linearly polarized wave, $\Phi=0$ with its polarization plane at an angle $\Psi=\omega zf(B^0)/2n_0c$ to the xz plane.

Here, we deal with circular birefringence (of circularly polarized waves). The first term of (32) determining the parameter $f(B^0)$, i.e., ${}_e^{(1)}\gamma_{e[xy]}^{(1)}(\omega)$, describes gyrotropic rotation;[15] the second term of (32) at $u=z$ (magnetic field acting along z) describes the well-known Faraday effect; and the third term at arbitrary orientation of the field $\mathbf{B}^0$ describes quadratic magnetic variations in gyrotropic rotation.[20]

B. Linear birefringence

The second case, which is given by (51), occurs if

$$f(B^0)=0 , \tag{58}$$

involving

$$A_5=\frac{\omega z}{n_0c}g(B^0)\left[1+\sum_{k=1}^{\infty}\frac{(-1)^k[2U(h,g)]^{2k}}{(2k+1)!}\right] , \tag{59}$$

$$\frac{A_3}{A_1}=\frac{g(B^0)[n_y^2-n_x^2+h(B^0)]\left[\dfrac{\omega z}{2n_0c}\right]^2\left[1+\sum_{k=1}^{\infty}\dfrac{(-1)^kU(h,g)^{2k}}{(2k+1)!}\right]^2}{1-2\left[\dfrac{\omega zg(B^0)}{2n_0c}\right]^2\left[1+\sum_{k=1}^{\infty}\dfrac{(-1)^kU(h,g)^{2k}}{(2k+1)!}\right]^2} , \tag{60}$$

where $U(h,g)$ is obtained from (57) on replacing f by g. The relation (51) states that two linearly polarized waves now propagate in the medium, respectively, in the directions $\hat{e}_+=\hat{e}_x+1/a\hat{e}_y$ and $\hat{e}_-=\hat{e}_x-a\hat{e}_y$, the two waves being mutually orthogonal: $\hat{e}_+\cdot\hat{e}_-=0$.

Their superposition in a point z (on having traversed a path z in the medium) is an elliptically polarized wave. Here, we deal with linear birefringence (of linearly polarized waves). In particular, if $a=1$ [this takes place if $n_x=n_y$ and $h(B^0)=0$] the parameters of the ellipse are equal to

$$\Phi = \frac{g(B^0)\omega z}{2n_0 c} , \qquad \Psi = 0 , \tag{61}$$

its major axis lying along the x axis. Obviously, the light wave will propagate in the medium with its state of polarization unchanged if $a=0$; this occurs if $g(B^0)=0$.

Let us consider the case when the incident wave is polarized linearly at an angle of 45° to the x axis in the xy plane. At the input, we now have $E_x(0,t)=E_y(0,t)$. If, moreover, the parameter a vanishes [this occurs if $f(B^0)=0$ and $g(B^0)=0$] two linearly polarized waves will again be propagating in the medium, in the directions $\hat{e}_x$ and $\hat{e}_y$, respectively, their superposition giving an elliptically polarized wave

$$\Psi = \pi/4 , \qquad \Phi = \frac{[n_y^2 - n_x^2 + h(B^0)]\omega z}{4n_0 c} , \tag{62}$$

the major axis of the ellipse coinciding with the polarization direction of the wave at the input to the crystal.

C. MATPE

The conditions (22) and (58) restrict the variety of antiferromagnetic crystals where linear birefringence can be expected to occur to those with directional symmetry of the following magnetic point groups: $4/mmm$, $\underline{4}/mm\underline{m}$, $\bar{3}m$, $\underline{6}/\underline{m}$, $6/mmm$, $\underline{6}/\underline{mm}m$, $m3$, $m3m$ and $m3\underline{m}$. From the tables[7] of i and c tensors of the second, third, and fourth rank we determined the parameters a, $h(B^0)$, $g(B^0)$, Ψ, and Φ for these crystallographical classes for $\mathbf{B}^0$ applied along the axis x or, respectively, along y [a disposition with the magnetic field parallel to the z axis would be forbidden with regard to the condition (58)]. In determining Ψ and Φ we restricted ourselves to the first term of the series expansion of $\arctan(A_3/A_1)$ and $\arcsin(A_5)$, an apparently satisfactory approximation since the term omitted in the two expansions is proportional to the third power of the argument of the respective function, thus giving a contribution to Ψ and Φ whose dependence on $\mathbf{B}^0$ has an exponent higher than that assumed in the expressions (5)–(8). For the same reasons, in the expression defining A_3/A_1 and A_5, we neglected the terms containing the parameter $U(h,g)$. Our results are given in Table I (where the fifth column specifies the configuration of the electric field of the light wave at the input to the crystal).

Table I clearly shows that by choosing appropriate configurations of the optical electric field at the input one can observe linear birefringence in antiferromagnetic insulators with the symmetries $4/mmm$, $\underline{4}/mm\underline{m}$, $6/mmm$, $m3$, $m3m$, $m3\underline{m}$, $\bar{3}m$, $\underline{6}/\underline{m}$, and $\underline{6}/\underline{mm}m$. Strictly, in the first five of these classes, the ellipticity is dependent on the square of the magnetic field. Here, we have the well-known magnetooptical effect of Voigt or, respectively, the Cotton-Mouton effect.

In the class $\bar{3}m$, if $\mathbf{B}^0$ is applied along the x axis, the ellipticity is dependent on the first power of B_x^0 and the major axis of the ellipse subtends an angle Ψ_1 with the x axis that is given by a rather complicated function which, in a first approximation, depends but on the third power of B_x^0 and thus exceeds the accuracy assumed for our calculations (which restricts us to quadtatic terms; thus, $\Psi_1 \approx 0$). The situation is similar for the class $\underline{6}/\underline{mm}m$ when $\mathbf{B}^0$ is applied along y (second part of Table I).

Birefringence proportional to the first power of B_x^0 will also occur in the classes $\underline{6}/\underline{m}$ and $\underline{6}/\underline{mm}m$ with $\mathbf{B}^0$ applied along x and the light wave at the input polarized linearly along x or in the $\hat{e}_x + \hat{e}_y$ direction. From the first part of Table I, the ellipticity for these two classes involves, moreover, a term proportional to the second power of the magnetic field. The linear term is easily separable from the quadratic term since it changes its sign on reversal of the magnetic field.

The situation is similar for the classes $\underline{6}/\underline{m}$ and $\bar{3}m$ if the magnetic field is applied along the y axis. The birefringence proportional to the first power of the static magnetic field which is at the core of our interest will occur as well in noncentrosymmetric magnetic crystals, possessing the symmetries 3, $\bar{3}$, 32, $3\underline{2}$, $3m$, $3\underline{m}$, $\bar{3}\underline{m}$, $\underline{6}$, $\bar{6}$, $\underline{6}2\underline{2}$, $\underline{6}m\underline{m}$, $\bar{6}2\underline{m}$, and $\bar{6}m\underline{2}$. In these classes, however, the condition (22) ceases to hold so that the birefringence in question will be accompanied by other optical effects, such as natural optical activity,[13] natural gyrotropic rotation and birefringence,[14,15] and their variations are proportional to the first and second power of the magnetic field strength[16–18] rendering difficult the observation of birefringence proportional to the first power of the static magnetic field.

At the configuration under consideration here (the laboratory coordinates coincide with the crystallographical set of axes and the field $\mathbf{B}^0$ is at right angles to the light propagation direction) no birefringence dependent on the first power of the field will appear in any of the magnetic crystals belonging to the regular system. If the system of laboratory coordinates coincides with the crystallographical axes, magnetic crystals having the symmetries 23, $m3$, $43\underline{2}$, $\bar{4}3\underline{m}$ and $m3\underline{m}$ can exhibit the effect only if the field is applied parallel to the direction of light propagation; however, the effect will always be masked by the linear magnetooptical effect of Faraday.

Dillon *et al.*[28] were the first to show that the birefringence linear in $\mathbf{B}^0$ can be expected in the longitudinal geometry $\mathbf{B}^0 \| [001] \| \mathbf{k}$ (Faraday configuration) in $Dy_3Al_5O_{12}$ crystals (magnetic symmetry class $m3\underline{m}$). It was subsequently measured in some antiferromagnetic crystals;[21] $DyFeO_3$ (symmetry mmm), $Ca_3Mn_2Ge_3O_{12}$ (symmetry $\underline{4}/m$), CoF_2 (symmetry $\underline{4}/mm\underline{m}$), α Fe_2O_3 and $CoCO_3$ (symmetry $\bar{3}m$). For a detailed discussion of these experiments see the paper of Eremanko and Kharchenko.[21] The values thus obtained for certain tensor components ${}^{(1)}_{e}\alpha^{(1)m}_{e(ij)k}(\omega)$ amounted to ${}^{(1)}_{e}\alpha^{(1)m}_{e(xy)z}(\omega) = 2.5\times10^{-7}$ Oe^{-1} at $\lambda = 5900$ Å for $DyFeO_3$, and ${}^{(1)}_{e}\alpha^{(1)m}_{e\,xxx}(\omega) = -{}^{(1)}_{e}\alpha^{(1)m}_{e\,yyx}(\omega) = 1.25\times10^{-8}$ Oe^{-1} at $\lambda = 11\,500$ Å for α Fe_2O_3. It is easy to check that for ${}^{(1)}_{e}\alpha^{(1)m}_{e(ij)k}(\omega)$ of the order of 10^{-7}–10^{-8} Oe^{-1}, a crystal

TABLE I. Explicit form of the parameters $h(B^0)$, $g(B^0)$, a, Ψ, and Φ for nine magnetic symmetry classes and for two selected directions of $\mathbf{B}^0$.

Class	$h(B^0)$	$g(B^0)$	a	$\mathbf{E}(0,t)$	Ψ	Φ
		$\mathbf{B}^0$ applied along the x axis				
$4/mmm$, $\underline{4}/mm\underline{m}$, $6/mmm$, $m3$, $m3m$, $m3\underline{m}$	$H_1(B_x^0)^2$	0	0	$(\hat{e}_x+\hat{e}_y)E(0,t)$	$\pi/4$	$\frac{\omega z}{4n_0c}H_1(B_x^0)^2$
$\bar{3}m$	$H_1(B_x^0)^2$	$g_1B_x^0$	a_1	$\hat{e}_xE(0,t)$	Ψ_1	$\frac{\omega z}{2n_0c}g_1B_x^0$
$\underline{6}/\underline{m}$	$(2h_1+H_1B_x^0)B_x^0$	$(g_1+G_1B_x^0)B_x^0$	a_2	$\hat{e}_xE(0,t)$	Ψ_2	$\frac{\omega z}{2n_0c}(g_1+G_1B_x^0)B_x^0$
$\underline{6}/\underline{mmm}$	$(2h_1+H_1B_x^0)B_x^0$	0	0	$(\hat{e}_x+\hat{e}_y)E(0,t)$	$\pi/4$	$\frac{\omega z}{4n_0c}(2h_1+H_1B_x^0)B_x^0$
		$\mathbf{B}^0$ applied along the y axis				
$4/mmm$, $\underline{4}/mm\underline{m}$, $6/mmm$, $m3$, $m3m$, $m3\underline{m}$	$-H_1(B_y^0)^2$	0	0	$(\hat{e}_x+\hat{e}_y)E(0,t)$	$\pi/4$	$-\frac{\omega z}{4n_0c}H_1(B_y^0)^2$
$\bar{3}m$	$-(2g_1+H_1B_y^0)B_y^0$	0	0	$(\hat{e}_x+\hat{e}_y)E(0,t)$	$\pi/4$	$-\frac{\omega z}{4n_0c}(2g_1+H_1B_y^0)B_y^0$
$\underline{6}/\underline{m}$	$-(2g_1+H_1B_y^0)B_y^0$	$(h_1-G_1B_y^0)B_y^0$	a_3	$\hat{e}_xE(0,t)$	Ψ_3	$\frac{\omega z}{2n_0c}(h_1-G_1B_y^0)B_y^0$
$\underline{6}/\underline{mmm}$	$-H_1(B_y^0)^2$	$h_1B_y^0$	a_4	$\hat{e}_xE(0,t)$	Ψ_4	$\frac{\omega z}{2n_0c}h_1B_y^0$

$$H_1=\frac{1}{\epsilon_0}[{}_e^{(1)}\alpha_{e\,yyxx}^{(1)mm}(\omega)-{}_e^{(1)}\alpha_{e\,xxxx}^{(1)mm}(\omega)],\quad G_1=\frac{1}{\epsilon_0}{}_e^{(1)}\alpha_{e(xy)xx}^{(1)mm}(\omega)$$

$$h_1=\frac{1}{\epsilon_0}{}_e^{(1)}\alpha_{e\,yyx}^{(1)m}(\omega),\quad g_1=\frac{1}{\epsilon_0}{}_e^{(1)}\alpha_{e(xy)x}^{(1)m}(\omega)$$

$$a_1=\frac{2g_1}{H_1B_x^0+[(H_1B_x^0)^2+4g_1^2]^{(1/2)}},\quad a_2=\frac{2(g_1+G_1B_x^0)}{2h_1+H_1B_x^0+[(2h_1+H_1B_x^0)^2+4(g_1+G_1B_x^0)^2]^{(1/2)}}$$

$$a_3=\frac{2(h_1-G_1B_y^0)}{-2g_1-H_1B_y^0+[(2g_1+H_1B_y^0)^2+4(h_1-G_1B_y^0)^2]^{(1/2)}},\quad a_4=\frac{2h_1}{-H_1B_y^0+[(H_1B_y^0)^2+4h_1^2]^{(1/2)}}$$

$$\Psi_1=-\frac{1}{2}\left[\frac{\omega zB_x^0}{2n_0c}\right]^2\frac{g_1H_1B_x^0}{1-2\left[\frac{\omega zg_1B_x^0}{2n_0c}\right]^2}+\cdots,\quad \Psi_2=-\frac{1}{2}\left[\frac{\omega zB_x^0}{2n_0c}\right]^2\frac{(g_1+G_1B_x^0)(2h_1+H_1B_x^0)}{1-2\left[\frac{\omega zB_x^0(g_1+G_1B_x^0)}{2n_0c}\right]^2}$$

$$\Psi_3=\frac{1}{2}\left[\frac{\omega zB_y^0}{2n_0c}\right]^2\frac{(2g_1+H_1B_y^0)(h_1-G_1B_y^0)}{1-2\left[\frac{\omega zB_y^0(h_1-G_1B_y^0)}{2n_0c}\right]^2}+\cdots,\quad \Psi_4=\frac{1}{2}\left[\frac{\omega zB_y^0}{2n_0c}\right]^2\frac{h_1H_1B_y^0}{1-2\left[\frac{\omega zB_y^0h_1}{2n_0c}\right]^2}+\cdots$$

with $l=0.1$ cm thick and a magnetic field H of n kOe (arbitrary n in cgs units, $\Phi=[4\pi\omega l/(2n_0c)]_e^{(1)}\alpha_{e(ij)k}^{(1)m}(\omega)H_k^0$), the ellipticity Φ will range from n rad to $n\times10^{-1}$ rad, respectively, magnitudes well accessible to measurement. The magnetic field acting on the antiferromagnetic crystals modifies the well-known ferromagnetic vector **M** and antiferromagnetic vector **L**. The components ${}_e^{(1)}\alpha_{e(ij)k}^{(1)m}(\omega)$ can be expressed via **L** and **M**:[21]

$$ {}_e^{(1)}\alpha_{e(ij)k}^{(1)m}(\omega)=Q_{(ij)rs}^{ML}(\omega)\chi_{rk}^{M}L_s+Q_{(ij)rs}^{LL}(\omega)\chi_{rk}^{L}L_s + Q_{(ij)rs}^{MM}(\omega)\chi_{rk}^{M}M_s , $$

where χ_{rk}^M is the magnetic susceptibility and χ_{rk}^L the antiferromagnetic susceptibility characteristic of induction of the r component of the antiferromagnetism vector **L** by the field $\mathbf{B}^0$. Antiferromagnetic crystals with the symmetries 23, $m3$, $\underline{4}3\underline{2}$, $\bar{\underline{4}}3\underline{m}$, and $m3\underline{m}$ can exhibit MATPE (birefringence linear in $\mathbf{B}^0$ in the transversal geometry $\mathbf{B}^0\perp\mathbf{k}\|\hat{\mathbf{z}}$) only if the laboratory coordinates x,y,z do not coincide with the crystallographical coordinates X,Y,Z. If x,y,z are chosen so that they shall go over into X,Y,Z after two rotations, the one about z by an angle ϕ and the other about the new axis $y'=Y$ by an angle θ, we get[29] for crystals with the symmetries 23, $m3$, $\underline{4}3\underline{2}$, $\bar{\underline{4}}3\underline{m}$, and $m3\underline{m}$:

$$ {}_e^{(1)}\alpha_{e\,yyy}^{(1)m}(\omega)=0 , $$

$$ \begin{aligned} {}_e^{(1)}\alpha_{e\,yxx}^{(1)m}(\omega)&={}_e^{(1)}\alpha_{e\,xyx}^{(1)m}(\omega)\\ &={}_e^{(1)}\alpha_{e\,xxy}^{(1)m}(\omega)\\ &=\sin(2\theta)\cos(2\phi){}_e^{(1)}\alpha_{e\,XYZ}^{(1)m}(\omega) , \end{aligned} $$

$$ {}_e^{(1)}\alpha_{e\,xxx}^{(1)m}(\omega)=-3\cos^2(\theta)\sin(\theta)\sin(2\phi){}_e^{(1)}\alpha_{e\,XYZ}^{(1)m}(\omega) , $$

$$ \begin{aligned} {}_e^{(1)}\alpha_{e\,xyy}^{(1)m}(\omega)&={}_e^{(1)}\alpha_{e\,yxy}^{(1)m}(\omega)\\ &={}_e^{(1)}\alpha_{e\,yyx}^{(1)m}(\omega)\\ &=\sin(\theta)\sin(2\phi){}_e^{(1)}\alpha_{e\,XYZ}^{(1)m}(\omega) , \end{aligned} $$

where ${}_e^{(1)}\alpha_{e\,XYZ}^{(1)m}(\omega)$ is the component of the susceptibility tensor ${}_e^{(1)}\alpha_e^{(1)m}(\omega)$ in the crystallographical set of coordinates.

The MATPE is sensitive to the crystal magnetic symmetry and to reorientation of the antiferromagnetic vector. Moreover, it undergoes a change in sign when the directions of the magnetic moments of a sublattice are inverted.[21] Owing to this, MATPE can be used to study the time-reversed domain structure of antiferromagnets, to determine the symmetry of magnetic ordering, and to study the magnetic crystal energy spectra by spectroscopic methods. MATPE can be used moreover for phase modulation of laser light and for controlling the efficiency of generation of squeezed states of the electromagnetic field by way of a static magnetic field.[30]

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