Evolution of the polarization state of an intense optical wave in uniaxial crystals

Marek Roman
Laboratory of Physicochemistry of Dielectrics, Department of Chemistry, The University of Warsaw, PL-02-089 Warsaw, Poland

Stanisław Kielich
Nonlinear Optics Division, Institute of Physics, Adam Mickiewicz University, Grunwaldzka 6, PL-60-780 Poznań, Poland

Wojciech Gadomski
Laboratory of Physicochemistry of Dielectrics, Department of Chemistry, The University of Warsaw, PL-02-089 Warsaw, Poland
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The evolution of the polarization state of an intense laser wave in uniaxial nonabsorbing crystals is considered with use of analytical and numerical methods. It has been found that, depending on the initial conditions, the polarization ellipse rotates or oscillates, changing its shape and handedness. A detailed study of various cases is performed.

I. INTRODUCTION

Since the discovery of self-induced rotation of the polarization ellipse of an intense laser wave by Maker *et al.* in 1964,[1] the self-induced changes of the polarization of intense laser beams have become the object of numerous studies in a wide class of materials.[2–13] It was found in isotropic media that the polarization ellipse rotates at an angular rate dependent on the laser-beam power density and the ellipticity of the optical wave.[2] In the absence of absorption and stimulated scattering the ellipticity remains constant. If the input power of the laser beam is greater than the threshold for stimulated Raman scattering, the ellipticity tends to unity. This phenomenon was observed in a number of liquids.[13]

The application of an external dc electric field to the isotropic medium causes a qualitative change in the evolution mechanism of the light polarization. Theoretical analysis of the propagation of an intense laser beam through a standard Kerr cell has shown[14–16] that in this case coupling occurs between the changes in light polarization due to dc-induced birefringence and self-induced ellipse rotation (SIER). This coupling leads directly to anomalies in the dc Kerr effect[14,15,17] and causes changes in the polarization of the optical wave[16–18] which do not occur in the previously considered systems.[7,12] In crystals, two mechanisms were found to be responsible for SIER, i.e., self-induced rotation and self-induced changes in optical natural gyration.[19–23] The latter effect occurs in crystals in which natural optical activity is present. It was also shown theoretically that, in magnetic crystals, several other mechanisms exist that produce SIER.[20] However, none of the theories describing the evolution of the polarization of the optical field in crystals take account of the coupling between SIER and the polarization changes due to optical birefringence. Therefore, they do not provide a correct description of the optical field polarization state in anisotropic materials.

In the present paper we report results of analytical and numerical analyses of SIER in uniaxial crystals with a center of symmetry. As was proved previously,[19] in such systems only the self-induced ellipse rotation and optical birefringence of the crystals have a significant influence on the light polarization.

Assuming that the optical field is a superposition of plane waves with amplitudes slowly varying in time and space, we have derived expressions describing the spatial distributions of the intensity and phases of the optical field components. We have shown both analytically and numerically that, depending on the initial conditions, the polarization ellipse of the light may rotate or oscillate, changing its shape and handedness. A detailed study is made of the conditions required for observing these new effects in experiments.

II. FUNDAMENTALS OF THE PHENOMENOLOGICAL THEORY

In linear optics, each Fourier component of the optical field in uniaxial, optically nonactive crystals can be represented as the sum of two linearly polarized monochromatic waves with wave vectors, given by $\mathbf{k}^{(s)}$, to be found from the equation[24]

$$\mathbf{k}^{(s)}\times\mathbf{k}^{(s)}\times\hat{\mathbf{e}}^{(s)}-\frac{\omega^2}{c^2}\overleftrightarrow{\epsilon}(\omega)\cdot\hat{\mathbf{e}}^{(s)}=\mathbf{0}\;, \tag{1}$$

where $\hat{\mathbf{e}}^{(s)}$ are the polarization unit vectors, $\overleftrightarrow{\epsilon}(\omega)$ is the frequency-dependent linear susceptibility tensor, and c denotes the light velocity. If the total optical field consists of a finite number of quasimonochromatic waves, i.e.,

$$\begin{aligned}\mathbf{E}(\mathbf{r},t)&=\tfrac{1}{2}\sum_{\omega_L}\mathbf{E}(\omega_L,\mathbf{r},t)\exp(i\omega_L t)+\text{c.c.}\;,\\ \mathbf{P}(\mathbf{r},t)&=\tfrac{1}{2}\sum_{\omega_L}\mathbf{P}(\omega_L,\mathbf{r},t)\exp(i\omega_L t)+\text{c.c.}\;,\end{aligned} \tag{2}$$

it is possible to extend this approach to nonlinear optics.

In this case the electric field vector describing each quasimonochromatic wave of basic frequency ω_L obeys the Maxwell wave equation[25]

$$\nabla\times\nabla\times\mathbf{E}(\omega_L,\mathbf{r},t)-\frac{4\pi}{c^2}\left[\omega_L+i\frac{\partial}{\partial t}\right]^2\mathbf{P}(\omega_L,\mathbf{r},t)=\mathbf{0}\ . \tag{3}$$

The vector $\mathbf{P}(\omega_L,\mathbf{r},t)$ describing the polarization of the medium is usually represented as the sum

$$\mathbf{P}(\omega_L,\mathbf{r},t)=\overleftrightarrow{\epsilon}\cdot\mathbf{E}(\omega_L,\mathbf{r},t)+\mathbf{P}^{\mathrm{NL}}(\omega_L,\mathbf{r},t)\ . \tag{4}$$

In our considerations we analyze the case when the quasimonochromatic wave propagating in the nonlinear medium is a plane wave of uniform power density distribution on the phase surface. Under such conditions the complex amplitudes $\mathbf{E}(\omega_L,\mathbf{r},t)$ and $\mathbf{P}(\omega_L,\mathbf{r},t)$ become functions of the one space variable. Further, we assume that the rise time of the nonlinear polarization of the medium is much shorter than the rise time of the optical field (steady-state approximation). Taking advantage of the assumptions mentioned above, we can write the vectors describing the electric field and the related nonlinear response in the form[7,26]

$$\mathbf{E}_L(\omega_L,\mathbf{r},t)=\sum_{s=1}^{2}\hat{\mathbf{e}}^{(s)}A^{(s)}(l,t)\exp(-ik^{(s)}l)\ , \tag{5}$$

$$\mathbf{P}^{\mathrm{NL}}(\omega_L,\mathbf{r},t)=\sum_{s,j,m=1}^{2}\chi^{(3)}\vdots\hat{\mathbf{e}}^{(s)}\hat{\mathbf{e}}^{(j)}\hat{\mathbf{e}}^{(m)}A^{(s)}(l,t)A^{(j)}(l,t)A^{(m)}(l,t)\exp[-i(k^{(s)}+k^{(j)}-k^{(m)})l]\ , \tag{6}$$

where l is the length of the propagation path in the medium, $\chi^{(3)}$ is the fourth-rank nonlinear susceptibility tensor, $\mathbf{k}^{(1)}$ and $\mathbf{k}^{(2)}$ are the solutions of (1), and $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$ denote the polarization unit vectors related to each of these solutions. The unit vectors $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$ are orthogonal and one of them is perpendicular to the optical axis of the crystal.

Since the term $\mathbf{P}^{\mathrm{NL}}(\omega_L,\mathbf{r},t)$ describing the nonlinear response of the medium to the optical field is small compared to the terms linear in $\mathbf{E}(\omega_L,\mathbf{r},t)$, the functions $A^{(s)}(l,t)$ are functions slowly varying with respect to $\exp[i(\omega_L t-k^{(s)}l)]$.[7,26] Because of this fact,

$$\left|\frac{\partial^2A^{(s)}(l,t)}{\partial l^2}\right|\ll\left|\frac{\partial A^{(s)}(l,t)}{\partial l}k^{(s)}\right|\ , \tag{7}$$

$$\left|\frac{\partial^2A^{(s)}(l,t)}{\partial t^2}\right|\ll\left|\omega_L\frac{\partial A^{(s)}(l,t)}{\partial t}\right|\ . \tag{8}$$

With the use of representation (3) and the neglect of the terms small in the meaning of (7) and (8), Maxwell's propagation equation (2) can be reduced to a set of partial first-order equations of the form

$$2i\sum_{s=1}^{2}\left[\left[\hat{\mathbf{e}}^{(s)}k^{(s)}-\frac{\mathbf{k}^{(s)}}{k^{(s)}}(\mathbf{k}^{(s)}\cdot\hat{\mathbf{e}}^{(s)})\right]\frac{\partial A^{(s)}(l,t)}{\partial l}-\frac{\overleftrightarrow{\epsilon}(\omega)\omega_L}{c^2}\frac{\partial}{\partial t}\hat{\mathbf{e}}^{(s)}A^{(s)}(l,t)\right]e^{(-ik^{(s)}l)}$$
$$=\frac{4\pi}{c^2}\left[\omega_L^2+2i\omega_L\frac{\partial}{\partial t}\right]\sum_{j,m,n=1}^{2}\chi^{(3)}\vdots\hat{\mathbf{e}}^{(j)}\hat{\mathbf{e}}^{(m)}\hat{\mathbf{e}}^{(n)}A^{(j)}(l,t)A^{(m)}(l,t)[A^{(n)}(l,t)]^*\exp[-i(k^{(j)}+k^{(m)}-k^{(n)})l]\ . \tag{9}$$

In our analysis we assume that the medium is thin with respect to the spatial width of the laser pulse, i.e.,

$$\left|\frac{\partial A^{(s)}(l,t)}{\partial l}\right|\gg\frac{n_0}{c}\left|\frac{\partial A^{(s)}(l,t)}{\partial t}\right|\ . \tag{10}$$

Therefore, we omit all terms containing time derivatives.

Now, projecting Eq. (9) on the directions given by $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$ we get the system of two equations

$$\begin{aligned}2im^{(1)}\frac{\partial A^{(1)}}{\partial l}&=\frac{4\pi\omega_L^2}{c^2}\{\chi^{(3)}_{1111}A^{(1)}A^{(1)}(A^{(1)})^*+\chi^{(3)}_{1122}A^{(1)}A^{(2)}(A^{(2)})^*\\&\quad+\chi^{(3)}_{1212}A^{(2)}A^{(1)}(A^{(2)})^*+\chi^{(3)}_{1212}A^{(2)}A^{(2)}(A^{(1)})^*\exp[2i(k^{(1)}-k^{(2)})l]\}\ ,\\2im^{(2)}\frac{\partial A^{(2)}}{\partial l}&=\frac{4\pi\omega_L^2}{c^2}\{\chi^{(3)}_{2222}A^{(2)}A^{(2)}(A^{(2)})^*+\chi^{(3)}_{2211}A^{(2)}A^{(1)}(A^{(1)})^*\\&\quad+\chi^{(3)}_{2121}A^{(1)}A^{(2)}(A^{(1)})^*+\chi^{(3)}_{2121}A^{(1)}A^{(2)}(A^{(1)})^*\exp[2i(k^{(2)}-k^{(1)})l]\}\ ,\end{aligned} \tag{11}$$

where, for simplicity, we denoted

$$m^{(i)}=k^{(i)}-\frac{(\hat{\mathbf{e}}^{(i)}\cdot\mathbf{k}^{(i)})^2}{k^{(i)}}\ ,$$

$$\chi^{(3)}_{ijkl}=\hat{\mathbf{e}}^{(s)}\cdot\chi^{(3)}\vdots\hat{\mathbf{e}}^{(j)}\hat{\mathbf{e}}^{(k)}\hat{\mathbf{e}}^{(l)}\ . \tag{12}$$

Owing to symmetry relations the following relations hold:[26]

$$\chi^{(3)}_{1212}=\chi^{(3)}_{2121},\ \chi^{(3)}_{1122}=\chi^{(3)}_{2211},\ \chi^{(3)}_{2112}=\chi^{(3)}_{1221}\ . \tag{13}$$

Thus, if we denote $A^{(j)}=|A^{(j)}|e^{-i\phi^{(j)}}$ and introduce new variables

$$\begin{aligned} l'&=\frac{4\pi\omega_L^2\chi^{(3)}_{2112}C_0 l}{c^2 m^{(1)} m^{(2)}}\ ,\\ s&=m^{(1)}|A^{(1)}|^2/C_0\ ,\\ u&=m^{(2)}|A^{(2)}|^2/C_0\ ,\\ \theta&=\phi^{(2)}-\phi^{(1)}+k^{(2)}l-k^{(1)}l\ . \end{aligned} \tag{14}$$

where

$$C_0=m^{(1)}|A_0^{(1)}|^2+m^{(2)}|A_0^{(2)}|^2$$

and $A_0^{(1)}$ and $A_0^{(2)}$ are the initial values of $A^{(1)}$ and $A^{(2)}$, respectively, we can transform the system of Eq. (11) to

$$\frac{\partial u}{\partial l'}=su\sin(2\theta)\ , \tag{15}$$

$$\frac{\partial \theta}{\partial l'}=(u-s)\sin^2\theta+D+2Mu\ , \tag{16}$$

with the Manley-Rowe constant of motion

$$u+s=1\ . \tag{17}$$

The constants D and M are expressed in terms of the previously used parameters as follows:

$$\begin{aligned} D&=\frac{(k^{(2)}-k^{(1)})c^2 m^{(1)} m^{(2)}}{4\pi\omega_L^2\chi^{(3)}_{1221}C_0}+\frac{\chi^{(3)}_{1122}+\chi^{(3)}_{1212}+\chi^{(3)}_{1221}}{2\chi^{(3)}_{1221}}-\frac{\chi^{(3)}_{1111}m^{(2)}}{2\chi^{(3)}_{1221}m^{(1)}}\ ,\\ M&=\frac{1}{2}\left[\frac{\chi^{(3)}_{2222}m^{(1)}}{2\chi^{(3)}_{1221}m^{(2)}}+\frac{\chi^{(3)}_{1111}m^{(2)}}{2\chi^{(3)}_{1221}m^{(1)}}-\frac{\chi^{(3)}_{1221}+\chi^{(3)}_{1212}+\chi^{(3)}_{1122}}{\chi^{(3)}_{1221}}\right]\ . \end{aligned} \tag{18}$$

As follows from (18) the parameter D can take arbitrary values depending on the optical features of the crystal and laser light intensity.[7,27] The value of the second parameter M is difficult to estimate from experimental data available up to now. However, there are some theoretical hints[7] that in most ionic and homopolar crystals M obeys the following relations:

$$|D|\gg|M|,\quad |M|<1\ . \tag{19}$$

III. ANALYTICAL AND NUMERICAL SOLUTIONS

The function $u(l')$ describes the relative intensities of the wave described by $\hat{\mathbf{e}}^{(2)}$, and owing to the definition (14) it obeys the relation

$$0\leq u\leq 1\ . \tag{20}$$

Since Eqs. (15)–(17) may be combined to yield a single elliptic integral of the form (see Appendix)

$$\int_{u_0}^{u}\frac{du}{\{(F+Du+Mu^2)[(1-D)u-F-(M+1)u^2]\}^{1/2}}=\pm 2l'\ , \tag{21}$$

the function $u(l')$ oscillates between the two biggest roots of the equation

$$(F+Du+Mu^2)[(1-D)u-F-(M+1)u^2]=0\ . \tag{22}$$

F denotes here the second constant of motion given by the expression

$$u(1-u)\sin^2\theta-Du-Mu^2=F\ . \tag{23}$$

In order to find the solutions for $u(l')$ we first simplify and factorize the polynomial in (21). If we choose $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$ properly, we can always keep $D>0$, $M>0$. Then, after omitting terms small in the meaning of (19), we get

$$M(M+1)D\left[u+\frac{D}{M}-\frac{F}{D}\right](u-u_3)(u-u_1)(u_2-u)=0\ , \tag{24}$$

where

$$u_3=-\frac{F}{D}\ , \tag{24a}$$

$$u_2=\frac{1-D+[(1-D)^2-4F(M+1)]^{1/2}}{2(M+1)}\ , \tag{24b}$$

$$u_1=\frac{1-D-[(1-D)^2-4F(M+1)]^{1/2}}{2(M+1)}\ . \tag{24c}$$

From (20) and (23) it follows immediately that

$$u+\frac{D}{M}-\frac{F}{D}\gg 1\ . \tag{25}$$

According to (25) the term $(u+D/M-F/D)^{-1/2}$ varies slowly as compared to the other terms in (21). With the use of this fact one can evaluate the elliptic integral as

$$\int_{u_0}^{u}\frac{du}{[(u-u_3)(u-u_1)(u_2-u)]^{1/2}}=\pm 2[D(M+1)]^{1/2}l'\ . \tag{26}$$

Depending on the value of D and the boundary conditions $u_0=u(0)$ and $\theta_0=\theta(0)$, Eq. (26) has various solutions expressed in terms of elliptical functions. The form of the solutions is directly related to the sign of F.

For $F>0$, $u(l')$ oscillates between u_2 and u_1 and is given by

$$u=u_2-(u_2-u_1)\mathrm{sn}^2\left[[D(u_2-u_3)(M+1)]^{1/2}(l'+l_1),\ \left[\frac{u_2-u_1}{u_2-u_3}\right]^{1/2}\right]\ ; \tag{27}$$

for $F=0$, $u(l')$ tends asymptotically to 0,

$$u=(1-D)\mathrm{sech}^2\{[D(1-D)]^{1/2}(l'+l_2)\}\ ; \tag{28}$$

and for $F<0$, $u(l')$ oscillates between u_2 and u_3,

$$u=u_2-(u_2-u_3)\mathrm{sn}^2\left[[D(u_2-u_1)(M+1)]^{1/2}(l'+l_2),\ \left[\frac{u_2-u_3}{u_2-u_1}\right]^{1/2}\right]. \tag{29}$$

The constants l_1, l_2, l_3 are obtained from the integral (21) by integration over the range 0 to u_0.

The oscillation periods of $u(l')$ are given by

$$K_1=\tfrac{1}{2}[D(u_2-u_3)(M+1)]^{-1/2} \times \int_0^1\left[(1-x^2)\left[1-\frac{u_2-u_1}{u_2-u_3}x^2\right]\right]^{-1/2}dx \tag{30}$$

if $F>0$ and by

$$K_2=\tfrac{1}{2}[D(u_2-u_1)(M+1)]^{-1/2} \times \int_0^1\left[(1-x^2)\left[1-\frac{u_2-u_3}{u_2-u_1}x^2\right]\right]^{-1/2}dx \tag{31}$$

when $F<0$.

The variations in handedness of the ellipticity of the optical wave are also directly related to the sign of F. The expressions (19), (22), and (24a) show that the handedness changes for $u=u_3$ only. According to (25), u attains u_3 if $F<0$. In the opposite case, i.e., for $F>0$, the handedness remains constant.

The terms containing M introduce only small changes in the periods and amplitudes of $u(l')$ and have no qualitative influence on the shape of this function.

The system of equations (15)—(17) was also solved using numerical methods for various values of u_0, θ_0, D, and M (Figs. 1—4). The accuracy of these calculations was tested each time by examining the value of F. For all numerical calculations made, the maximum error was never greater than 10^{-8} of the value of F. It should be noted that, depending on the sign of F, a qualitatively different evolution of the function $\theta(l')$ occurs [Figs. 1(a), 1(c), 2, 3(b), and 4]. The results of our numerical calculations also confirm the weak influence of the parameter M on the shape of $u(l')$ and $\theta(l')$.

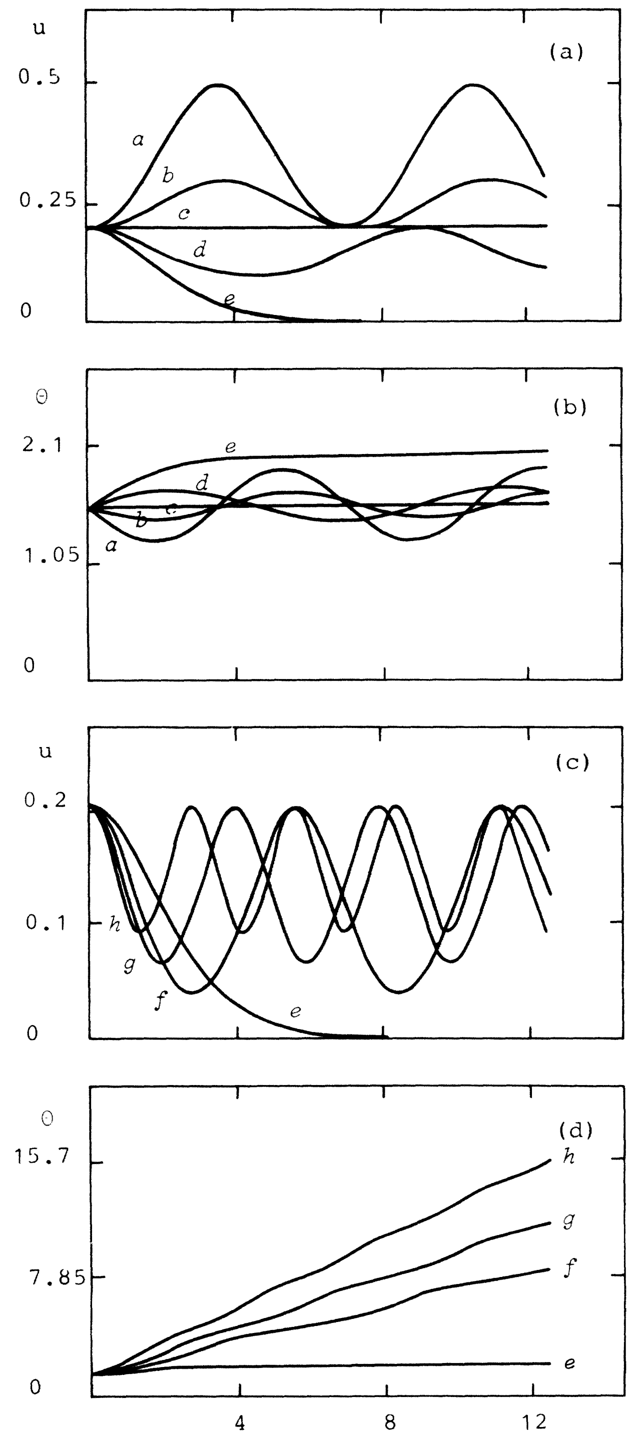

FIG. 1. Changes in the intensity of extraordinary waves, (a) and (c), and the phase difference between the ordinary and extraordinary waves, (b) and (d), for $F>0$ and $F<0$; $u_0=0.2$, $\theta_0=\pi/2$. a, $D=0.3$, $F>0$; b, $D=0.5$, $F>0$; c, $D=0.6$, $F>0$; d, $D=0.7$, $F>0$; e, $D=0.8$, $F=0$; f, $D=1.0$, $F<0$; g, $D=1.2$, $F<0$; h, $D=1.5$, $F<0$, $u_2=u_1$.

IV. EVOLUTION OF THE POLARIZATION STATE OF LIGHT

As follows from the properties of the function $u(l')$, the self-action of light causes, for $F\neq 0$, a periodic transfer of energy between the ordinary and extraordinary beams. In the crystals under consideration the self-action of light may cause, depending on the initial conditions, a qualitatively different evolution of the laser-beam polarization.

Since the terms containing M do not introduce any significant changes in the shape of $u(l')$ and $\theta(l')$, as was stated in Sec. III, in the present consideration we omit these terms.

Let a and b be the longer and shorter axes of the laser

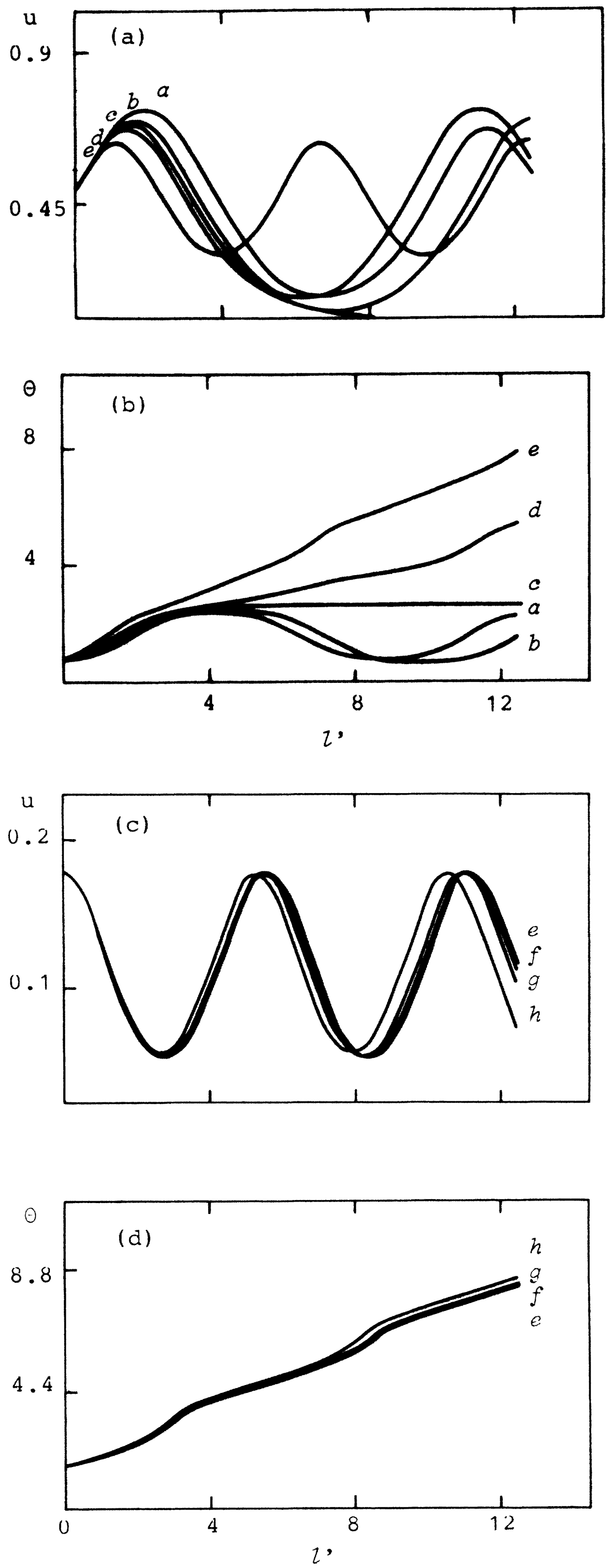

FIG. 2. Changes in the intensity of extraordinary waves, (a) and (c), and the phase difference between the ordinary and extraordinary waves, (b) and (d), for $u_0=0.2$ and $\theta_0=\pi/2$ as functions of M. a, $D=0.3$, $M=0$; b, $D=0.3$, $M=0.003$; c, $D=0.3$, $M=0.009$; d, $D=0.3$, $M=0.03$; e, $D=1$, $M=0$; f, $D=1$, $M=0.01$; g, $D=1$, $M=0.03$; h, $D=1$, $M=0.1$.

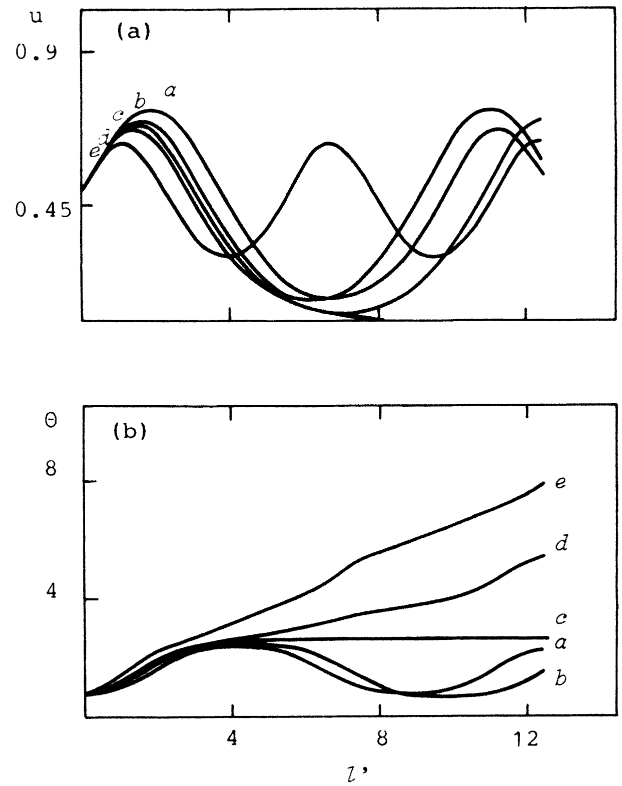

FIG. 3. Changes in the intensity of the extraordinary waves, (a), and the phase difference between the ordinary and extraordinary waves (b), for $F>0$ and $F<0$; $u_0=0.5$, $\theta_0=\pi/4$. a, $D=0.1$, $F>0$; b, $D=0.2$, $F>0$; c, $D=0.25$, $F=0$; d, $D=0.3$, $F<0$; e, $D=0.5$, $F<0$.

light ellipse, respectively. Further, let α be the angle between a and the axis perpendicular to the optical axis of the crystal, measured counterclockwise. Using (19) and (24a)–(24c) we can write

$$a^2=\tfrac{1}{2}C_0[1+(1-4F-4Du)^{1/2}]\ , \tag{32}$$

$$b^2=\tfrac{1}{2}C_0[1-(1-4F-4Du)^{1/2}]\ , \tag{33}$$

$$\tan(2\alpha)=\frac{-[(u-u_1)(u_2-u)]^{1/2}}{u-\frac{1}{2}}\ , \tag{34}$$

and

$$\frac{d\alpha}{dl'}=\frac{d\alpha_1}{dl'}+\frac{d\alpha_2}{dl'}$$
$$=\frac{\sqrt{us}\sin\theta}{1-4F-4Du}[1-4us\sin^2\theta+2D(u-\tfrac{1}{2})]\ . \tag{35}$$

Here, the term

$$\frac{d\alpha_1}{dl'}=\frac{\sqrt{us}\sin\theta}{1-4F-4Du}(1-4us\sin^2\theta) \tag{36}$$

describes the changes in inclination due to self-action of light and

$$\frac{d\alpha_2}{dl'}=\frac{2D\sqrt{us}\sin\theta}{1-4F-4Du}(u-\tfrac{1}{2}) \tag{37}$$

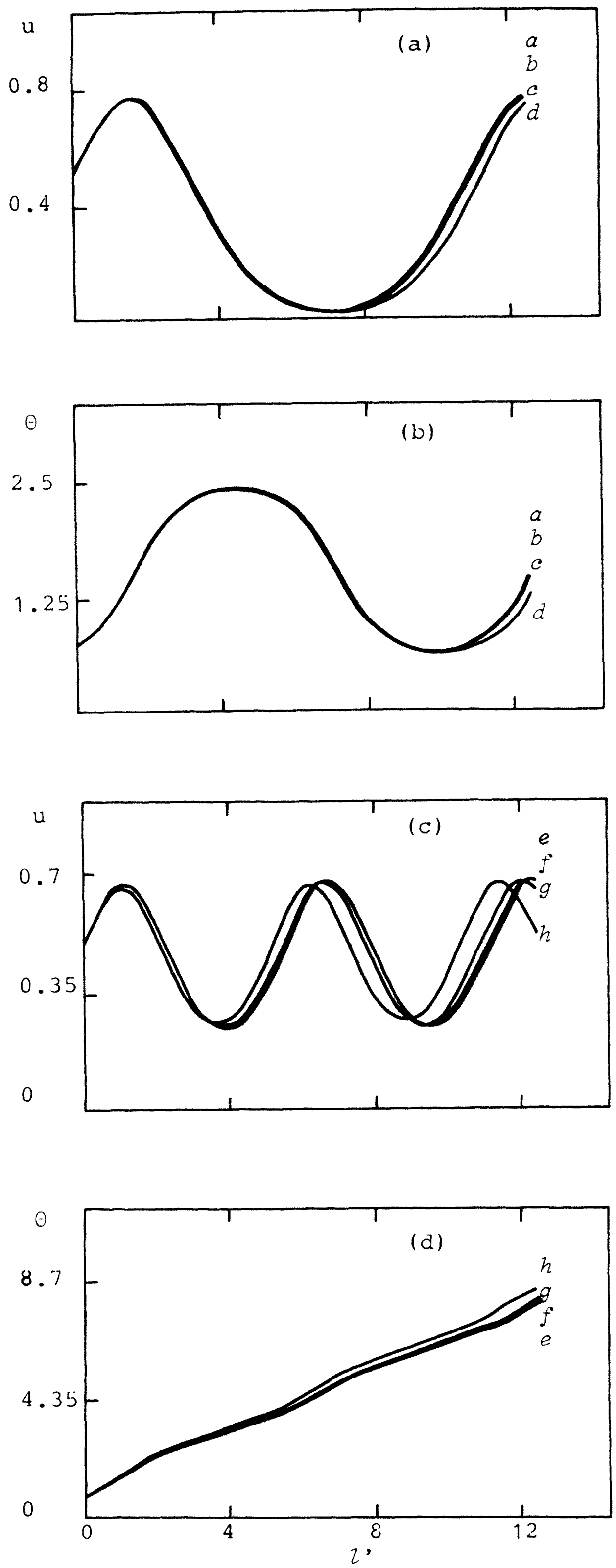

FIG. 4. Changes in the intensity of extraordinary waves, (a) and (c), and the phase difference between the ordinary and extraordinary waves, (b) and (d), for $u_0=0.5$ and $\theta_0=\pi/4$ as functions of M. a, $D=0.2$, $M=0$; b, $D=0.2$, $M=0.002$; c, $D=0.2$, $M=0.006$; d, $D=0.2$, $M=0.02$; e, $D=0.5$, $M=0$; f, $D=0.5$, $M=0.005$; g, $D=0.5$, $M=0.015$; h, $D=0.5$, $M=0.05$.

gives the influence of linear birefringence of the medium. As follows from expressions (35)–(37), the angle of inclination may decrease or increase depending on the handedness of light ellipticity and on the contributions of the mechanisms described by (36) and (37). The behavior of the polarization ellipse may be discussed in terms of a parameter B defined by the initial conditions

$$B=1-4u_0(1-u_0)\sin^2\theta_0+2D(2u_0-1)\ . \tag{38}$$

If $B>0$, the term in parentheses in (35) is always positive, causing the decrease or increase in inclination to be controlled by the handedness of ellipticity only. In the opposite case, i.e., at $B<0$, the contribution of linear birefringence dominates, and for an arbitrary value of F the maximum angle of inclination is given by

$$|\alpha_m|=\tfrac{1}{2}\arctan\left[\frac{\sqrt{(u_3-u_1)(u_2-u_3)}}{\tfrac{1}{2}-u_3}\right]. \tag{39}$$

Our detailed studies of the behavior of the polarization ellipse have shown that the following cases occur.

(1) $F>0$. The handedness of the optical wave remains constant during propagation.

(1a) $B>0$. The polarization ellipse rotates at a varying angular rate [Fig. 5(a)].

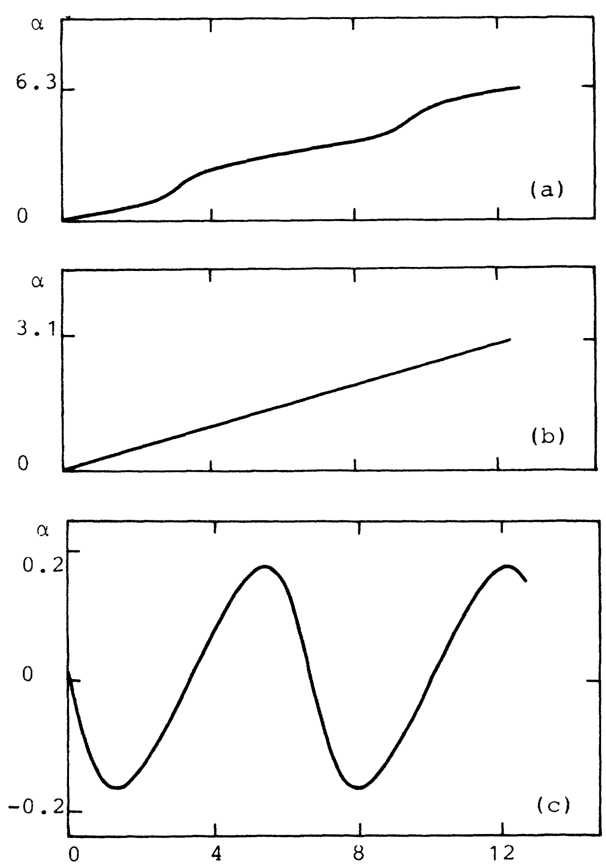

FIG. 5. Angle of inclination as a function of propagation length if the handedness of the optical wave remains constant ($F>0$): (a) $B>0$, (b) $B=0$, (c) $B<0$.

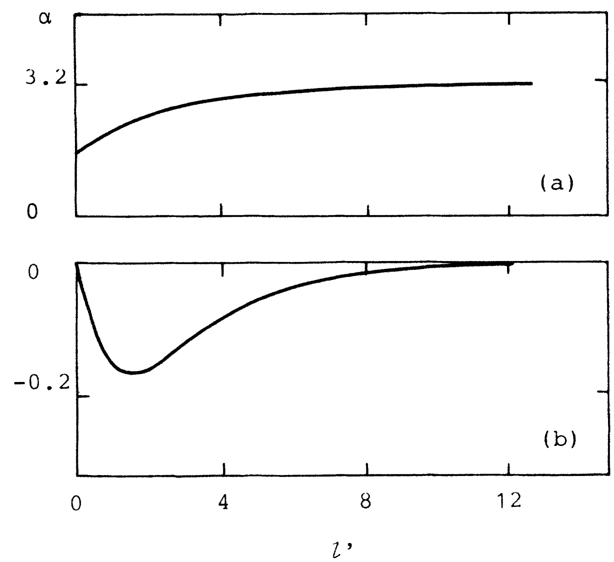

FIG. 6. Angle of inclination as a function of propagation length if the polarization of the optical wave tends to linear polarization ($F=0$): (a) $B>0$, (b) $B<0$.

(1b) $B=0$. The polarization ellipse rotates at an almost constant angular rate [Fig. 5(b)].

(1c) $B<0$. The polarization ellipse oscillates [Fig. 5(c)].

(2) $F=0$. The elliptically polarized wave becomes linearly polarized. The final polarization is perpendicular to the optical axis of the crystal.

(2a) $B>0$. The function $\alpha(l')$ changes monotonically [Fig. 6(a)].

(2b) $B<0$. The function $\alpha(l')$ tends to 0 [Fig. 6(b)].

(3) $F<0$. The handedness of ellipticity of the optical wave changes periodically in space.

(3a) $B>0$. The polarization ellipse oscillates around the optical axis of the crystal [Fig. 7(a)].

(3b) $B=0$. The polarization ellipse oscillates around an axis inclined by an angle $\pi/4$ to the optical axis of the crystal [Fig. 7(b)].

(3c) $-1<B<0$. The ellipse makes a double oscillation around an axis perpendicular to the optical axis of the crystal [Fig. 7(c)].

(3d) $B\leq-1$. The ellipse makes simple oscillations around an axis perpendicular to the optical axis of the crystal [Fig. 7(d)].

V. DISCUSSION

It follows from our analytical and numerical solutions of the Maxwell nonlinear wave equation that the coupling of the polarization changes which result from linear birefringence with those resulting from SIER leads to modulation of the well-known effects such as the light ellipse rotations[1–3] and it leads to new phenomena such as double oscillations, single oscillations controlled by the handedness of ellipticity, and the linearization of the polarization of the laser beam. Simple rotation of the polarization ellipse and the linear birefringence effect are asymptotic cases of the presented model, which correspond to $D\rightarrow0$ and $D\rightarrow\infty$, respectively. According to these asymptotic behaviors a separation in linear-birefringence-dominated ($F<0$) and SIER-dominated

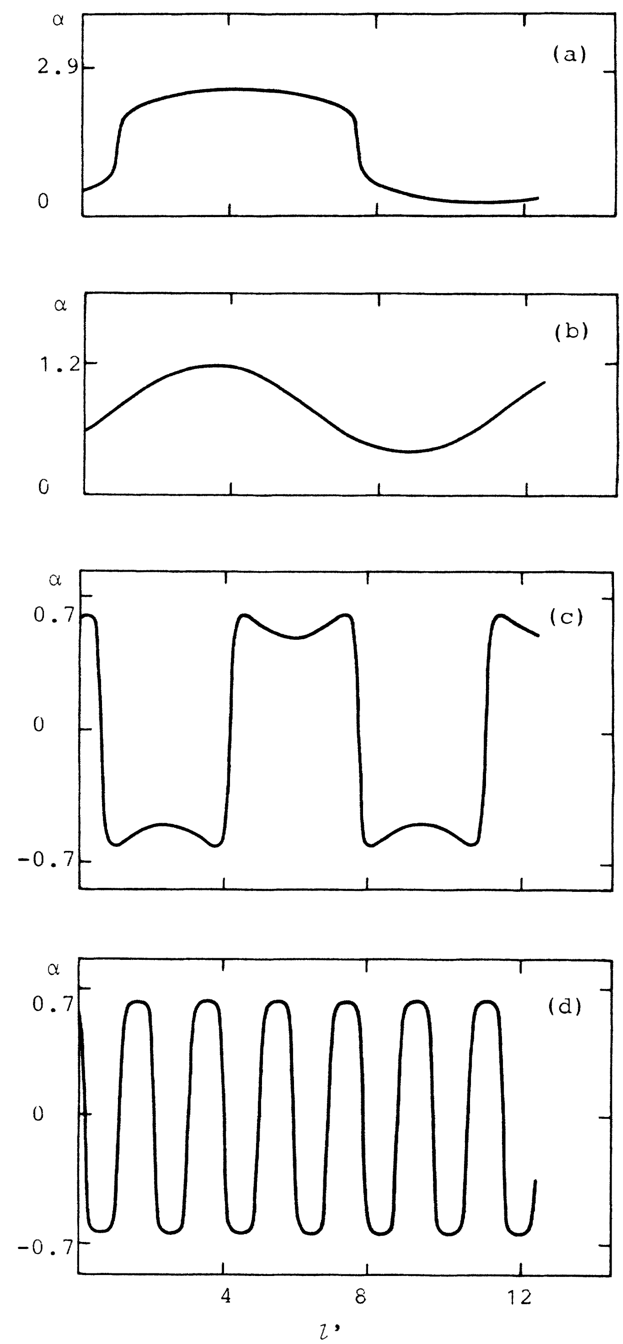

FIG. 7. Angle of inclination as a function of propagation length if the handedness of the optical wave changes periodically in space ($F<0$): (a) $B>0$, (b) $B=0$, (c) $-1<B<0$, (d) $B<-1$.

($F>0$) cases can be made. In these two cases a qualitatively different behavior of the function $\theta(l')$ occurs. In the first case $\theta(l')$ changes monotonically, while in the second case it oscillates with constant period [Figs. 1(b), 1(d), and 3(b)].

The results of the calculations proved that the changes of the light polarization in uniaxial crystals cannot be described as a trivial superposition of the asymptotic cases. Such a superposition leads to variations of inclination angle described by a function given by

$$\alpha=\sin\theta_0(us)^{1/2}l' + \frac{1}{2}\arctan\left[\frac{2A_0^{(1)}A_0^{(2)}}{(A_0^{(1)})^2-(A_0^{(2)})^2}\cos[2\pi(k^{(1)}-k^{(2)})l]\right] \quad (40)$$

which does not agree qualitatively with our results.

Although the assumption of a lossless medium and the application of the plane-wave approximation involve a limitation of the light power below the appropriate thresholds for self-focusing, Brillouin and Raman stimulated scatterings, the predicted changes in polarization may be observed by means of experimental setups often used in previous SIER studies.[2–10,12,13]

Let us consider the propagation of a linearly polarized laser beam of $\omega_L=2.7\times10^{15}$ s^{-1} (698 nm) in sapphire. In this case $\chi^{(3)}_{1221}\approx10^{-13}$ esu (Ref. 7) and $(k^{(2)}-k^{(1)})/k^{(1)}=4\times10^{-3}$. If we take $D=0.5$, the power density of the laser beam as 500 MW/cm^2, and the relative intensities of the ordinary and extraordinary waves as being equal, then the period of the function u is approximately equal to 3 cm and its amplitude exceeds 0.33. Such intensity changes can be measured in the Rivoire *et al.* experimental system[13] with an accuracy of several percent. Knowing $u(l')$, the phase difference can be measured directly by placing the crystal between crossed polarizers.

As follows from our numerical calculations, we can choose the initial values of u_0 and θ_0 in such a manner that the particular cases for $F>0$ or $F<0$ can be obtained by changing the value of D only. This is easy to obtain in the experiment by varying the initial power density of the laser beam.

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APPENDIX

In order to deduce the elliptical integral (21), let us first consider Eqs. (15) and (16) after the variable $s=1-u$ is eliminated:

$$\frac{du}{dl'}=(1-u)u\sin(2\theta)\,, \quad (A1)$$

$$\frac{d\theta}{dl'}=(2u-1)\sin^2\theta+D+2Mu\,. \quad (A2)$$

Elimination of the differential dl' from Eqs. (A1) and (A2) leads to the following equation:

$$[(2u-1)\sin^2\theta+D+2Mu]du-(1-u)u\sin(2\theta)d\theta=0\,. \quad (A3)$$

Differentiating the first term in regard to θ and the second term in regard to u, we find it to be the exact differential with the integral given by

$$u(1-u)\sin^2\theta-Du-Mu^2=F\,. \quad (A4)$$

Obtaining from (A4) the expression of $\sin^2\theta$, we eliminate $\sin(2\theta)$ from (A1). Then we get

$$\frac{du}{dl'}=\pm2\{(F+Du+Mu^2)\times[(1-D)u-F-(M+1)u^2]\}^{1/2}\,. \quad (A5)$$

The simple separation of the variables gives us the elliptical integral in the form (21).

[1]P. D. Maker, R. W. Terhune, and C. M. Savage, Phys. Rev. Lett. 12, 507 (1964).

[2]P. D. Maker and R. W. Terhune, Phys. Rev. 137, A801 (1965).

[3]C. Wang, Phys. Rev. 152, 149 (1966).

[4]R. W. Hellwarth, J. Cherlow, and T. T. Yang, Phys. Rev. B 11, 964 (1975).

[5]A. Owyoung, IEEE J. Quantum Electron. QE-9, 1064 (1973).

[6]A. Owyoung, R. W. Hellwarth, and N. George, Phys. Rev. B 5, 628 (1972).

[7]R. W. Hellwarth, Prog. Quantum Electron. 5, 1 (1977).

[8]L. Dahlström, Opt. Commun. 4, 214 (1971).

[9]J. M. Thorne, T. R. Loree, and G. H. McCall, Appl. Phys. Lett. 22, 259 (1973); J. Appl. Phys. 45, 3072 (1974).

[10]R. Hellwarth, J. Cherlow, and T. T. Yang, Phys. Rev. B 11, 964 (1975).

[11]G. B. Altshuller, V. S. Ermolaev, V. B. Karasev, S. A. Kozlov, K. I. Krylov, and L. I. Pavlov, Appl. Phys. B32, 97 (1983).

[12]Ng. Phu-Xuan, J. L. Ferrier, J. Gasengel, and G. Rivoire, Opt. Commun. 46, 329 (1983).

[13]Ng. Phu-Xuan and G. Rivoire, Opt. Acta 25, 233 (1978).

[14]W. Gadomski and M. Roman, Acta Phys. Pol. A55, 713 (1979).

[15]W. Gadomski and M. Roman, Opt. Commun. 33, 331 (1980). We have been alerted to the letter by E. G. Winful in Appl. Phys. Lett. 47, 213 (1985) which presents another approach to the case being discussed in the paper cited above.

[16]M. Roman and M. Gadomski, in Abstracts of Winter College on Lasers, Atomic and Molecular Physics, Trieste, 1981 (unpublished).

[17]M. Roman, thesis, Warszawa-Poznań, 1982.

[18]K. L. Sala, Phys. Rev. A 29, 1944 (1984).

[19]S. Kielich and R. Zawodny, Opt. Commun. 15, 267 (1975).

[20]S. Kielich and R. Zawodny, Physica 89B, 122 (1977).

[21]S. Kielich, Proceedings of the VIIth Conference Quantum Electronics and Nonlinear Optics, EKON-76, 1 (VAM,

Poznań, 1977), in Polish.
[22]S. A. Akhmanov, G. A. Lyakhov, V. A. Makarov, and V. I. Zharikov, Opt. Acta **29**, 1359 (1982).
[23]A. D. Petrenko and N. I. Zheludev, Opt. Acta **31**, 1177 (1984).
[24]V. M. Agranovich and V. L. Ginzburg, *Spatial Dispersion in Crystal Optics and Theory of Excitons* (Wiley, New York, 1965).
[25]J. H. Marburger, Prog. Quantum Electron. **4**, 35 (1975).
[26]N. Bloembergen, *Nonlinear Optics* (Benjamin, New York, 1965).
[27]M. Thalhammer and A. Peuzkofer, Appl. Phys. **B32**, 137 (1983).