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Squeezed states in N -atom time-dependent resonance fluorescence (*)

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Résumé. — On discute de la dépendance de la compression des fluctuations du champ en fonction du nombre d'atomes dans le cas de la fluorescence de résonance collective dépendant du temps, lorsque N -atomes à deux niveaux sont soumis de manière cohérente à un champ laser résonnant et intense. On montre que, dans le cas du champ intense, la compression n'apparaît que dans le régime transitoire de la fluorescence de résonance. La localisation temporelle de la compression ne dépend ni du nombre d'atomes, ni de la population atomique initiale. On trouve que, bien que la compression des fluctuations diminue pour chaque atome pris séparément, la compression augmente pour le champ de fluorescence total lorsque N croît et atteint sa valeur maximale pour des valeurs modérées de N . De plus, la valeur maximale de la compression dépend de l'intensité du champ incident : lorsque cette dernière augmente, la compression maximale augmente et est obtenue pour un nombre plus grand d'atomes. A la limite des très grands N , cependant, la compression est complètement supprimée.

Abstract. — The dependence of squeezing on the number of atoms is discussed in the time-dependent collective resonance fluorescence of N two-level atoms, coherently driven by a strong resonant laser field. It is shown that, for strong driving field, squeezing appears only in the transient regime of resonance fluorescence. The location of squeezing in time is independent of the number of atoms and the initial atomic populations. It is found that despite the reduction in the amount of squeezing per single atom, squeezing increases in the whole resonance fluorescence field as the number of atoms increases, and attains its maximal value for moderate N . Moreover, the maximal value of squeezing is dependent on the intensity of the driving field. As the latter increases, the maximum of squeezing increases and shifts towards larger numbers of atoms. For a very large number of atoms, however, squeezing is altogether removed.

1. Introduction.

The generation of squeezed states has recently attracted considerable attention in quantum optics research. This effect consists in the reduction of fluctuations in one of the two quadrature components (the in-phase and the out-of-phase components) of the electric field below the minimum reached for a coherent state, at the expense of increased fluctuations in the other component. Squeezed states have a negative normally ordered variance of one of the two field components, meaning that they have no classical analog in the sense

that their diagonal coherent-state representation cannot be non-negative [1]. Various methods of producing squeezed states have been shown to be theoretically possible. Several review articles [2, 3] covering the subject of squeezing are now available and extensive references on squeezed states are to be found therein. However, so far experimental detection of reduced fluctuations has been performed for the process of four-wave mixing [4, 5] and parametric down conversion [6].

Particular attention has been paid to resonance fluorescence from a single atom, coherently driven by an external field. In particular, the existence of squeezing along with other quantum mechanical features, such as photon antibunching [7-8], sub-Poissonian photon statistics [9], correlation between

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the photons emitted in the two sidebands of the fluorescent triplet [10], has been demonstrated.

Walls and Zoller [11] have shown that either of the fluorescent field components can become squeezed under certain conditions in steady state if the Rabi frequency Ω of the exciting field is sufficiently low. For time dependent resonance fluorescence, however, as it has been shown in [12], squeezing occurs for an arbitrary intensity of the exciting field. With increasing intensity of the exciting field, squeezing shifts towards shorter times and its maximal value increases to become, for very strong exciting fields, twice greater than its maximal value in the steady state. Arnoldus and Nienhuis [13] have considered the conditions for squeezing in steady state resonance fluorescence showing that these conditions for squeezing turn out to be equivalent to the requirement that the intensity of the coherent Rayleigh line shall be more than half the total intensity of the fluorescent field.

The possibilities of detection of squeezed states in resonance fluorescence have been considered by Mandel [14]. In particular, he has shown that the use of a one-port homodyne detection scheme to detect squeezing in steady state resonance fluorescence always leads to sub-Poissonian photon statistics if the detected component is squeezed. Homodyne and heterodyne schemes of detection of squeezed light have been discussed by Heidmann *et al.* [15].

Loudon [16] considered the two-time photon-number correlation function for fluorescent light homodyned with coherent light, showing that squeezing is removed at simultaneous observation of fluorescent light from many randomly distributed atoms.

The influence of multiatom effects on squeezing in resonance fluorescence has been the subject of several papers [17-20]. Furthermore, Vogel and Welsch [21] have shown that in resonance fluorescence from N independent atoms, distributed regularly in a linear chain, squeezing is N times stronger than for a single atom. Heidmann and Reynaud [22] considered resonance fluorescence radiated in the forward direction by an ensemble of N independent atoms showing that squeezing is not modified by multiatom effects. In all the above cited papers the atoms are assumed to radiate independently. In particular, it is desirable to know how the cooperative effects affect squeezing in resonance fluorescence.

Bogolubov *et al.* [23] considered strong-field cooperative resonance fluorescence and have shown that squeezing is present in the mixture of the two spectrum sidebands but is absent for the whole field of resonance fluorescence. Lakshmi and Agarwal [24] have shown that cooperative effects reduce the magnitude of squeezing from that obtained for the single atom. For a large number of atoms, or

strong external laser fields, there is no squeezing in steady state resonance fluorescence.

In the present paper we consider the possibility of obtaining squeezed states in the transient regime of cooperative N -atom resonance fluorescence. We show that despite the reduction of squeezing per single atom in the transient regime, large squeezing is present in the whole field of resonance fluorescence if the Rabi frequency of the exciting field is large. As the number of atoms increases, squeezing increases, reaches its maximal value, and then decreases for large numbers of atoms.

2. Strong field resonance fluorescence.

To describe resonance fluorescence of a cooperative system of N identical two-level atoms, we start with the master equation [25]

$$\frac{\partial \rho}{\partial t} = \frac{1}{2} i \Omega [S^+ + S^-, \rho] - \gamma (S^+ S^- \rho + \rho S^+ S^- - 2 S^- \rho S^+), \quad (1)$$

where Ω is the Rabi frequency and 2γ the Einstein A coefficient. The operators S^+ and S^- are the usual collective atomic dipole operators, satisfying the well-known commutation relations:

$$[S^+, S^-] = 2 S^z, \quad [S^z, S^\pm] = \pm S^\pm. \quad (2)$$

For simplicity, the laser frequency ω_L is assumed to be exactly equal to the atomic transition frequency ω_0 , i.e. detuning is zero. Moreover, the phase of the laser field is chosen as zero. It is well known that squeezing is sensitive to phase [2, 12], but the maximal value is reached for the particular choice of phase $\varphi = 0$ or $\pi/2$ [14, 16]. The influence of cooperative effects on this maximal value of squeezing is the subject of our calculations.

For high field strengths, $\Omega \gg N\gamma$, an approximation technique has been suggested by Agarwal *et al.* [26] and Kilin [27], which greatly simplified the master equation (1). A transformation is made to the new collective operators R^\pm , R^z by

$$\begin{aligned} S^\pm &= \pm \frac{1}{2} i (R^+ + R^-) + R^z, \\ S^z &= -\frac{1}{2} i (R^+ - R^-). \end{aligned} \quad (3)$$

The R operators are a rotation of the S operators and thus obey the angular momentum commutation relations (2). For strong fields, the R^\pm vary with time approximately as $\exp(\pm i\Omega t)$, while R^z is slowly varying. Substituting S^\pm and S^z from equations (3) into the master equation (1) and dropping rapidly oscillating terms, such as $R^\pm R^z$, $R^+ R^+$, etc., leads to the approximate master equation:

$$\frac{\partial \rho}{\partial t} = i\Omega [R^z, \rho] - \gamma \{ (R^z R^z \rho + \rho R^z R^z - 2 R^z \rho R^z) + \frac{1}{4} [(R^+ R^- \rho + \rho R^+ R^- - 2 R^- \rho R^+) + (R^- R^+ \rho + \rho R^- R^+ - 2 R^+ \rho R^-)] \}. \quad (4)$$

The equation of motion for the expectation value of an arbitrary (fixed) operator Q follows from (4) :

$$\langle \dot{Q} \rangle = i\Omega \langle [Q, R^z] \rangle - \gamma \{ \langle [[Q, R^z], R^z] \rangle + \frac{1}{4} (\langle [[Q, R^+], R^-] \rangle + \langle [[Q, R^-], R^+] \rangle) \}. \quad (5)$$

Equation (5), for an arbitrary combination Q of atomic operators, can be solved exactly. The solutions for atomic operators will be used in our calculations of squeezing in a system of N atoms coherently driven by a strong resonant laser field.

3. Squeezing in transient regime.

Defining the Hermitian atomic operators

$$S_1 = \frac{1}{2} (S^+ + S^-), \quad (6)$$

$$S_2 = -\frac{1}{2} i (S^+ - S^-),$$

with the commutation relations (2), we obtain the following commutation rule

$$[S_1, S_2] = iS_3. \quad (7)$$

Squeezed states are defined by the condition that [2, 11] :

$$\langle (\Delta S_{1,2})^2 \rangle < \frac{1}{2} | \langle S_3 \rangle | \quad (8)$$

or

$$\langle (\Delta S_{1,2})^2 \rangle - \frac{1}{2} | \langle S_3 \rangle | < 0. \quad (9)$$

The expectation values for the atomic operators occurring in (9) are readily obtained from the equation of motion (5). The solutions for the variances of the atomic operators are

$$\langle (\Delta S_1(t))^2 \rangle = \frac{N(N+2)}{12} - \frac{N(N-1)}{12} e^{-3\gamma t}, \quad (10)$$

$$\langle (\Delta S_2(t))^2 \rangle = \frac{N(N+2)}{12} + \frac{N(N-1)}{24} e^{-3\gamma t} - \frac{N(N-1)}{8} e^{-5\gamma t} \cos 2\Omega t - \frac{1}{4} N^2 e^{-3\gamma t} \sin^2 \Omega t, \quad (11)$$

and for the expectation value of atomic energy

$$\langle S_3(t) \rangle = \langle S_3(0) \rangle e^{-\frac{3}{2}\gamma t} \cos \Omega t. \quad (12)$$

We note from (10) and (11) that the variances of the atomic operators are independent of the initial population of the atomic states, whereas the expectation value of atomic energy (12) depends on the initial population. If the atoms are initially in their

ground (excited) states, then $\langle S_3(0) \rangle = -\frac{N}{2} \left(\langle S_3(0) \rangle = \frac{N}{2} \right)$. Since the quantity on the left of the inequality (9) depends only on $| \langle S_3(t) \rangle |$, it has the same value for initial conditions where the atoms are either all in their ground states or all in their excited states.

By equations (9)-(12) we find for the fluctuations in atomic variables :

$$F_1(t) = \langle (\Delta S_1(t))^2 \rangle - \frac{1}{2} | \langle S_3(t) \rangle | = \frac{N(N+2)}{12} - \frac{N(N-1)}{12} e^{-3\gamma t} - \frac{1}{4} N e^{-\frac{3}{2}\gamma t} | \cos \Omega t |, \quad (13)$$

$$F_2(t) = \langle (\Delta S_2(t))^2 \rangle - \frac{1}{2} | \langle S_3(t) \rangle | = \frac{N(N+2)}{12} + \frac{N(N-1)}{24} e^{-3\gamma t} - \frac{N(N-1)}{8} e^{-5\gamma t} \cos 2\Omega t - \frac{1}{4} N^2 e^{-3\gamma t} \sin^2 \Omega t - \frac{1}{4} N e^{-\frac{3}{2}\gamma t} | \cos \Omega t |. \quad (14)$$

Our solutions (13) and (14) describe the time-evolution of the fluctuations in the atomic operators of cooperative N -atom resonance fluorescence for strong external resonant laser field. Equations (13)-(14) are valid for arbitrary initial conditions. For $N = 1$, they are similar to the equations obtained previously in [12]. In the steady state ($t \rightarrow \infty$), the values of fluctuations in F_1 and F_2 are positive and equal ($F_1 = F_2 = N(N + 2)/12$), and the same as those obtained by Lakshmi and Agarwal [24]. It is clear from equation (14) that in addition to the frequency components at $\omega_L \pm \Omega$, the fluctuations F_2 contain moreover components at $\omega_L \pm 2\Omega$ which are absent in one-atom resonance fluorescence. The existence of these additional frequencies has recently been discussed in connection with the appearance of additional sidebands in the cooperative resonance fluorescence spectrum [28-31]. The F_1 -component contains, by comparison with one-atom resonance fluorescence, an additional contribution from the central line of the resonance fluorescence spectrum. Numerical analyses of equation (13) show that F_1 is always positive, independently of N and the time t . Squeezing occurs only in the F_2 -component.

Firstly, we discuss the effect of cooperativity on the fluctuations in single-atom fluorescence at coupling with the remaining $(N - 1)$ atoms. These fluctuations are described by $F_1(t)/N$ and $F_2(t)/N$. The dependence of the fluctuation F_2/N on the number of atoms is illustrated in figure 1, where F_2/N is plotted for $\Omega = 200\gamma$ and different numbers of atoms. Figure 1 shows that the amount of squeezing per atom goes down as the number of atoms increases. So, the effect of cooperativity reduces the magnitude of squeezing per atom.

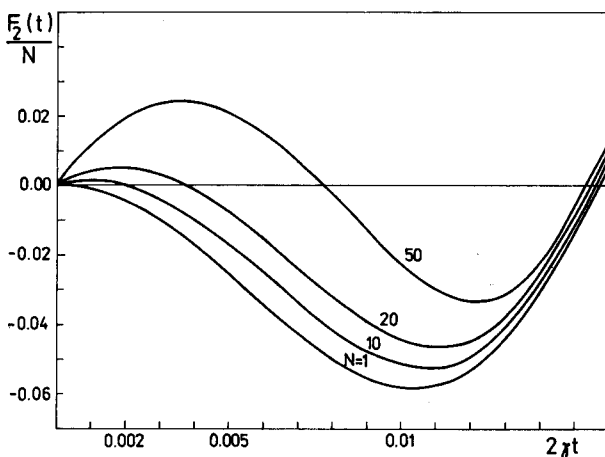


Fig. 1. — Time-dependence of $F_2(t)/N$ for $\Omega = 200\gamma$ and different numbers of atoms N .

The situation is quite different if we take the whole resonance fluorescence field of N atoms. The fluorescent field can be, in the far-field limit, ex-

pressed in terms of the atomic operators S^\pm by using the following relation [25]:

$$E^{(+)}(r, t) = E_0^{(+)}(r, t) + \frac{1}{2} \psi(r) S^- \left(t - \frac{r}{c} \right), \quad (15)$$

with $\psi^2(r) = (3 \hbar \omega_0 \gamma / 2 cr^2) \sin^2 \theta$, where θ is the angle between the observation direction r and the atomic transition dipole moment μ , and $S^-(t)$ is the collective atomic operator, which together with the operator $S^+(t)$ obeys the commutation relation (2). Since the normally ordered variance of the fluorescent field is given by:

$$\langle : (\Delta E_{1,2}(t))^2 : \rangle = \psi^2(r) [F_{1,2}(t) + \frac{1}{2} | \langle S_3(t) \rangle | (1 + \text{sgn} \langle S_3(t) \rangle)], \quad (16)$$

where $\text{sgn} \langle S_3(t) \rangle = -1$ for $\langle S_3(t) \rangle < 0$ whereas $\text{sgn} \langle S_3(t) \rangle = 1$ for $\langle S_3(t) \rangle > 0$. It is obvious from (16) that the normally ordered variance of the fluorescent field depends on the fluctuations in the atomic operators. For $\langle S_3(t) \rangle < 0$ squeezing in the atomic variables ($F_{1,2}(t) < 0$) leads to the negative value of the normally ordered variance of the fluorescent field, whereas for $\langle S_3(t) \rangle > 0$ we get always positive $\langle : (\Delta E_{1,2}(t))^2 : \rangle$ despite the presence of squeezing in the atomic variables. As it has been shown by Mandel [14], the negative value of the normally ordered variance of the fluorescent field always leads to sub-Poissonian photon statistics of the fluorescent field homodyned with a local oscillator, assumed to be in coherent state with amplitude greatly exceeding the amplitude of the fluorescent field. From (16) we find that for time intervals for which $\langle S_3(t) \rangle > 0$ the presence of squeezing in the atomic variables does not lead to the sub-Poissonian photons statistics. For time intervals for which $\langle S_3(t) \rangle < 0$, squeezing in $F_2(t)$ does lead to sub-Poissonian photon statistics.

The time-dependence of fluctuations in the whole resonance fluorescence field is shown in figure 2, where $F_2(t)$ is plotted for Rabi frequency $\Omega = 200\gamma$ and various numbers of atoms N . The atoms start at $t = 0$ from their ground state. We restrict ourselves to time intervals in which $F_2(t)$ has its first minimum. In these intervals of time $\langle S_3(t) \rangle < 0$, and squeezing in $F_2(t)$ leads to negative $\langle : (\Delta E_2(t))^2 : \rangle$. It is obvious from figure 2 that, as the number of atoms increases, so does the minimum in F_2 (the maximum of squeezing). Moreover, the position of this minimum in time is independent of the number of atoms. For a great number of atoms, F_2 is positive for short times and, after reaching a peak, goes down to take negative values, and shows an oscillatory behaviour reflecting the Rabi oscillations. The first minimum of F_2 is reached for $\Omega t = \pi/3$ i.e. as the intensity of the

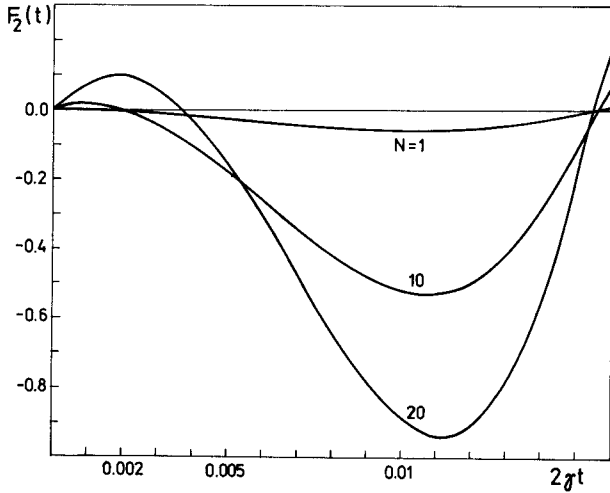


Fig. 2. — Time-dependence of

$$F_2(t) = \langle (\Delta E_2)^2 \rangle / \psi^2(r)$$

for $\Omega = 200 \gamma$ and different numbers of atoms. Initially all atoms were in their ground state.

laser field increases, squeezing in F_2 shifts to the region of shorter times. The dependence of the first minimum of F_2 on the number of atoms and the laser intensity is shown in figure 3. It is clear that squeezing in the whole resonance fluorescence field increases as the number of atoms increases, and attains a maximum for moderate N . For large N , however, squeezing decreases and is removed for very large numbers of atoms. As the strength of the driving field increases, the maximum of squeezing shifts to the region of larger N , and its value increases.

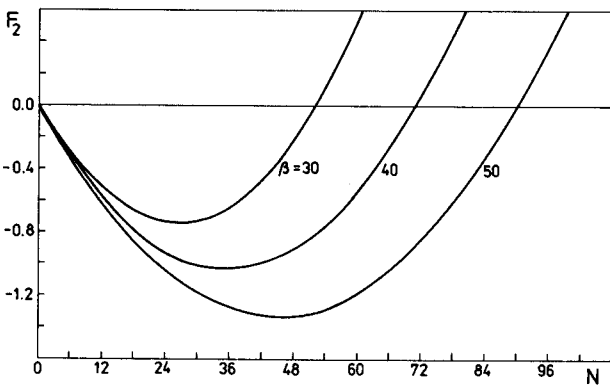


Fig. 3. — Dependence of the minimum value of $F_2(t)$ on the number of atoms, for different strengths of the exciting field ($\beta = \Omega / 4 \gamma$).

The increase in squeezing with the increase in number of atoms, in spite of the decrease in squeezing calculated per atom, results from the fact that with increasing N the energy of the atoms increases

faster than the variance $\langle (\Delta S_2(t))^2 \rangle$. This is shown in figure 4, where we have plotted $\frac{1}{2} |\langle S_3 \rangle|$ and $\langle (\Delta S_2)^2 \rangle$ versus N for $\Omega = 200 \gamma$ and $t = 0.0105$. For large N the variance $\langle (\Delta S_2)^2 \rangle$ increases faster than $\frac{1}{2} |\langle S_3 \rangle|$ because collective effects become important and in consequence squeezing decreases and is removed altogether for large N .

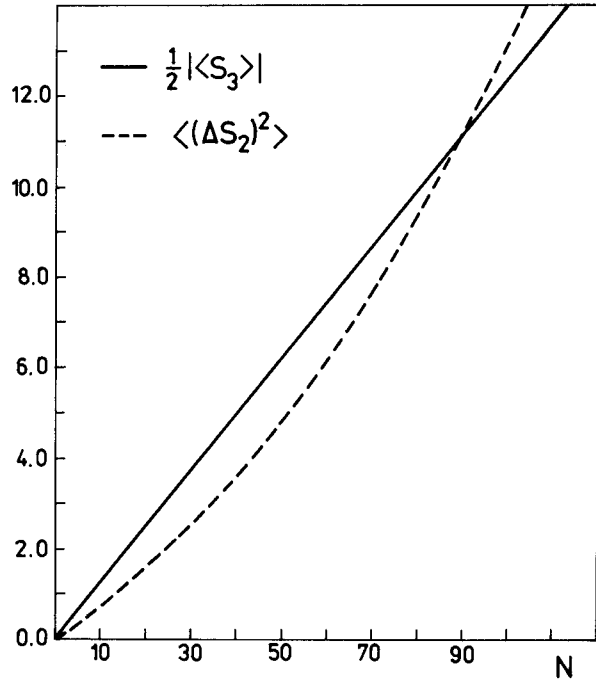


Fig. 4. — Dependence of one half the absolute value of the total energy $\langle S_3(t) \rangle$ and the variance $\langle (\Delta S_2)^2 \rangle$ on the number of atoms N , for $\Omega = 200 \gamma$ and $t = 0.0105$.

4. Conclusions.

We have studied the problem of the occurrence of squeezing in cooperative time-dependent resonance fluorescence from N atoms, distributed in a region much smaller than the resonant wavelength and coherently driven by a strong resonant laser field. The secular approximation proposed by Agarwal *et al.* [26] and Kilin [27] has been used to obtain analytical solutions for the atomic operators.

The dependence of the quantities

$$F_1(t) = \langle (\Delta S_1(t))^2 \rangle - \frac{1}{2} |\langle S_3(t) \rangle|$$

and

$$F_2(t) = \langle (\Delta S_2(t))^2 \rangle - \frac{1}{2} |\langle S_3(t) \rangle|$$

on the time and the number of atoms has been calculated. These quantities, if negative, signify squeezing in the atomic variables. Our results show

that in the steady state ($t \rightarrow \infty$) resonance fluorescence squeezing is not present (in the steady state $F_1 = F_2 = N(N+2)/12$), and appears in time-dependent resonance fluorescence, albeit only in one component F_2 . In time-dependent resonance fluorescence calculated per atom, squeezing is reduced as the number of atoms increases. This is in agreement with the conclusion of Lakshmi and Agarwal [24] that cooperative effects reduce squeezing in resonance fluorescence. The situation is quite different if we take the whole resonance fluorescence field. In this case squeezing increases for a small number of atoms, and reaches its maximal value for moderate N . As the Rabi frequency of the driving field increases, the maximum of squeezing (minimum of F_2) shifts towards larger N (Fig. 3).

From the experimental point of view, it is interesting to note that the time-location of minimum F_2 is independent of the number of atoms (Fig. 1, 2). The fluctuations in atomic operators are closely related with the normally ordered variance of the fluorescent field. For time intervals for which $\langle S_3(t) \rangle$ is negative, squeezing in the atomic operators leads to the negative value of the normally order variance of the fluorescent field. In the opposite case, i.e. for $\langle S_3(t) \rangle > 0$ the normally ordered variance of the fluorescent field is positive despite the presence of squeezing in the atomic operators.

We infer that it should be easier to detect N atom-squeezed states in transient regime of resonance fluorescence. This, however, would require very fast detection technique, such as picosecond technique.

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