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## SELF-SQUEEZING AS A NOVEL POTENT SOURCE OF QUANTUM FIELD

### Introduction

In recent years, an increasing amount of work has been devoted to studies of the quantum and stochastic properties of the electromagnetic field, in particular to the feasibility of producing quantum fields exhibiting photon anticorrelation [1-5] and squeezing [5-7]. The effect of photon antibunching, revealing univocally the quantum nature of light, has hitherto been observed in experiment in processes of atomic resonance fluorescence [1] only. These experiments have directed attention to the possibilities of producing squeezed states of the radiation field [6, 8, 9].

Squeezed states of light are characterized by a decrease in quantum fluctuations in one component of the radiation field at the expense of an increase in fluctuations in the other (non-commuting) component. The crucial role of nonlinearity in photon anticorrelation is a well established fact. Hence, as one would expect, theoretical work on squeezing started from processes involving a nonlinear response of the quantum system to the field signal. Here, several nonlinear optical processes should be mentioned: parametric amplification [8, 11, 12], resonance fluorescence [13-17], four-wave mixing [18, 19], harmonics ge-

neration [20, 21], multi-photon absorption [22-25], hyper-Raman scattering [26], and nonlinear propagation of light [27].

The effect of photon anticorrelation is dependent on a number of favourable as well as destructive factors [4, 28]. Squeezing is particularly sensitive to the phase of the field: the fluctuations in phase (or amplitude) strongly reduce squeezing of the signal field [29], whereas interference acts favourably [30]. Other factors as well have an influence on these quantum effects [16, 17, 31]. Despite the differences between photon anticorrelation and squeezing, the two effects have one property in common: they are purely quantum in nature, and fields with such properties have no counterparts in classical optics. The two effects can exist concurrently in certain regions, or can exist independently of each other in different regions [15], whereas certain processes are accompanied by one of the two effects only.

The efficiency of individual nonlinear optical effects in producing photon antibunching and squeezing varies from one case to another. Here, it would appear that a particular role is played by self-induced effects like self-squeezing, which resides in the circumstance that a strong laser beam traversing an atomic medium endows it with optical nonlinearity owing to which the light undergoes self-squeezing of its quantum fluctuations. Herein may well reside a novel method of producing quantum fields with an efficiency of almost 100 per cent of that predicted by quantum mechanics. The process of self-squeezing is universal in nature, because it takes place in all matter even in systems composed of centrosymmetric molecules, or atoms.

#### Effective interaction Hamiltonian

Consider  $N$  noninteracting microsystems (atoms or molecules) in a volume  $V$ , acted on by an electromagnetic field with electric vector  $\vec{E}$  and magnetic vector  $\vec{B}$ . The total Hamiltonian of the system as a whole is, in standard form

$$H = H_N + H_F + H_I, \quad (1)$$

with  $H_N$  the Hamiltonian of the system of  $N$  atoms (molecules), and  $H_F$  that of the free radiation field.

We are interested in the explicit form of the Hamiltonian  $H_I$  describing the interaction between the material system and the field of ra-

diation. The interaction is, in general, nonlinear and involves all multipolar electric and magnetic transitions [32, 33]. Since the system is optically isotropic in the absence of external fields, the effective Hamiltonian is a function in even powers of  $\vec{E}$  and  $\vec{B}$ . Hence, the second-order Hamiltonian  $H_I^{(2)}$  is the lowest non-zero contribution to  $H_I$ ; if spatial dispersion is weak,  $H_I^{(2)}$  can be written as follows [32] (we omit the purely diamagnetic term, proportional to  $B^2$  [33]):

$$H_I^{(2)} = -\frac{N}{2} \left\{ \alpha_{ij} E_i E_j + 2 \varrho_{ij} E_i B_j + \frac{2}{3} \eta_{ijk} E_i \nabla_k E_j + \dots \right\}, \quad (2)$$

where the Einstein convention regarding  $ijk$  is assumed.

In (2), the second-rank tensor  $\alpha_{ij}$  defines the linear electric polarizability of the molecule for an electric dipole-dipole quantum transition. The second-rank pseudotensor  $\varrho_{ij}$  describes the electro-magnetic polarizability for an electric dipole-magnetic dipole quantum transition. Whereas the third-rank tensor  $\eta_{ijk}$  accounts for the linear electric polarizability for an electric dipole-electric quadrupole quantum transition.

In the same multipole approximation the fourth-order Hamiltonian (which is the first nonlinear contribution) has the following form [32] (we retain terms linear in  $\vec{B}$  only):

$$H_I^{(4)} = -\frac{N}{24} \left\{ \gamma_{ijkl} E_i E_j E_k E_l + 4 \sigma_{ijkl} E_i E_j E_k B_l + \frac{4}{3} \varkappa_{ijkm} E_i E_j E_k \nabla_m E_l + \dots \right\}, \quad (3)$$

where the fourth-rank tensor  $\gamma_{ijkl}$  defines the nonlinear electric polarizability for four-fold electric dipole quantum transitions. The fourth-rank pseudotensor  $\sigma_{ijkl}$  determines the nonlinear electro-magnetic polarizability for three-fold electric dipole-magnetic dipole quantum transitions. And the tensor of rank 5,  $\varkappa_{ijkm}$  expresses the nonlinear electric polarizability for three-fold electric dipole-electric quadrupole quantum transitions.

In the classical case, one can split the electric vector of the electromagnetic field at the space-time point  $(\vec{r}, t)$  into two complex parts [34]:

$$\vec{E}(\vec{r}, t) = \vec{E}^{(+)}(\vec{r}, t) + \vec{E}^{(-)}(\vec{r}, t), \quad (4)$$

where the component  $\vec{E}^{(+)}(\vec{r}, t)$  is related with the time-function  $\exp(-i\omega t)$  ( $\omega$  positive), with  $\vec{E}^{(-)}(\vec{r}, t) = \left\{ \vec{E}^{(+)}(\vec{r}, t) \right\}^*$ .

The transversal electric field can be expressed as a superposition of plane waves:

$$\vec{E}(\vec{r}, t) = \sum_{\vec{k}} \left\{ \vec{E}^{(+)}(\vec{k}) e^{i(\vec{k} \cdot \vec{r} - \omega t)} + \vec{E}^{(-)}(\vec{k}) e^{-i(\vec{k} \cdot \vec{r} - \omega t)} \right\} \quad (5)$$

The same holds with regard to the magnetic vector  $\vec{B}(\vec{r}, t)$ , with

$$B_i^{(+)} = \frac{c}{\omega} \varepsilon_{ijk} k_j E_k^{(+)}, \quad (6)$$

where  $\varepsilon_{ijk}$  is the antisymmetric Levi-Civita tensor.

We assume the configuration in which the light beam propagates along the Z-axis of the Cartesian reference frame XYZ and introduce the circular representation of the field with the unit vectors (on the angular momentum convention):

$$\vec{e}_{\pm} = \frac{1}{\sqrt{2}} (\vec{x} \pm i\vec{y}), \quad (7)$$

where  $\vec{e}_{+}$  and  $\vec{e}_{-}$  refer, respectively, to the right- and left-polarized waves, with  $\vec{x}$  and  $\vec{y}$  unit vectors in the direction of X and Y.

In quantum electrodynamics, the field vectors (4)-(6) are dealt with as operators in Hilbert space; we have

$$\vec{E}^{(+)}(\vec{k}) = i \sum_{\lambda} c(\omega_k) \vec{e}^{(\lambda)}(\vec{k}) \hat{a}_{\vec{k}\lambda}^{\dagger}. \quad (8)$$

Above, in cgs units, the normalizing factor is  $c(\omega_k) = (2\pi\hbar\omega_k/V)^{1/2}$  with V the quantisation volume.

In (8),  $\hat{a}_{\vec{k}\lambda}^{\dagger}$  is the annihilation operator of a photon with the momentum  $\hbar\vec{k}$  and polarisation state  $\lambda$  given by the unit vector  $\vec{e}^{(\lambda)}(\vec{k})$ . The photon annihilation and creation operators  $\hat{a}_{\vec{k}\lambda}$  and  $\hat{a}_{\vec{k}\lambda}^{\dagger}$  fulfill the boson commutation rules

$$\left[ \hat{a}_{\vec{k}\lambda}, \hat{a}_{\vec{k}'\lambda'}^{\dagger} \right] = \delta_{\vec{k}\vec{k}'} \delta_{\lambda\lambda'}, \quad \left[ \hat{a}_{\vec{k}\lambda}, \hat{a}_{\vec{k}\lambda} \right] = \left[ \hat{a}_{\vec{k}\lambda}^{\dagger}, \hat{a}_{\vec{k}\lambda}^{\dagger} \right] = 0,$$

The unit vectors describing the state of polarisation of the wave are in general complex quantities satisfying the ortho-normalisation conditions

$$e_{\vec{k}\sigma}^{(\lambda)*} e_{\vec{k}\tau}^{(\lambda')} = \delta_{\sigma\tau} \delta_{\lambda\lambda'}, \quad e_{\vec{k}\sigma}^{(\lambda)} k_{\sigma} = 0.$$

For a quasi-monochromatic wave with the frequency  $\omega$  propagating along the Z-axis one may omit in (5) the summation over the index  $k$ . Thus, by (8), we get the field component in the form:

$$E_{\sigma}^{(+)}(z, t) = i c(\omega) \exp \left\{ -i(\omega t - kz) \right\} \sum_{\lambda=1,2} e_{\sigma}^{(\lambda)} \hat{a}_{\lambda}, \quad (9)$$

where  $k = \omega/c$ .

In fact, the formula (9) provides a two-mode description of a field being the coherent superposition of two modes with mutually orthogonal polarisations.

By analogy with the circular representation of the polarisation of a classical field we may write the transformations for the field operators as:

$$\hat{a}_{+} = \frac{1}{\sqrt{2}} (\hat{a}_{x} - i \hat{a}_{y}), \quad \hat{a}_{-} = \frac{1}{\sqrt{2}} (\hat{a}_{x} + i \hat{a}_{y}). \quad (10)$$

Both representations are well adapted to the description of the interaction between elliptically polarized light and a material medium. However, as shown previously [27], the circular representation  $(\hat{a}_{+}, \hat{a}_{-})$  is radically more convenient than the Cartesian representation  $(\hat{a}_{x}, \hat{a}_{y})$  in that the former enables us to obtain the solution of the equation of motion for the field operators in general propagator form.

Since the microsystems are freely oriented in space, we may perform an unweighted averaging of the interaction Hamiltonians (2) and (3) over all their possible orientations. By (4), (9) and (10) and taking the operators in normal order we obtain the effective Hamiltonian in the form [35]

$$H_I^{(2)} = -\tilde{\chi}_R^L (\hat{a}_{+}^{\dagger} \hat{a}_{+} + \hat{a}_{-}^{\dagger} \hat{a}_{-}) - i \tilde{\chi}_A^L (\hat{a}_{+}^{\dagger} \hat{a}_{+} - \hat{a}_{-}^{\dagger} \hat{a}_{-}), \quad (11)$$

$$H_I^{(4)} = -\frac{1}{2} \tilde{\chi}_R^{NL} (\hat{a}_{+}^{+2} \hat{a}_{+}^2 + \hat{a}_{-}^{+2} \hat{a}_{-}^2) - \tilde{\chi}_R^{NL} \hat{a}_{+}^{\dagger} \hat{a}_{+}^{\dagger} \hat{a}_{-} \hat{a}_{-} - \frac{1}{2} \tilde{\chi}_A^{NL} (\hat{a}_{+}^{+2} \hat{a}_{+}^2 - \hat{a}_{-}^{+2} \hat{a}_{-}^2), \quad (12)$$

where we have introduced the coupling constants [36]

$$\tilde{\chi}_R^L = \frac{1}{3} c(\omega)^2 N \operatorname{Re} \alpha_{\alpha\alpha}, \quad \tilde{\chi}_A^L = \frac{i}{3\omega} c(\omega)^2 N \operatorname{Im} \rho_{\alpha\alpha}, \quad (13)$$

$$\begin{aligned}
\tilde{\chi}_R^{NL} &= c(\omega)^4 \frac{N}{15} \operatorname{Re}(3 \gamma_{\alpha\beta\alpha\beta} - \gamma_{\alpha\alpha\beta\beta}), \\
\tilde{\varkappa}_R^{NL} &= c(\omega)^4 \frac{N}{15} \operatorname{Re}(\gamma_{\alpha\beta\alpha\beta} + 3 \gamma_{\alpha\alpha\beta\beta}), \\
\tilde{\chi}_A^{NL} &= -i c(\omega)^4 \frac{4 N k_z}{15} \left\{ \frac{1}{\omega} \operatorname{Im}(3 \sigma_{\alpha\beta\alpha\beta} - \sigma_{\alpha\alpha\beta\beta}) + \right. \\
&\quad \left. + \frac{1}{3} \operatorname{Re} \varkappa_{\alpha\beta\gamma\beta\delta} \varepsilon_{\alpha\gamma\delta} \right\}.
\end{aligned} \tag{14}$$

Thus, the effective Hamiltonians (11) and (12) derived by us are expressed in general by 5 invariants of the field operators (2 linear and 3 nonlinear) and 5 rotational molecular invariants (2 linear (13) and 3 nonlinear). The coupling parameters  $\tilde{\chi}_R^{NL}$  and  $\tilde{\varkappa}_R^{NL}$  are related with self-induced rotation of the polarisation ellipse whereas the parameters  $\tilde{\chi}_A^L$  and  $\tilde{\chi}_A^{NL}$  - with optical activity and its nonlinear variation, respectively.

#### Solution of the Equation of Motion for the Field Operators

The evolution in time of the field operators is described by Heisenberg's equation of motion:

$$\frac{\partial \vec{E}(\pm)(\vec{r}, t)}{\partial t} = \frac{1}{i\hbar} \left[ \vec{E}(\pm)(\vec{r}, t), H \right]. \tag{15}$$

In the case of a field, the Hamiltonian  $H$  occurring above is nothing but that ( $H_F$ ) of the free field of radiation. In this case the solution of Eq. (15) is given by the electric field (5) corresponding to "rapidly variable" time-evolution.

Due to interaction between the field of radiation and the material medium the Hamiltonian  $H$  of Eq. (15) contains, in addition to  $H_F$ , the interaction Hamiltonian

$$H_I = H_I^{(2)} + H_I^{(4)} + \dots \tag{16}$$

In the present case the solution of (15) is no longer given by the free field (15). Additional time-dependent terms appear, and this additional "slowly variable" time-dependence due solely to the interaction (16) causes the amplitudes  $\vec{E}(\pm)(\vec{k})$  given by Eq. (8) to become functions of time.

Generally, considerations bear on the time-evolution of the operators of a field in a cavity of volume  $V$ . In our case, however, we deal with the propagation of a wave through an active medium in which it traverses a path of well defined length  $z$  i.e. the field emerging from the medium is dependent on  $z$ . In the case of plane waves the transition from the problem of a field in a cavity to that of propagation requires but the replacement of the time  $t$  by  $+z/c$ . Once this is done, the Heisenberg equations of motion (15) become equations describing the  $z$ -dependence of the operators of the  $k$ -th mode of the field:

$$\frac{\partial \hat{E}^{(\pm)}(k, z)}{\partial z} = \frac{1}{i\hbar c} \left[ \hat{E}^{(\pm)}(k, z), H \right]. \quad (17)$$

The next step, which is crucial, consists in the choice of the interaction Hamiltonian (16) in a form enabling us to solve Eq. (17) analytically without loss of generality. It has been shown [27, 35] that the general analytical solution is obtained directly if the interaction between the field and the system is described by the effective Hamiltonian (16) in conjunction with the components (11) and (12). On eliminating free evolution, the equation of motion for the field operators reads:

$$\begin{aligned} \frac{d}{dz} \hat{a}_{\pm}(z) = \frac{i}{\hbar c} \left\{ \tilde{\chi}_R^L \pm i \tilde{\chi}_A^L + \tilde{\omega}_R^{NL} \hat{a}_{\mp}(z) \hat{a}_{\mp}(z) + \right. \\ \left. + (\tilde{\chi}_R^{NL} \pm i \tilde{\chi}_A^{NL}) \hat{a}_{\pm}^+(z) \hat{a}_{\pm}(z) \right\} \hat{a}_{\pm}(z). \end{aligned} \quad (18)$$

Since the photon number operators  $\hat{a}_{\pm}^+ \hat{a}_{\pm}$  and  $\hat{a}_{\mp}^+ \hat{a}_{\mp}$  are constants of motion (they commute with the Hamiltonian), the above equation possesses a strict formal solution in the form of the translation operator:

$$\begin{aligned} \hat{a}_{\pm}(z) = \exp \left\{ iz \left[ \varphi_{\pm} + \varepsilon_{\pm} \hat{a}_{\pm}^+(0) \hat{a}_{\pm}(0) + \right. \right. \\ \left. \left. + \delta \hat{a}_{\mp}^+(0) \hat{a}_{\mp}(0) \right] \right\} \hat{a}_{\pm}(0), \end{aligned} \quad (19)$$

where we have introduced the following notation:

$$\begin{aligned} \varphi_{\pm} &= \frac{1}{\hbar c} (\tilde{\chi}_R^L \pm i \tilde{\chi}_A^L), \\ \varepsilon_{\pm} &= \frac{1}{\hbar c} (\tilde{\chi}_R^{NL} \pm i \tilde{\chi}_A^{NL}), \\ \delta &= \frac{1}{\hbar c} \tilde{\omega}_R^{NL}. \end{aligned} \quad (20)$$



## Squeezed States of the Field

The preceding solutions show that the photon statistics of a circularly polarized beam remains unchanged on traversal of the light wave through an isotropic nonlinear medium. However, this does not mean that the state of the field remains unaffected by the nonlinear interaction. The latter, in fact, can cause the field to go over into a squeezed state, having no classical counterpart. In order to prove this, we introduce two Hermitian field operators  $\hat{Q}_\sigma$  and  $\hat{P}_\sigma$ , defined as follows:

$$\hat{Q}_\sigma = \hat{a}_\sigma + \hat{a}_\sigma^\dagger, \quad \hat{P}_\sigma = -i(\hat{a}_\sigma - \hat{a}_\sigma^\dagger), \quad (21)$$

with  $\sigma$  standing for  $+$ ( $-$ ) in circular basis, or  $x$ ( $y$ ) in Cartesian basis. The operators (21) satisfy the commutation rule

$$[\hat{Q}_\sigma, \hat{P}_\sigma] = 2i\delta_{\sigma\sigma}.$$

The definition of the squeezed state of the electromagnetic field is the following: it is a field state in which the square of the uncertainty in  $\hat{Q}_\sigma$  or  $\hat{P}_\sigma$  is less than unity [13]:

$$\langle (\Delta \hat{Q}_\sigma)^2 \rangle < 1 \quad \text{or} \quad \langle (\Delta \hat{P}_\sigma)^2 \rangle < 1, \quad (22)$$

where  $\Delta \hat{Q}_\sigma = \hat{Q}_\sigma - \langle \hat{Q}_\sigma \rangle$ .

On normal ordering of the creation and annihilation operators, the definition (22) can be re-written in the form [13, 14]

$$\langle : (\Delta \hat{Q}_\sigma)^2 : \rangle < 0 \quad \text{or} \quad \langle : (\Delta \hat{P}_\sigma)^2 : \rangle < 0. \quad (23)$$

To calculate the quantities occurring in the above definition (in the case of propagation in a nonlinear medium) we have to insert the operator solution (19) into the definition (23) and calculate the mean value in the quantum state of the incident beam which we assume as the coherent state  $\hat{a}(0) | \alpha \rangle = \alpha | \alpha \rangle$ . If one of the normally ordered variances turns out to be negative, the respective field component of the emerging light is in a squeezed state. Such calculations for normally ordered variances of the beam emerging from the medium give

$$\langle : [\Delta \hat{Q}_\pm(z)]^2 : \rangle = \langle : [\hat{a}_\pm(z) + \hat{a}_\pm^\dagger(z)]^2 : \rangle -$$

$$\begin{aligned}
& - \langle \hat{a}_{\pm}(z) + \hat{a}_{\pm}^+(z) \rangle^2 = \\
& = 2 \operatorname{Re} \left\{ \alpha_{\pm}^2 \exp \left[ 2iz\varphi_{\pm} + iz\varepsilon_{\pm} + (e^{2iz\varepsilon_{\pm}} - 1) |\alpha_{\pm}|^2 + \right. \right. \\
& \quad \left. \left. + (e^{2iz\delta} - 1) |\alpha_{\mp}|^2 \right] - \alpha_{\pm}^2 \exp \left[ 2iz\varphi_{\pm} + \right. \right. \\
& \quad \left. \left. + 2(e^{iz\varepsilon_{\pm}} - 1) |\alpha_{\pm}|^2 + 2(e^{iz\delta} - 1) |\alpha_{\mp}|^2 \right] \right\} + \\
& \quad + 2 |\alpha_{\pm}|^2 \left\{ 1 - \exp \left[ 2(\cos z\varepsilon_{\pm} - 1) |\alpha_{\pm}|^2 + \right. \right. \\
& \quad \left. \left. + 2(\cos z\delta - 1) |\alpha_{\mp}|^2 \right] \right\}, \tag{24}
\end{aligned}$$

where

$$\alpha_{\pm} = \frac{\alpha}{\sqrt{2}} (\cos \eta \pm \sin \eta) e^{\mp i\theta}$$

$\eta$  and  $\theta$  being the ellipticity and azimuth of the elliptically polarized wave.

Of especial interest is the case when the incident beam is polarized circularly:  $\eta = \pi/4$ ,  $\theta = 0$ . We have  $|\alpha_{+}|^2 = |\alpha|^2$ ,  $|\alpha_{-}|^2 = 0$  and (24) reduces to

$$\begin{aligned}
& \langle : [\Delta \hat{Q}_{+}(z)]^2 : \rangle = \\
& = 2 |\alpha|^2 \left\{ \exp \left[ |\alpha|^2 (\cos 2z\varepsilon_{+} - 1) \right] \cos(z\varphi_{+} + z\varepsilon_{+} + \right. \\
& \quad \left. + |\alpha|^2 \sin 2z\varepsilon_{+}) - \exp \left[ 2 |\alpha|^2 (\cos z\varepsilon_{+} - 1) \right] \cos(z\varphi_{+} + \right. \\
& \quad \left. + 2 |\alpha|^2 \sin z\varepsilon_{+}) \right\} + 2 |\alpha|^2 \left\{ 1 - \exp \left[ 2 |\alpha|^2 (\cos z\varepsilon_{+} - 1) \right] \right\} \tag{25}
\end{aligned}$$

We get a similar expression for the other component:

$$\langle : [\Delta \hat{P}_{+}(z)]^2 : \rangle = -2 |\alpha|^2 \{ \dots \} + 2 |\alpha|^2 \{ \dots \}. \tag{26}$$

On putting the initial phase of the field  $\varphi_0$  in a manner to have  $\varphi_{+}(z) + \varphi_0 = 0$  and on taking a numerical value [37] of  $\varepsilon_{+} z = 1 \times 10^{-6}$ , we are in a position to compute the expression (25) numerically, leading to the graphs of Fig. 1,

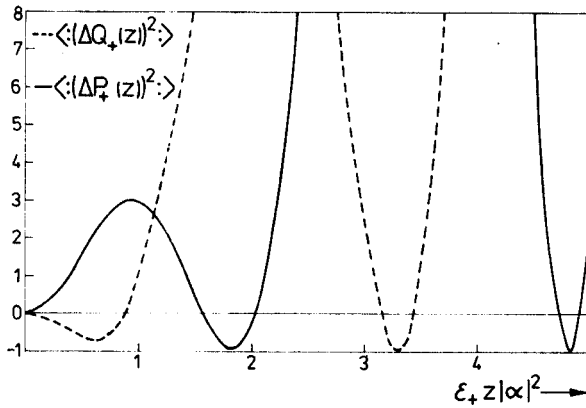


Fig. 1.

where the quantities  $\langle : [\Delta \hat{Q}_+(z)]^2 : \rangle$  and  $\langle : [\Delta \hat{P}_+(z)]^2 : \rangle$  are found to be oscillatory in shape depending on  $\xi_+ z |\alpha|^2$  and to take positive as well as negative values. In the regions where these values are negative, the respective field component  $\hat{Q}_+(z)$  or  $\hat{P}_+(z)$  is in a squeezed state. We note that the field can be in squeezed state despite the Poisson photon statistics. Also notable is the magnitude of the squeezing possible to achieve in the process considered above. On our definition of  $\hat{Q}_+$  and  $\hat{P}_+$  the value permitted for (25) and (26) by quantum mechanics is -1, meaning zero fluctuations of the field. In Fig. 1 the first minimum of (25) amounts to -0.66 whereas its second minimum amounts to as much as -0.97 i.e. 97 per cent of the magnitude permitted by quantum mechanics. As to the first minimum of (26), it amounts to -0.92 also signifying an almost complete elimination of quantum fluctuations.

Finally, it may be worth mentioning that on performing an appropriate interchange of the variables the expressions (25) and (26) become identical with the respective expressions for the isotropic anharmonic oscillator [38].

### Conclusions

We have shown previously that the process of light propagation in an isotropic optically active nonlinear medium can be a source of non-classical fields. The photon antibunching effect in this process is insignificant, but can undergo amplification by interference with another

beam [39]. In contradistinction to this, the effect of self-squeezing can be almost total and takes place as well for a circularly polarized incident beam, in which case no change in the photon statistics occurs. Hence squeezed states can exist for fields with a Poissonian photon distribution, characteristic for fields in coherent state. Moreover, our results draw a boundary between the phenomenon of photon antibunching and that of squeezing.

We wish to stress once again that we have obtained a strict analytical solution in translation operator form valid for arbitrary polarisation of the incident beam and involving explicitly the nonlinear and multipolar properties of the molecules of the optically active medium [3, 40].

The latest communications [41-44] suggest that the study of quantum optical effects, in particular squeezed states of light, is no longer a matter of purely academic interest but is fast approaching the stage of experimental testing and applications in telecommunication. In the nearest future some well designed experiments may lead to a dramatic breakthrough in our understanding of the nature of light. Quite recently, theoretical papers [45-56] have appeared analyzing various subtler details of squeezed light and its interaction with matter [57].

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