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SQUEEZING CONDITIONS IN TRANSIENT REGIME OF RESONANCE FLUORESCENCE OF TWO-LEVEL ATOM*

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The dependence of squeezing on the initial phase of the exciting field is given explicitly. It is shown that, contrary to the steady-state régime in which squeezing can occur at low or moderate Rabi frequencies only, there is no such restriction on the Rabi frequencies in the transient régime. As the intensity of the exciting field increases, squeezing shifts towards shorter times and its maximum value increases to become twice as large as the steady-state maximum. The dependence of squeezing on the initial atomic state is also discussed. It is shown that in the transient régime of resonance fluorescence from a two-level atom intervals of time exist in which, despite the presence of squeezing in the atomic variables, the normally ordered variance of the fluorescent field becomes positive.

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1. Introduction

Resonance fluorescence from a two-level atom interacting with a coherent laser field has been the subject of considerable interest since the early work of Newstein [1] and Mollow [2][1]. A new and interesting question regarding the phenomenon of resonance fluorescence concerns the generation of special states of the electromagnetic field referred to as squeezed states. Walls and Zoller [4] have shown, when considering the steady-state régime, that either the in-phase component E_1 or the out-of-phase component E_2 of the fluorescent field can become squeezed if the Rabi frequency Ω of the exciting field is sufficiently low. To satisfy this condition when the exciting field is tuned perfectly to the atomic transitions, the Rabi frequency Ω has to be less than $\sqrt{2}\gamma$, with $2\gamma = A$ — the Einstein coefficient for spontaneous emission. Off-resonance excitation further lowers this critical value of Ω. Walls and Zoller [4] have also indicated, without giving the details, that in the transient régime the squeezing effect can become twice as large as in the steady state. Mandel

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[1] The literature on this subject is quite extensive. For a survey of recent publications and some new results see the review article of Knight and Milonni [3].

[5] has shown that the use of a homodyne detection scheme to detect squeezing in steady-state resonance fluorescence leads always to sub-Poissonian photon statistics if the detected component is squeezed. Arnoldus and Nienhuis [6] have considered the conditions for sub-Poissonian photon statistics and squeezing in steady-state resonance fluorescence showing that these conditions for the two effects are largely complementary if their dependence on the detuning from resonance, the linewidth and the Rabi frequency is taken into account.

The possibilities of generating squeezed states in various nonlinear processes have been the subject of numerous papers [7–18]. Yuen and Chan [19] have proposed a new homodyne scheme of detection that allows one to eliminate the local oscillator noise thus increasing the chances for the detection of squeezed states.

In this paper we give a detailed analysis of the possibility of obtaining squeezed states in the transient régime of resonance fluorescence from a coherently driven two-level atom. We show that, contrary to the steady-state régime, there is no restriction on the required values of Ω and that squeezing can occur for any value of the Rabi frequency. Its maximum value increases as the intensity of the exciting field increases and becomes twice larger than the maximum value in the steady state for very strong exciting fields ($\Omega = 200\gamma$) provided the atom started in its ground state and the initial phase of the exciting field was zero. The initial field phase dependence of squeezing is discussed both for the steady-state and transient régime. It is shown that intervals of time exist in which the normally ordered variance of the fluorescent field is positive despite the circumstance that squeezing is exhibited at the same time by the atomic variables.

2. *Resonance fluorescence squeezed states*

On defining the in-phase component E_1 and out-of-phase component E_2 of the electromagnetic field amplitude with the positive (negative) frequency parts $E^{(+)}(E^{(-)})$

$$E_1 = E^{(+)}+E^{(-)}, \quad E_2 = -i(E^{(+)}-E^{(-)}), \tag{1}$$

we have the commutation rule

$$[E_1, E_2] = 2iC, \tag{2}$$

where C is a positive c-number if the field is described by boson operators. Squeezed states of the electromagnetic field are defined by the requirement that the variance of one of the two noncommuting observables (E_1 and E_2) shall be less than C, i.e. [4, 20]:

$$\langle(\Delta E_1)^2\rangle < C \quad \text{or} \quad \langle(\Delta E_2)^2\rangle < C. \tag{3}$$

Since the following relations hold for the field operators [5]:

$$\begin{aligned}\langle(\Delta E_1)^2\rangle &= C+\langle:(\Delta E_1)^2:\rangle,\\ \langle(\Delta E_2)^2\rangle &= C+\langle:(\Delta E_2)^2:\rangle,\end{aligned} \tag{4}$$

where the colon stands for normal ordering of the operators; the squeezing conditions [3] imply negative values of the normally ordered variance for the field component that is squeezed. This reflects the nonclassical nature of the field.

In resonance fluorescence from a coherently driven two-level atom the positive-frequency part of the fluorescent field can be, in the far-field limit, expressed in terms of the atomic operators as follows [21, 22]:

$$E^{(+)}(\boldsymbol{r}, t) = E_0^{(+)}(\boldsymbol{r}, t)+\tfrac{1}{2}\,\psi(\boldsymbol{r})S^-\left(t-\frac{r}{c}\right), \tag{5}$$

with $\psi^2(\boldsymbol{r}) = (3\hbar k\gamma/2r^2)\sin^2\theta$, where θ is the angle between the observation direction $\boldsymbol{r}$ and the atomic transition dipole moment $\boldsymbol{\mu}$, and $S^-(t)$ is the operator lowering the energy of the atom which together with the raising operator $S^+(t)$ obeys the well-known commutation rules

$$[S^+(t), S^-(t)] = 2S_3(t), \quad [S_3(t), S^\pm(t)] = \pm S^\pm(t), \tag{6}$$

where $S_3(t)$ describes the energy of the atom. Defining the Hermitian operators

$$S_1(t) = \tfrac{1}{2}(S^+(t)+S^-(t)), \quad S_2(t) = -\frac{i}{2}(S^+(t)-S^-(t)), \tag{7}$$

we have

$$[S_1(t), \quad S_2(t)] = iS_3(t). \tag{8}$$

Unlike the commutation rule (2) where C is a positive c-number, in the relation (8) $S_3(t)$ is an operator. Nonetheless, one can define atomic squeezed states by the relations [4]

$$\langle(\Delta S_{1,2}(t))^2\rangle < \tfrac{1}{2}|\langle S_3(t)\rangle|, \tag{9}$$

or

$$F_{1,2}(t) = \langle(\Delta S_{1,2}(t))^2\rangle-\tfrac{1}{2}|\langle S_3(t)\rangle| < 0. \tag{10}$$

It can also be shown, using the relation (5), that the normally ordered variance of the fluorescent field is given by:

$$\langle:(\Delta E_{1,2}(t))^2:\rangle = \psi^2(\boldsymbol{r})\,[\langle(\Delta S_{1,2}(t))^2\rangle+\tfrac{1}{2}\langle S_3(t)\rangle]. \tag{11}$$

In the steady state $\langle S_3(\infty)\rangle < 0$ and $\langle S_3(\infty)\rangle = -|\langle S_3(\infty)\rangle|$ and squeezing in the atomic variables given by the condition (10) implies a negative value of the normally ordered variance of the corresponding component of the fluorescent field. In the transient régime, however, $\langle S_3(t)\rangle$ evolves in time and can take positive as well as negative values. It is obvious that intervals of time exist in which the atomic squeezing does not necessarily lead to a nonclassical nature of the emitted field. In the following we discuss these problems in detail.

The time dependence of the atomic variables is well known in the literature [22, 23] and gives:

$$\langle S^{-}(t)\rangle = \frac{-\frac{1}{2}\beta e^{-i\varphi}(1-i\Delta)}{(\frac{1}{4}+\frac{1}{4}\Delta^2+2\beta^2)} + 2\beta e^{-i\varphi} w_0 \sum_{\substack{i=1\\ i\neq j\neq k}}^{3} \frac{[z_i+\frac{1}{2}(1-i\Delta)]e^{z_i t}}{(z_i-z_j)(z_i-z_k)}$$

$$-\beta e^{-i\varphi} \sum_{\substack{i=1\\ i\neq j\neq k}}^{3} \frac{(z_i+1)[z_i+\frac{1}{2}(1-i\Delta)]e^{z_i t}}{z_i(z_i-z_j)(z_i-z_k)}, \tag{12a}$$

$$\langle S_3(t)\rangle = -\frac{\frac{1}{8}(1+\Delta^2)}{(\frac{1}{4}+\frac{1}{4}\Delta^2+2\beta^2)} + w_0 \sum_{\substack{i=1\\ i\neq j\neq k}}^{3} \frac{[(z_i+\frac{1}{2})^2+\frac{1}{4}\Delta^2]e^{z_i t}}{(z_i-z_j)(z_i-z_k)}$$

$$+2\beta^2 \sum_{\substack{i=1\\ i\neq j\neq k}}^{3} \frac{(z_i+\frac{1}{2})e^{z_i t}}{z_i(z_i-z_j)(z_i-z_k)}, \tag{12b}$$

where z_1, z_2 and z_3 are the three roots (assumed as unequal) of the cubic equation

$$(z+1)[(z+\tfrac{1}{2})^2+\tfrac{1}{4}\Delta^2]+4\beta^2(z+\tfrac{1}{2}) = 0. \tag{12c}$$

In equations (12) we have used the notation

$$\beta = \frac{\Omega}{4\gamma}, \quad \Delta = \frac{\omega-\omega_0}{\gamma}, \quad w_0 = \langle S^{+}(0)S^{-}(0)\rangle, \tag{13}$$

where Ω is the Rabi frequency of the driving field which is real because the factor of $\exp(-i\varphi)$ including the initial phase of the field ensures this, Δ is the detuning of the laser frequency from the atomic resonance, w_0 is the initial population of the excited state of the atom, and 2γ is equal to the Einstein A coefficient. If the atom is initially in its ground state then $w_0 = 0$ whereas $w_0 = 1$ if the atom starts in its excited state.

A. Steady-state régime

For steady-state resonance fluorescence ($t \to \infty$), we find from the definition (10) and equations (12) taking into account that for a two level atom $S_1^2 = S_2^2 = 1/4$:

$$F_{1,2}(\infty) = \frac{2\beta^2}{(1+\Delta^2+8\beta^2)^2}\{8\beta^2 \mp (1-\Delta^2)\cos 2\varphi \pm 2\Delta \sin 2\varphi\}. \tag{14}$$

Equation (14) is the exact formula describing the fluorescent field fluctuations in steady-state resonance fluorescence. For $\varphi = 0$ this formula goes over into that obtained by Walls and Zoller [4]. It is clear from (14) that a change by $\pi/2$ of the initial phase of the exciting field interchanges F_1 with F_2. At this point we should also note that because of our different choice of phase, our F_1 corresponds to what would be F_2 for Walls and Zoller [4]. $F_1(\infty)$ is plotted in Fig. 1 versus the detuning Δ for $\beta = 0.2$ and three different values of the initial phase φ. Figure 1 shows how $F_1(\infty)$ goes over into $F_2(\infty)$ as the initial phase is changed. The maximum of squeezing in steady-state resonance fluorescence turns out to occur for $\Delta = \varphi = 0$ and $\beta^2 = \frac{1}{24}$ when $F_1 = -\frac{1}{32}$, which is the value obtained by Walls and Zoller [4].

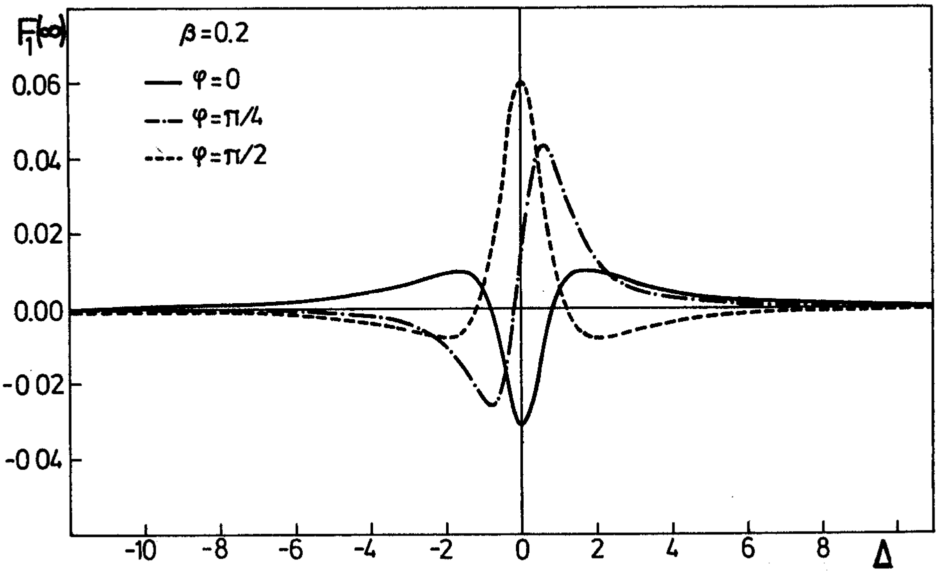

Fig. 1. Steady-state values of $F_1(\infty)$ versus detuning Δ for $\beta = 0.2$ and different values of the initial phase φ

To obtain squeezing in the steady state the Rabi frequency has to be sufficiently low. In fact, from (14) for $\varphi = 0$ we have $8\beta^2+\Delta^2 < 1$ for squeezing in F_1, and $\Delta^2 > 8\beta^2+1$ for squeezing in F_2. Recently, it has been shown that quite similar squeezing conditions hold for the case of two interacting atoms [17].

B. Transient régime

Using Eq. (10) and Eqs. (12) we obtain general formulae for the time evolution of $F_1(t)$ and $F_2(t)$. We restrict our considerations, however, to the case of zero detuning $\Delta = 0$. In this case we have (for $\beta > \frac{1}{8}$):

$$F_1(t) = \tfrac{1}{4} - \frac{\beta^2\cos^2\varphi}{(\frac{1}{2}+4\beta^2)^2}\left\{-1+\left[\cos at+\frac{A}{a}\sin at\right]e^{-\frac{3}{4}t}\right\}^2$$
$$-\tfrac{1}{2}\frac{1}{(\frac{1}{2}+4\beta^2)}\left|-\tfrac{1}{4}+\left[D\cos at-\frac{B}{a}\sin at\right]e^{-\frac{3}{4}t}\right|, \tag{15}$$

$$F_2(t) = \tfrac{1}{4} - \frac{\beta^2 \sin^2 \varphi}{(\frac{1}{2}+4\beta^2)^2} \left\{ -1 + \left[\cos at + \frac{A}{a} \sin at \right] e^{-\frac{3}{4}t} \right\}^2 - \tfrac{1}{2} \frac{1}{(\frac{1}{2}+4\beta^2)} \left| -\tfrac{1}{4} + \left[D \cos at - \frac{B}{a} \sin at \right] e^{-\frac{3}{4}t} \right|, \tag{16}$$

where

$$a = \sqrt{4\beta^2 - \tfrac{1}{16}},$$

$$A = [(w_0 + \tfrac{1}{4}) + 4\beta^2(2w_0 - 1)],$$

$$B = [\tfrac{1}{4} w_0(\tfrac{1}{2} + 4\beta^2) + \tfrac{3}{2}\beta^2],$$

$$D = [w_0(\tfrac{1}{2} + 4\beta^2) - 2\beta^2], \tag{17}$$

and t is in units of $(2\gamma)^{-1}$. For $\beta < \frac{1}{8}$, the equations for $F_1(t)$ and $F_2(t)$ have the same form as (15) and (16) with the replacement $\cos at \to \cosh a't$, $\sin at \to \sinh a't$ with $a' = \sqrt{\frac{1}{16} - 4\beta^2}$.

The time dependence of $F_1(t)$ is illustrated graphically in Figs 2–4 for various values of the parameters β, φ and w_0. It is seen from Fig. 2 that, for a weak exciting field ($\beta = 0.2$) and $\varphi = 0$, F_1(t) is always negative if the atom was in its ground state at $t = 0$. If the atom was in its excited state at $t = 0$, $F_1(t)$ is positive for short times and, after reaching a peak, goes down to take the negative steady-state value for very long times. For $\varphi = \pi/4$, $\cos^2 \varphi = \sin^2 \varphi = \frac{1}{2}$ and $F_1(t) = F_2(t) > 0$ for any values of the remaining parameters, as is also

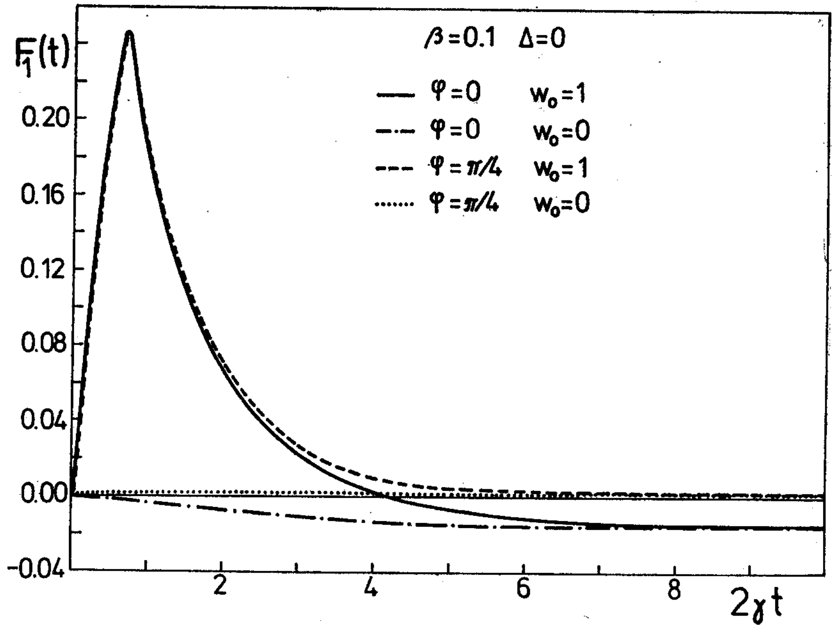

Fig. 2. Time dependence of $F_1(t)$ for $\beta = 0.1$, different values of φ and different w_0

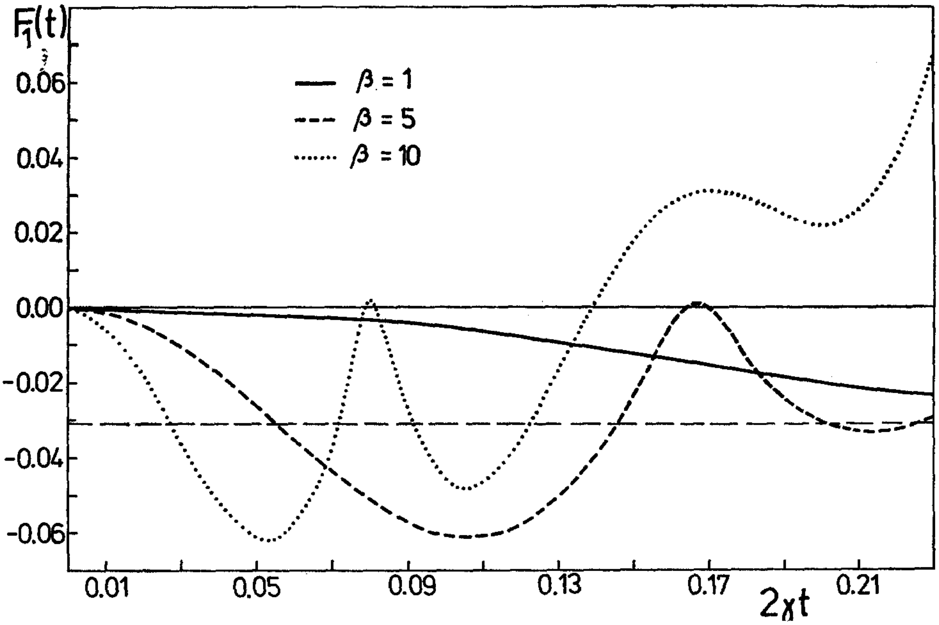

Fig. 3. Time dependence of $F_1(t)$ for $\beta = 1$; 5; 10, $w_0 = 0$, and different φ. Dashed line marks the maximum value of squeezing in steady state

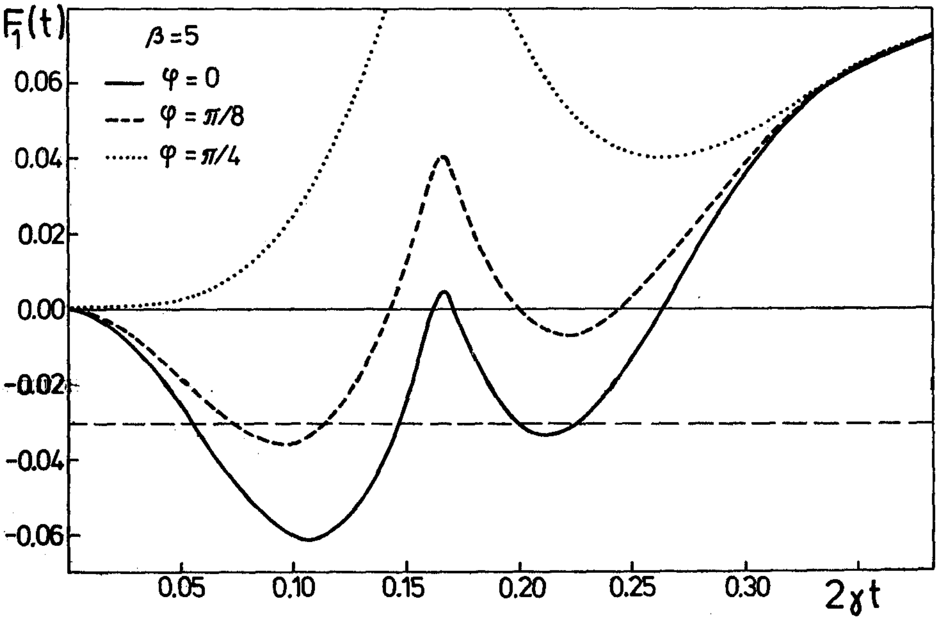

Fig. 4. Time dependence of $F_1(t)$ for very strong exciting field ($\beta = 5$), $w_0 = 0$ and different φ. Dashed line marks the maximum value of squeezing in steady state

seen in Fig. 2. Again, like in the steady state, a change in phase by $\pi/2$ interchanges $F_1(t)$ and $F_2(t)$ as is obvious directly from the formulae (15) and (16). For a weak field and zero detuning (as assumed by us) $F_2(t)$ is always positive.

As the intensity of the driving field increases, squeezing in $F_1(t)$ shifts to the region of shorter times, and $F_1(t)$ itself shows an oscillatory behaviour reflecting the Rabi oscillations. This is shown in Figs 3 and 4. The maximum value of squeezing that can be obtained in the transient régime becomes larger than the steady-state maximum which is marked by the dashed line in Figs 3 and 4. For very strong incident fields squeezing in

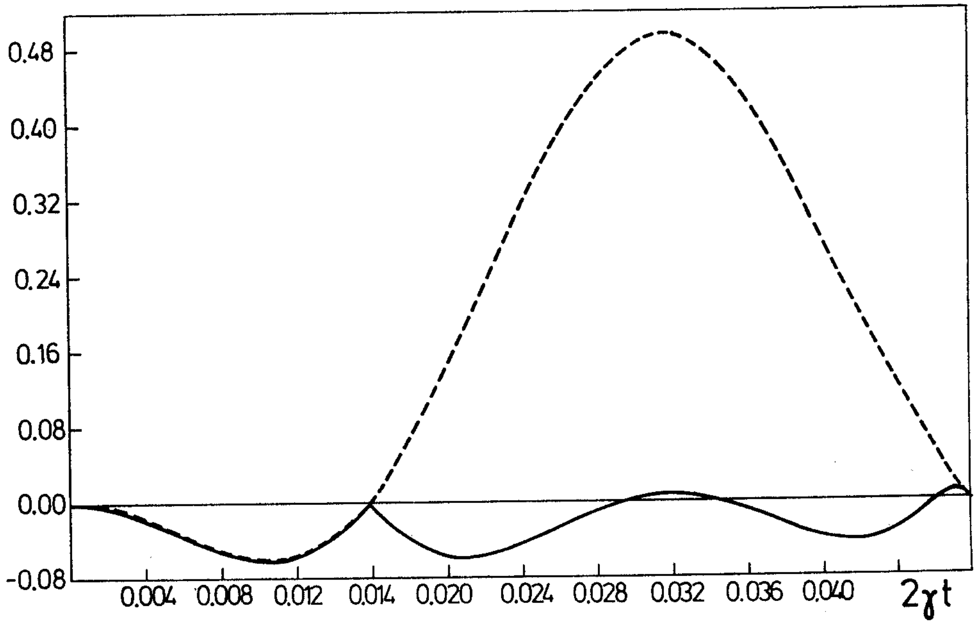

Fig. 5. Time dependence of $F_1(t)$ (solid curve) and $\langle:(\Delta E_1)^2:\rangle/\psi^2 r)$ (dashed curve) for $\beta = 50$, $\varphi = 0$ and $w_0 = 0$. For $0 \leqslant 2\gamma t \leqslant 0.015$ the two curves merge

$F_1(t)$ becomes twice as large as that which can be obtained in the steady state. This is the result given by Walls and Zoller [4] and it is convincingly illustrated in Fig. 5. It requires, however, $w_0 = 0$; $\varphi = 0$ and for $\beta = 50$ gives $F_1(t) = -\frac{1}{16}$ at the time $t = 0.0105$. This is the absolute maximum of squeezing (the absolute minimum of $F_1(t)$) that can be obtained in one-atom resonance fluorescence. The twofold increase of squeezing in the transient régime with respect to the steady state is an echo of the corresponding increase of the fluorescent light intensity. This limit is related to the fact that the two-level atom cannot reach an energy level higher than that of its excited state. The reduction of the atomic Hilbert space to two states determines, by way of the relation (5), the operator character of the fluorescent field which causes that, in the transient régime, squeezing in the atomic variables not necessarily signifies negative values of the normally ordered variance of the corresponding component of the fluorescent field. That this is so is clarly seen from Fig. 5.

The first minimum in $F_1(t)$, however, appears to exhibit both atomic squeezing and negative values of the normally ordered variance of the fluorescent field. In the vicinity of this minimum the nonclassical character of the field is thus evident.

2. *Conclusions*

We have considered the problem of the occurrence of squeezed states in time-dependent resonance fluorescence from a two-level atom interacting with the resonant laser field. Analytical formulae describing fluctuations in the in-phase and out-of-phase components of the fluorescent field have been obtained. Our results show that in the transient régime of resonance fluorescence there is no restriction on the values of the Rabi frequency Ω for which squeezing can occur. We have also shown that, as the Rabi frequency of the driving field increases, the region of squeezing shifts towards shorter times and the maximum of squeezing that can be obtained increases. For very strong driving fields it can attain a value two times larger than for the steady state, but the detection of this squeezing would require very short detection times. We also show that in the transient régime time intervals exist in which squeezing in the atomic variables does not proceed in step with the negative values of the normally ordered variance of the fluorescent field. It is shown, however, that in the interval of maximal squeezing the two coincide and the nonclassical nature of the field is clearly demonstrated.

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