Analytical solutions for light absorption spectra of two driven atoms†

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Abstract. The absorption spectra of a pair of two-level atoms driven on resonance by a coherent laser field are considered analytically including the dipole–dipole interaction between the atoms. Analytical formulae for the absorption spectra are derived for four regions of the dipole–dipole interaction Ω_{12}:

$$\Omega \gg \gamma \gg |\Omega_{12}| \qquad \Omega \gg |\Omega_{12}| \gg \gamma$$

$$\Omega \simeq |\Omega_{12}| \gg \gamma \qquad |\Omega_{12}| \gg \gamma \gg \Omega$$

(2γ being Einstein's coefficient A, and Ω the Rabi frequency). The formulae, containing both absorption-like and dispersion-like terms, reveal explicitly the resonant structure of the absorption spectra. At great intensities of the driving field the absorption is negative in a certain range of frequencies. The dipole–dipole interaction between the atoms is shown to modify considerably the form of this absorption spectrum. For sufficiently strong dipole–dipole interaction it exhibits dispersion-like behaviour in the vicinity of 2Ω with a new region of negative absorption. The spectrum is asymmetric, albeit slightly. For strong dipole–dipole interaction and a weak driving field the spectrum consists of one absorption-like peak, located at $\omega - \omega_0 = \Omega_{12}$.

1. Introduction

The problem of the energy absorption from a weak probe field by a single atom driven by a coherent laser field has been considered theoretically by Mollow (1972a, b) and experimentally verified by Wu *et al* (1977). A similar experimental system has been used for observation of the 'AC Stark effect' (Schuda *et al* 1974, Walther 1975, Wu *et al* 1975).

It has been shown (Mollow 1972b, Haroche and Hartmann 1972, Freedhoff 1978) that the absorption spectrum changes drastically with the intensity of the driving field. For low driving field intensities the absorption peak occurs when the probe field is at resonance with the atomic transition frequency. At high driving field intensities the absorption becomes negative in a certain range of frequencies, indicating that stimulated emission takes place, amplifying the probe beam at the expense of the driving field.

Recently, interest has shifted to the absorption spectra of two or more atoms, coherently driven by a resonant laser field. It has been shown (Suguna and Agarwal 1979) that the two-atom absorption spectrum differs from that for one atom by a

scaling factor only, if the driving field is strong. Hassan *et al* (1982) have shown, for detuning of the driving field from atomic resonance but without dipole–dipole interaction, that the structure of the absorption spectrum in the higher harmonic regions is asymmetric and exhibits 'bumps' in its wings. The absorption spectra of optically bistable systems have also been analysed (Hassan and Walls 1978, Narducci *et al* 1979). Agarwal *et al* (1981) have provided a numerical analysis of the two-atom absorption spectrum including dispersion forces (retarded dipole–dipole interaction) between the atoms and have found that this interaction modifies the spectrum in the low harmonic regions. This modification is connected with a splitting of the usual Rabi sideband of the resonance fluorescence spectrum into a doublet (Cordes and MacAulay 1982, Griffin and Harris 1982, Ficek *et al* 1983).

The purpose of this paper is to present analytical results for the absorption spectrum of two atoms, coherently driven on resonance by a laser beam. The dipole–dipole interaction between the atoms is included and the interatomic separation is assumed to be much smaller than the resonant wavelength. We adapt Lehmberg's approach (Lehmberg 1970) to the case of coherent pumping and obtain a closed system of 15 equations describing the time evolution of the atomic variables. For interatomic separations much smaller than the resonant wavelength, this set of equations reduces to nine equations. Apart from the Rabi frequency Ω describing the interaction of either atom with the driving field, the time evolution of the system depends on the collective parameter Ω_{12} which describes the collective shift of energy levels due to the dipole–dipole interaction. We also make use of the quantum regression theorem (Lax 1968) to write equations for the two-time correlation functions. Applying the Laplace transform method, we obtain an analytical expression for the steady-state absorption spectrum of such a system. In the strong field limit the roots of the denominator defining the positions and widths of the spectral lines are found approximately for various ranges of Ω_{12}. We have also found the roots for the case of strong dipole–dipole interaction and weak driving field. This enables us to derive analytical formulae for the absorption spectrum in a form showing its dispersion-like and/or absorption-like structure.

2. Model and method

We consider two identical two-level atoms, confined to a region much smaller than the resonant wavelength (Dicke model), in the field of a linearly polarised, coherent laser beam, the frequency ω_L of which is assumed as exactly equal to the atomic transition frequency ω_0 i.e. detuning is zero. The atoms are coupled to all other modes of the electromagnetic field which are assumed as being initially in their vacuum state. A weak probe beam is introduced after the system has attained steady-state conditions. The absorption lineshape $U(\Delta\omega)$ of the probe beam of amplitude $\boldsymbol{E}'$ and frequency ω is given by (Kubo 1957, Mollow 1972a):

$$U(\Delta\omega) = \frac{2\omega}{\hbar}|\boldsymbol{\mu}\cdot\boldsymbol{E}'|^2 \operatorname{Re} \int_0^\infty \mathrm{d}t' \exp(-\mathrm{i}\omega t') \lim_{t\to\infty} \langle [S^-(t+t'), S^+(t)]\rangle \qquad (1)$$

where $\Delta\omega = \omega - \omega_0$, $\boldsymbol{\mu}$ is the transition electric dipole moment, $S^\pm = S_1^\pm + S_2^\pm$ are the collective dipole operators for two atoms in the Heisenberg picture, and the commutator is calculated in the absence of the probe beam, but the laser field is always present.

Henceforth we shall concentrate only on the function

$$P_A(\Delta\omega) = \text{Re} \int_0^\infty dt' \exp(-i\omega t') \lim_{t\to\infty} \langle [S^-(t+t'), S^+(t)] \rangle. \tag{2}$$

In order to obtain the time evolution of the commutator in (1) we apply Lehmberg's approach (Lehmberg 1970) which, for small interatomic separations, gives us the following equations of motion for the pseudo-spin operators of atom 1 (the slowly varying parts of the operators):

$$\frac{d}{d\tau} S_1^\pm = -\tfrac{1}{2} S_1^\pm \pm 2i\beta S_1^Z + (1 \mp i\ell) S_1^Z S_2^\pm$$

$$\frac{d}{d\tau} S_1^Z = -(S_1^Z + \tfrac{1}{2}) + i\beta(S_1^+ - S_1^-) - \tfrac{1}{2}[(1 + i\ell) S_1^+ S_2^- + \text{HC}], \tag{3}$$

where S_1^+ and $S_1^- = (S_1^+)^+$ are operators raising and lowering the energy of atom 1, and S_1^Z describes its energy. These operators fulfil the well known commutation relations

$$[S_i^+, S_j^-] = 2S_i^Z \delta_{ij} \qquad [S_i^Z, S_j^\pm] = \pm S_i^\pm \delta_{ij} \qquad i,j = 1,2 \tag{4}$$

and

$$\tau = 2\gamma t \qquad \beta = \Omega/4\gamma \qquad \ell = \Omega_{12}/\gamma. \tag{5}$$

In (5), 2γ is the Einstein A coefficient for spontaneous emission, Ω the Rabi frequency describing the interaction of either atom with the laser beam†, and

$$\Omega_{12} \simeq \frac{3\gamma}{2(k_0 r_{12})^3} [1 - 3(\hat{\boldsymbol{\mu}} \cdot \hat{\boldsymbol{r}}_{12})^2] \qquad (k_0 r_{12} \ll 1) \tag{6}$$

represents the dipole–dipole interaction (Stephen 1964, Lehmberg 1970, Agarwal 1974), where $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{r}}_{12}$ are unit vectors along the transition electric dipole moment and the vector $\boldsymbol{r}_{12} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, respectively, and $k_0 = \omega_0/c$, $r_{12} = |\boldsymbol{r}_{12}|$.

The equations for atom 2 have the same form as (3) except for the interchange of indices $1 \leftrightarrow 2$. Since $(S_i^\pm)^2 \equiv 0$, the set of equations (3) leads to a finite hierarchy of equations for the atomic variables. For two atoms, we have a closed system of fifteen first-order equations. The solving of these equations is facilitated by taking suitable linear combinations of the operators. These combinations lead to a set of nine operators which are symmetric under interchange of the atoms and a set of six operators which are antisymmetric. When the atomic separations r_{12} are much less than the resonant wavelength the antisymmetric operators no longer contribute to the spectrum (Ficek *et al* 1983), and we have to solve a set of nine coupled equations in symmetric operators only. We now write this set of equations in matrix form:

$$\frac{d}{d\tau} \boldsymbol{X} = \check{\mathbf{A}} \boldsymbol{X} + \boldsymbol{\alpha} \tag{7}$$

† For the role of the Rabi frequency in resonance fluorescence, see the review article of Knight and Milonni (1980).

where $\check{A}$ is the following real 9×9 matrix:

$$\check{A} = \begin{pmatrix} -1 & 4\beta & 0 & 0 & 1 & 0 & \frac{1}{2}\ell & -\ell & 0 \\ -\beta & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta & -1 & -1 & 0 & 2\beta & 4 & 0 & 0 & 0 \\ -\beta & 0 & 0 & -1 & 2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\beta & -2\beta & -2\beta & -2 & 8\beta & 0 & -\frac{1}{2}\ell & 0 \\ 0 & 0 & 0 & 0 & -\beta & -2 & 0 & 0 & 0 \\ -\frac{1}{2}\ell & 0 & 0 & 0 & \ell & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}\ell & 0 & 0 & -2 & -2\beta \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta & 2\beta & -1 \end{pmatrix} \quad (7a)$$

The column vector X has the components

$$X_1 = S_1^+ + S_2^+ + S_1^- + S_2^-$$

$$X_2 = S_1^+ S_1^- + S_2^+ S_2^-$$

$$X_3 = S_1^+ S_2^- + S_2^+ S_1^-$$

$$X_4 = S_1^+ S_2^+ + S_1^- S_2^-$$

$$X_5 = S_1^+ S_1^- S_2^- + S_1^+ S_2^+ S_1^- + S_2^+ S_2^- S_1^- + S_1^+ S_2^+ S_2^- \quad (7b)$$

$$X_6 = S_1^+ S_2^+ S_1^- S_2^-$$

$$X_7 = -i(S_1^- + S_2^- - S_1^+ - S_2^+)$$

$$X_8 = -i(S_1^+ S_1^- S_2^- - S_1^+ S_2^+ S_1^- + S_2^+ S_2^- S_1^- - S_1^+ S_2^+ S_2^-)$$

$$X_9 = -i(S_1^- S_2^- - S_1^+ S_2^+).$$

The components of the column vector α are given by

$$\alpha_i = -4\beta \delta_{1i}. \quad (7c)$$

The system of equations (7) can be transformed, using the Laplace transform method, into an easily solvable system of algebraic equations in transformed variables. The solution of equations (7) will be used in our calculations of the absorption spectrum.

3. Absorption spectra

To study the absorption spectrum, we have to find the correlation functions $\langle S^+(t)S^-(t+t')\rangle$ and $\langle S^-(t+t')S^+(t)\rangle$. From the quantum regression theorem (Lax 1968), it is well known that for $t' > 0$ the two-time average $\langle S^+(t)S^-(t+t')\rangle$ satisfies the same equation of motion as the one-time average $\langle S^-(t)\rangle$. By equations (7), the Laplace transform of the steady-state value of the commutator $\langle [S^-(t+t'), S^+(t)]\rangle$ leads to

$$P_A(z) = \int_0^\infty dt' \exp(-zt') \lim_{t\to\infty} \langle [S^-(t+t'), S^+(t)]\rangle$$

$$= \frac{z}{D} \frac{L(z) - i\ell R(z)}{zM(z)} \quad (8)$$

with

$$L(z) = [(z+1)^2(z+2)+2\beta^2(2z+1)][(z+1)(z+2)^3+2\beta^2(3z+4)^2$$
$$+2\beta^2(z+1)(z+2)^3+4\beta^2(z+2)^2-8\beta^4(z+2)^2(2z+1)-32\beta^4(3z+4)$$
$$-128\beta^6(2z+3)]+[(z+1)(z+2)+2\beta^2(z^2+3z+5)+8\beta^4]$$
$$\times[(z+1)^2(z+2)^3+4\beta^2z(z+1)^2+2\beta^2(z+2)(8z^2+17z+8)$$
$$+32\beta^4(2z+3)]+\ell^2\{\tfrac{1}{2}(z+1)^3(z+2)^2[(z+2)^2+1]$$
$$+\tfrac{1}{2}(z+1)(z+2)[(z+1)(z+2)(2z+3) \tag{8a}$$
$$+(z+1)(10z^2+22z+16)+(z+2)(10z^2+31z+25)]\beta^2$$
$$+2[(z+1)(z+2)(2z^2+14z+10)+(z+2)(11z+8)+z(2z^2+9z+8)]\beta^4$$
$$-32z(z+3)\beta^6\}+\tfrac{1}{8}\ell^4(z+1)(z+2)^2[(z+1)^2+8\beta^2]$$

$$R(z) = (z+1)^2(z+2)^4-4(z+1)(z+2)(2z^3+5z^2+2z-2)\beta^2$$
$$+4[(z+1)(z+2)(6z^2-19z-22)-(z+1)^2(5z+4)+z(2z+1)]\beta^4$$
$$+32(3z^2-2z-6)\beta^6+\ell^2[\tfrac{1}{4}(z+1)^2(z+2)^2(z^2+4z+5)$$
$$+(z+1)(z+2)(3z^2+4z+2)\beta^2-4(z+1)(2z+3)\beta^4]$$
$$+\tfrac{1}{16}\ell^4(z+1)^2(z+2)^2 \tag{8b}$$

$$M(z) = [(z+1)^2(z+2)+2\beta^2(2z+1)][(z+1)^2(z+2)^3+4\beta^2z(z+1)^2$$
$$+2\beta^2(z+2)(8z^2+17z+8)+32\beta^4(2z+3)]$$
$$+\ell^2[\tfrac{1}{4}(z+1)^2(z+2)^2(2z^2+6z+5)+(z+1)(z+2)(6z^2+11z+4)\beta^2$$
$$+4(z+1)(z+3)\beta^4]+\tfrac{1}{16}\ell^4(z+1)^2(z+2)^2 \tag{8c}$$

and $D = 12\beta^4+4\beta^2+\tfrac{1}{4}\ell^2+1$.

Formula (8) is the exact analytical expression for the absorption spectrum of the system of two atoms including their dipole–dipole interaction. It is valid for arbitrary incident field strengths β and for arbitrary dipole–dipole interaction ℓ.

The actual absorption spectrum can be easily plotted with the help of a computer by having recourse to the definition

$$P_A(\Delta\omega) = \operatorname{Re} P_A(z)|_{z=i(\omega-\omega_0)/2\gamma}. \tag{9}$$

Instead of doing this, however, we calculated analytical formulae for a strong laser field and different ranges of values of the dipole–dipole interaction parameter ℓ, as well as for strong dipole–dipole interaction and a weak driving field. In the strong driving field limit our formulae include all terms up to the order of $1/\beta^2$. In the case of strong dipole–dipole interaction, $|\ell| \gg 1$, and a weak driving field, $\beta \ll 1$, we include all terms up to the order of β^2/ℓ.

Thus we consider the absorption spectra for the four cases: (i) $\beta \gg 1 \gg |\ell|$; (ii) $\beta \gg |\ell| \gg 1$; (iii) $\beta \simeq |\ell| \gg 1$; (iv) $|\ell| \gg 1 \gg \beta$.

(i) $\beta \gg 1 \gg |\ell|$. In this case, interaction of the atoms with the driving laser field is very strong but the dipole–dipole interaction between the atoms is very weak and the

absorption spectrum given by equation (8) takes the following form:

$$P_{\text{A}}(\nu) = \frac{1}{3}\left[\frac{1}{4\beta^2}\frac{1}{\nu^2+\frac{1}{4}}+\frac{1}{\beta}\left(1+\frac{\ell^2}{16}\right)\frac{(\nu-2\beta)}{(\nu-2\beta)^2+\frac{1}{16}(3+\ell^2/4)^2}+\beta\rightarrow-\beta\right.$$

$$+\frac{1}{4\beta^2}\left(1+\frac{\ell^2}{32}\right)\frac{\frac{1}{4}(3+\ell^2/4)}{(\nu-2\beta)^2+\frac{1}{16}(3+\ell^2/4)^2}+\beta\rightarrow-\beta$$

$$\left.+\frac{\ell^2}{16\beta}\frac{(\nu+2\beta)}{(\nu+2\beta)^2+\frac{1}{16}(7-\ell^2/4)^2}+\beta\rightarrow-\beta\right] \tag{10}$$

where

$$\nu = \frac{\omega-\omega_0}{2\gamma}. \tag{10a}$$

Formula (10) shows explicitly the resonance structure of the absorption spectrum. For $\ell=0$, it goes over into that obtained by Mollow (1972b) for the one-atom case, except for a so-called scaling factor (Amin and Cordes 1978). It has been shown (Ficek *et al* 1981, Freedhoff 1982), however, that this scaling factor appears only in S^2-conserving systems (with interatomic separations much less than the resonant wavelength). Otherwise, the factor $\frac{1}{3}$ is replaced by $\frac{1}{4}$. In this case the absorption spectrum shows dispersion-like behaviour around $\pm2\beta$; this is connected with the existence of stimulated emission, amplifying the probe beam at the expense of the driving field.

The dipole–dipole interaction between the atoms introduces new terms (those that vanish for $\ell=0$) and modifies the widths of the lines. However, as long as $|\ell|\ll1$, the modifications in the absorption spectrum are very small and can be neglected. Nevertheless, the formula (10) visualises the tendency of the modifications due to the dipole–dipole interaction.

(ii) $\beta\gg|\ell|\gg1$. In this case, the dipole–dipole interaction, still being much weaker than the interaction with the field, becomes more pronounced and the absorption spectrum takes the form:

$$P_{\text{A}}(\nu) = \frac{1}{3}\left\{\frac{1}{4\beta^2}\frac{1}{\nu^2+\frac{1}{4}}+\left[\frac{1}{2\beta}\left(1-\frac{\ell}{4\beta}\right)\frac{(\nu-2\beta-\frac{1}{4}\ell)}{(\nu-2\beta-\frac{1}{4}\ell)^2+\frac{25}{16}}+\ell\rightarrow-\ell\right]\right.$$

$$+\left[-\frac{1}{2\beta}\left(1-\frac{\ell}{4\beta}\right)\frac{(\nu+2\beta+\frac{1}{4}\ell)}{(\nu+2\beta+\frac{1}{4}\ell)^2+\frac{25}{16}}+\ell\rightarrow-\ell\right]$$

$$+\left[\frac{1}{4\beta}\left(\frac{1}{2\beta}+\frac{4}{\ell}\right)\frac{\frac{5}{4}}{(\nu+2\beta+\frac{1}{4}\ell)^2+\frac{25}{16}}+\ell\rightarrow-\ell\right]$$

$$\left.+\left[\frac{1}{4\beta}\left(\frac{1}{2\beta}+\frac{4}{\ell}\right)\frac{\frac{5}{4}}{(\nu-2\beta-\frac{1}{4}\ell)^2+\frac{25}{16}}+\ell\rightarrow-\ell\right]\right\}. \tag{11}$$

The formula (11) for the absorption spectrum exhibits some interesting features. First of all, the dispersion-like terms at $\nu=\pm2\beta$ split into doublets with components distant by $\frac{1}{2}\ell$.

Since $|\ell|\gg1$, this splitting is so large that the two components can be resolved. Also, the dipole–dipole interaction gives some new absorption-like terms. The spectrum calculated using equation (11) for $\beta=50$ and $|\ell|=10$ is plotted in figure 1,
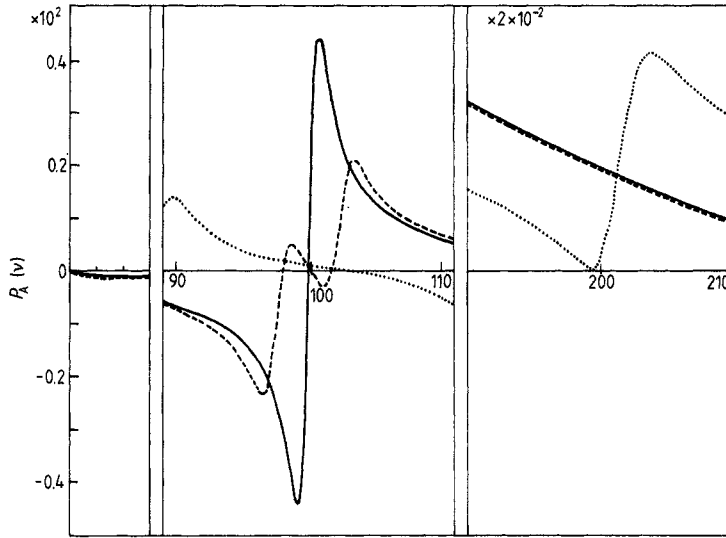
Figure 1. Absorption spectrum for $\beta = 50$ and: $\ell = 0$ (equation (10)), full curve; $\ell = 10$ (equation (11)), broken curve; $\ell = 50$ (equation (12)), dotted curve.

showing negative values of the absorption spectrum and thus pointing to amplification of the probe beam.

It is seen that the spectrum is modified around $\nu = \pm 2\beta$. This modification is connected with the splitting of the usual Rabi side band of the resonance fluorescence spectrum into a doublet and is in agreement with the numerical result obtained by Agarwal *et al* (1981).

(iii) $\beta \simeq |\ell| \gg 1$. As the interatomic distance decreases, the dipole–dipole interaction becomes comparable with the Rabi frequency and the absorption spectrum takes the form:

$$P_A(\nu) = \tfrac{1}{3}\Bigg(\frac{1}{u\beta} \frac{(1 - \tfrac{1}{4}\alpha u - \tfrac{1}{8}\alpha^2)(\nu - u\beta - \tfrac{1}{2}\alpha\beta)}{(\nu - u\beta - \tfrac{1}{2}\alpha\beta)^2 + \tfrac{1}{16}(5 + \alpha/2u)^2} + \beta \to -\beta$$

$$+ \frac{1}{u\beta} \frac{(1 + \tfrac{1}{4}\alpha u - \tfrac{1}{8}\alpha^2)(\nu - u\beta + \tfrac{1}{2}\alpha\beta)}{(\nu - u\beta + \tfrac{1}{2}\alpha\beta)^2 + \tfrac{1}{16}(5 - \alpha/2u)^2} + \beta \to -\beta$$

$$+ \frac{\alpha^2}{4u\beta} \frac{(\nu - 2u\beta)}{(\nu - 2u\beta)^2 + \tfrac{1}{4}(3 + 8/u^2)^2} + \beta \to -\beta$$

$$+ \frac{\alpha^2[w + (u^3/\alpha)v]}{16u^4(18 + \alpha^2)\beta^2} \frac{\tfrac{1}{2}(3 + 8/u^2)}{(\nu + 2u\beta)^2 + \tfrac{1}{4}(3 + 8/u^2)^2} + u \to -u$$

$$+ \frac{\alpha m + n/u}{8u^2\alpha(u + \tfrac{1}{2}\alpha)(\alpha + 6u)\beta^2} \frac{\tfrac{1}{4}(5 + \alpha/2u)}{(\nu + u\beta + \tfrac{1}{2}\alpha\beta)^2 + \tfrac{1}{16}(5 + \alpha/2u)^2} + \alpha \to -\alpha$$

$$+ \frac{\alpha p + q/u}{32u^2\alpha(u + \tfrac{1}{2}\alpha)(\alpha + 6u)\beta^2} \frac{\tfrac{1}{4}(5 + \alpha/2u)}{(\nu - u\beta - \tfrac{1}{2}\alpha\beta)^2 + \tfrac{1}{16}(5 + \alpha/2u)^2} + \alpha \to -\alpha$$

$$+ \frac{[s^2(8 + 7\alpha^2 + 2s) + 6\alpha^4(1 - \tfrac{1}{4}\alpha^2)]}{8u^4 s\beta^2} \frac{(8 + \alpha^2 - s)/2u^2}{\nu^2 + (8 + \alpha^2 - s)^2/4u^4} + s \to -s \Bigg) \quad (12)$$

where we have introduced the notation

$$\ell = 2\alpha\beta \qquad u^2 = 4 + \tfrac{1}{4}\alpha^2 \qquad s^2 = 16 + \alpha^2 + \tfrac{1}{4}\alpha^4$$

$$w = 648 + \tfrac{189}{2}\alpha^2 + \tfrac{11}{2}\alpha^4 + \tfrac{1}{8}\alpha^6 \qquad v = 72 - 2\alpha^2 - \alpha^4$$

$$m = 224 + 234\alpha^2 + 29\alpha^4 + \tfrac{33}{32}\alpha^6$$

$$n = 768 + 296\alpha^2 + 149\alpha^4 + \tfrac{33}{2}\alpha^6 + \tfrac{33}{64}\alpha^8 \tag{13}$$

$$p = 896 - 472\alpha^2 - 36\alpha^4 + \tfrac{1}{8}\alpha^6$$

$$q = 3072 - 372\alpha^2 - 316\alpha^4 - 17\alpha^6 + \tfrac{1}{16}\alpha^8.$$

The parameter $\alpha = \ell/2\beta$ provides a measure of the strength of the dipole–dipole interaction with respect to the Rabi frequency. For $\alpha = 1$ we have $\ell = 2\beta$, and the dipole–dipole interaction (in units of frequency) is equal to the Rabi frequency.

In this case, the absorption spectrum is considerably modified with respect to case (ii). The components of the doublet in the vicinity of the Rabi frequency are very well resolved, with splitting equal to $\alpha\beta$. New dispersion-like terms appear which are connected with the existence of resonances at $\pm 2u\beta$. These new dispersion-like terms introduce modifications of the absorption spectrum and, as is shown in figure 2, lead to a negative value of the absorption spectrum in the high harmonic region. The experimental verification of these negative values, once performed, will provide evidence of the dipole–dipole interaction between the atoms.

In addition, the spectrum is now nonsymmetric around $\nu = 0$, but this asymmetry is very small if α is of the order of unity.

(iv) $|\ell| \gg 1 \gg \beta$. For very small interatomic separations the dipole–dipole interaction can become predominant in equation (8) and, for weak laser fields, leads to the
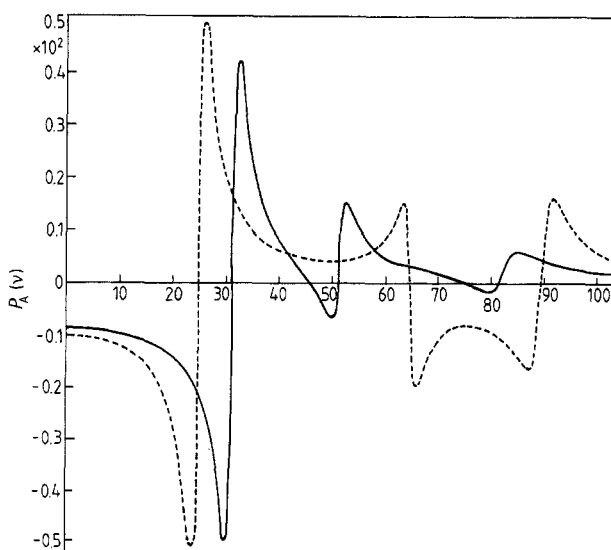
Figure 2. Absorption spectrum given by equation (12) for $\beta = 20$, $\alpha = 1$ (full curve) and $\alpha = 2$ (broken curve). Only one side of the spectrum is shown since the asymmetry discussed in the text is very small for these values of α and cannot be visualised on this scale.

following formula for the absorption spectrum:

$$P_A(\nu) = \left(\frac{2}{(\nu - \frac{1}{2}\ell)^2 + 1} + \frac{28\beta^2}{\ell} \frac{(\nu - \frac{1}{2}\ell)}{(\nu - \frac{1}{2}\ell)^2 + 1} \right.$$

$$\left. - \frac{28\beta^2}{\ell} \frac{(\nu - \frac{1}{2}\ell)}{(\nu - \frac{1}{2}\ell)^2 + 4} - \frac{15\beta^2}{\ell} \frac{(\nu + \frac{1}{2}\ell)}{(\nu + \frac{1}{2}\ell)^2 + 4} \right). \tag{14}$$

In this case the spectrum exhibits one very pronounced absorption peak, located at $\nu = \frac{1}{2}\ell$. This means that we have attenuation of the probe field by the atomic system. There are still some dispersion-like terms, but their amplitudes (which are proportional to β^2/ℓ) are very small. The spectrum given by equation (14) is illustrated in figure 3 for $\ell = 10$ and two different values of the field strength. The comparison of our expression (14) with those obtained by Mollow (1972b), Suguna and Agarwal (1979) and Yeh (1982) shows that our spectrum resembles that of the one-atom case (or the two-atom case without dipole–dipole interaction) with great detuning. So, in this case, the dipole–dipole interaction plays a role similar to that of detuning.

4. Summary

We have considered in this paper the problem of energy absorption from a weak probe radiation beam by a pair of two-level atoms, resonantly driven by another, coherent laser beam. Particular attention has been paid to the role of the dipole–dipole interaction between the atoms in the absorption spectrum of such a system. The set of nine equations describing the time evolution of the system in which the atoms are confined to a region much smaller than the resonant wavelength has been solved with the use of the Laplace transform method. An exact analytical formula for the steady-state absorption spectrum has been obtained using the quantum regression theorem. Moreover, analytical expressions revealing explicitly the resonant structure of the spectrum have been calculated for various ranges of values of the dipole–dipole interaction parameter ℓ and of the driving field strength β. These expressions permit the evaluation of the contributions to the absorption spectrum from the particular dispersion-like and absorption-like terms.

We have shown that, for weak dipole–dipole interaction, the absorption spectrum has the same form as that obtained by Mollow (1972b) for the one-atom case apart from the scaling factor. In a certain range of frequencies, the absorption is negative indicating that stimulated emission takes place amplifying the probe field at the expense of the strong driving field. Our formula shows explicitly the trend of the modification of the spectrum when the dipole–dipole interaction increases, i.e. the interatomic separation decreases; here, we can have a situation in which $\beta \gg |\ell| \gg 1$. In this case the absorption spectrum differs significantly from the previous one. The components at $\nu = \pm 2\beta$ split into doublets with separation equal to $\frac{1}{2}\ell$, and some additional absorption-like terms appear.

For still smaller interatomic distances the dipole–dipole interaction can become comparable with the interaction of the atoms with the driving field and the doublet splitting becomes clearly visible and additional dispersion-like terms occur at $\nu = \pm 2u\beta$. These extra terms correspond to the additional sidebands in the resonance fluorescence spectrum of such a system, but the Lorentzian shape of the fluorescence lines transforms into the dispersion-like shape apparent in our absorption spectrum. The numerical

results of Hassan *et al* (1982) have shown 'bumps' in the higher harmonic region in the case of great detuning; these 'bumps', signifying a reduction in absorption, were however not deep enough to reach negative values and the authors suggested that the 'bumps' could attain negative values (i.e. amplification of the beam) for a greater number of atoms or greater detuning. As we have shown if the dipole–dipole interaction is strong enough, regular dispersion-like behaviour of the absorption spectrum is obtained in the vicinity of $\pm 4\beta$ with a region of negative absorption. Also, the spectrum becomes asymmetric around $\nu = 0$ although this asymmetry is small as long as the dipole–dipole interaction is comparable with the atom–field interaction. All these features of the absorption spectrum are graphically illustrated in figures 1 and 2.

For very strong dipole–dipole interaction and weak driving field the absorption spectrum exhibits one very pronounced absorption peak at $\nu = \frac{1}{2}\ell$ as well as some dispersion-like terms which do not contribute significantly to the spectrum. This is shown in figure 3. In this case the spectrum is similar to the absorption spectrum for large detuning and a weak driving field.
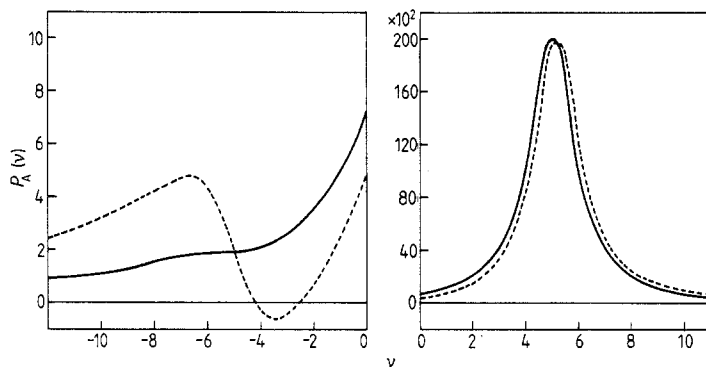
Figure 3. Equation (14), plotted against ν for $\ell = 10$, $\beta = 0.1$ (full curve) and $\beta = 0.3$ (broken curve).

The formulae, derived by us in this paper for the absorption spectrum allow us to keep track of the changes in the absorption spectrum due to changes in interatomic interaction and driving field strength as well.

Though it is an open question whether these effects of dipole–dipole interaction on the absorption spectrum are accessible to observation, experiments at very low frequency have been performed for very small numbers of atoms in an atomic beam (Raimond *et al* 1982). Also, experiments with atoms in a resonant field in the presence of a conducting metallic surface have been reported (Kuhn 1970)—the case of one atom at such a surface (Morawitz 1969) is equivalent to the problem of two atoms considered in the present paper.

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