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SQUEEZING IN TWO-ATOM RESONANCE FLUORESCENCE

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INTRODUCTION

Squeezing —— a new nonclassical effect in radiation theory —— has recently become the subject of extensive discussion (Milburn and Walls 1981 and 1983, Walls and Zoller 1981, Mandel 1982a and b, Lugiato and Strini 1982, Kozierowski and Kielich 1983). It is characterized by a field state in which the variance of two noncommuting observables is less than one half of the absolute value of their commutator. The obtaining of squeezed states in experiment would give the opportunity to reduce quantum fluctuations in one quadrature component of the field at the expense of increased fluctuations in the other component. The interest in such states is stimulated by their potential use to reduce fluctuations due to quantum noise in the Michelson interferometer to detect gravity waves (Caves 1981) as well as their possible applications in optical communication systems (Yuen and Shapiro 1979).

In this paper we show that squeezed states arise in resonance fluorescence of two interacting atoms.

RESONANCE FLUORESCENCE AND SQUEEZED STATES

The two quadrature components of the electromagnetic field: the in-phase component E_1 and the out-of-phase component E_2, can be expressed by the positive- and negative-frequency parts $E^{(+)}$ and $E^{(-)}$ as:

$$E_1 = E^{(+)} + E^{(-)} \quad \text{and} \quad E_2 = -i(E^{(+)} - E^{(-)}). \tag{1}$$

For quantized fields these two operators do not commute and their commutator takes the form

$$[E_1 , E_2] = 2iC , \tag{2}$$

where C is a positive c number.

The electromagnetic field is said to be in a squeezed state (Walls and Zoller 1981, Mandel 1982a) whenever the variance of E_1 or E_2 is less than C. Since the variances can be written as

$$\begin{aligned} \langle(\Delta E_1)^2\rangle &= C + \langle:(\Delta E_1)^2:\rangle \\ \langle(\Delta E_2)^2\rangle &= C + \langle:(\Delta E_2)^2:\rangle , \end{aligned} \tag{3}$$

squeezing requires either $\langle:(\Delta E_1)^2:\rangle$ or $\langle:(\Delta E_2)^2:\rangle$ to be negative (the colon stands for normal ordering of the operators).

The fluorescent field from N identical two-level atoms, in the far field zone, is given by (Lehmberg 1970 , Agarwal 1974):

$$E^{(+)}(\vec{r},t) = E_0^{(+)}(\vec{r},t) - k^2 \sum_{i=1}^{N} \frac{[\hat{r}\times[\hat{r}\times\vec{\mu}]]}{r} S_i^-(t-\tfrac{r}{c})e^{i\vec{k}\cdot\vec{r}_i}, \tag{4}$$

$E_0^{(+)}(\vec{r},t)$ being the free field. S^+ and S^- are atomic raising and lowering operators; $\hat{\mu}$ and $\hat{r}$ are unit vectors along the transition dipole moment $\vec{\mu}$ and the observation point vector $\vec{r}$, respectively.

For the fluorescent field, from eqs (1) and (4), we obtain

$$\begin{aligned} \langle:(\Delta E_1)^2:\rangle &= \psi^2(\vec{r})\left[\langle(\Delta R_1)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle|\right] , \\ \langle:(\Delta E_2)^2:\rangle &= \psi^2(\vec{r})\left[\langle(\Delta R_2)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle|\right] , \end{aligned} \tag{5}$$

where $\psi(\vec{r}) = (k^2/2\pi\varepsilon_0 r)[\hat{r}\times(\hat{r}\times\vec{\mu})]$ and $R_1 = (S^+ + S^-)/2$, $R_2 = (S^+ - S^-)/2i$, $R_3 = [S^+ , S^-]/2$ are Dicke's collective-spin operators, fulfilling the commutation relation

$$[R_1 , R_2] = iR_3 . \tag{6}$$

Relation (5) expresses the squeezing condition for the fluorescent light in terms of the operators of the atomic system. To study the problem of squeezing in two-atom resonance fluorescence, we apply Lehmberg's master equation approach (Lehmberg 1970) which leads to the following equations of motion for the pseudo-spin operators of individual atoms:

$$\frac{d}{d\tau} S_i^{\pm} = -\tfrac{1}{2}(1 \mp i\Delta) S_i^{\pm} + \left[\pm i\beta + \tfrac{1}{2}(a \mp ib) S_j^{\pm} \right] \left(S_i^+ S_i^- - S_i^- S_i^+ \right), \quad i \neq j, \tag{7}$$

where we have introduced the notation:

$$\tau = 2\gamma t, \quad \beta = \frac{\Omega}{4\gamma}, \quad a = \frac{\gamma_{ij}}{\gamma}, \quad b = \frac{\Omega_{ij}}{\gamma}, \quad \Delta = \frac{\delta}{\gamma}. \tag{8}$$

Ω is the Rabi frequency, γ_{ij} the collective damping parameter, Ω_{ij} the level shift due to the dipole-dipole interaction between the atoms, 2γ the Einstein A coefficient, and $\delta = \omega_0 - \omega_L$ is the detuning of the laser frequency ω_L from the atomic transition frequency ω_0. Both γ_{ij} and Ω_{ij} depend on the interatomic separation r_{ij} and angular orientation functions (Stephen 1964, Lehmberg 1970).

For two atoms, this set of equations generates a closed system of 15 equations describing the time evolution of the atomic variables: 9 equations for symmetric and 6 for antisymmetric combinations of the atomic operators. We have derived explicitly this system of equations in an earlier paper (Ficek et al. 1983). Here, we give only the steady-state solutions that are needed to calculate the squeezing effect; these are:

$$\begin{aligned}
\langle S_1^+ + S_2^+ + S_1^- + S_2^- \rangle &= -4\sqrt{z}\left[2z + (1+a)(1+\Delta^2)\right]/D, \\
\langle S_1^+ + S_2^+ - S_1^- - S_2^- \rangle &= -4i\sqrt{z}\left[2\Delta z + (\Delta+b)(1+\Delta^2)\right]/D, \\
\langle S_1^+ S_1^- + S_2^+ S_2^- \rangle &= 2z\left[2z + (1+\Delta^2)\right]/D, \\
\langle S_1^+ S_2^- + S_2^+ S_1^- \rangle &= 2z(1+\Delta^2)/D, \\
\langle S_1^+ S_2^+ + S_1^- S_2^- \rangle &= 2z\left[1 + a - \Delta(\Delta+b)\right]/D, \\
\langle S_1^+ S_2^+ - S_1^- S_2^- \rangle &= 2iz\left[\Delta(1+a) + (\Delta+b)\right]/D,
\end{aligned} \tag{9}$$

where $D = 4z^2 + (1+\Delta^2)\left[4z + (1+a)^2 + (\Delta+b)^2\right]$, (10)

with $z = 4\beta^2 = (\Omega/2\gamma)^2$.

From eqs (5) and the steady-state solutions (9) we find for two atoms:

$$F_1 = \langle(\Delta R_1)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle| = z(N_1 + N_3)/D^2 , \qquad (11)$$

$$F_2 = \langle(\Delta R_2)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle| = z(N_2 - N_3)/D^2 , \qquad (12)$$

where we have introduced

$$N_1 = 8z^3 + 4\left[(2+a) + \Delta(2\Delta-b) + (\Delta^2-1)\cos k\vec{r}_{12}\cdot\hat{r}\right]z^2 + 2(1+\Delta^2)\left[1+a^2+\Delta^2+b^2+2(\Delta^2-2a-1)\cos k\vec{r}_{12}\cdot\hat{r}\right]z , \qquad (13)$$

$$N_2 = 8z^3 + 4\left[(2-a) + \Delta(2\Delta+b) + (1-\Delta^2)\cos k\vec{r}_{12}\cdot\hat{r}\right]z^2 + 2(1+\Delta^2)\left[1+a^2+\Delta^2+b^2+2(1-2\Delta b-\Delta^2)\cos k\vec{r}_{12}\cdot\hat{r}\right]z , \qquad (14)$$

$$N_3 = (1+\Delta^2)\left\{a(1+a)^2 + (2+a)(\Delta+b)^2 - \Delta\left[(b+2\Delta)(1+a)^2 + b(\Delta+b)^2\right] + (1+\Delta^2)\left[(\Delta+b)^2-(1+a)^2\right]\cos k\vec{r}_{12}\cdot\hat{r}\right\} , \qquad (15)$$

D being the same as in eqs (10).

These formulae are exact analytical expressions describing the field fluctuations in the steady-state resonance fluorescence of two interacting atoms. Negative values of F_1 or F_2 mean squeezing in the corresponding component of the field. In the absence of interatomic interactions a = b = 0 , and eqs (10) and (11) go over into the equations obtained by Walls and Zoller (1981) for one-atom resonance fluorescence except for the factor 2 standing for two atoms. Due to the factor 2, our numerical values are two times greater than those of Walls and Zoller. However, the mean value of the field is also two times greater, and the fluctuations scaled to the mean value of the field remain the same.

RESULTS AND DISCUSSION

F_1 , as given by eq. (11), is plotted in Fig.1 against the detuning Δ for $\vec{r}_{12}\perp\hat{r}$ and for various interatomic separations r_{12}.

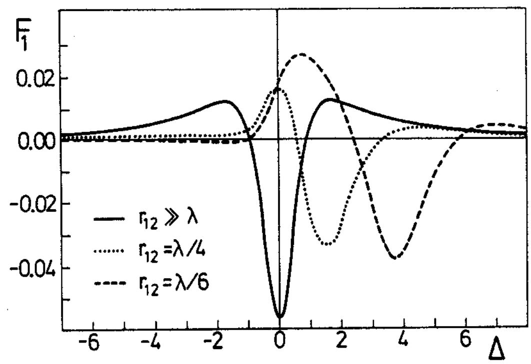

Fig. 1. F_1 against Δ for $\vec{r}_{12} \perp \hat{r}$ and for various r_{12}; the field strength is such that z = 0.1.

Fig.1 shows clear evidence of squeezing in F_1 which is the most pronouced when $\Delta = 0$ and the atoms are mutually independent($r_{12} \to \infty$) . As the inter-atomic separation r_{12} decreases, the squeezing shifts to the region of finite Δ and its maximum (the minimum of the curves) occurs when the detuning Δ and the dipole-dipole interaction b cancel out mutually.

The component F_2 given by eq. (12) is illustrated graphically in Fig.2.

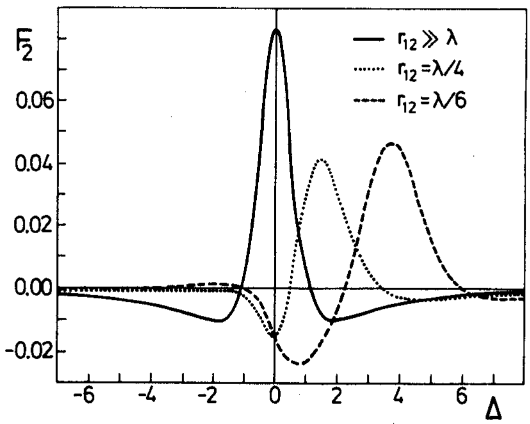

Fig.2. F_2 against Δ ; other parameters as in Fig.1.

It is seen from Fig.2 that, although there is no squeezing in F_2 if $\Delta = 0$ and if the atoms are far apart, there are regions of Δ where F_2 becomes negative, thus squeezed. Again, the dipole-dipole interaction between the atoms shifts the region of squeezing and, in contrast to F_1, enhances the amount of squeezing although the squeezing in this component is less pronounced than in F_1.

If the interatomic separation r_{12} decreases, the dipole-dipole interaction increases. For very strong interactions($|b| \rightarrow \infty$) and a not excessively strong field, both F_1 and F_2 tend to zero. This is shown in Fig.3, where F_1 and F_2 are plotted against the distance separating the atoms for given values of the field strength and detuning.

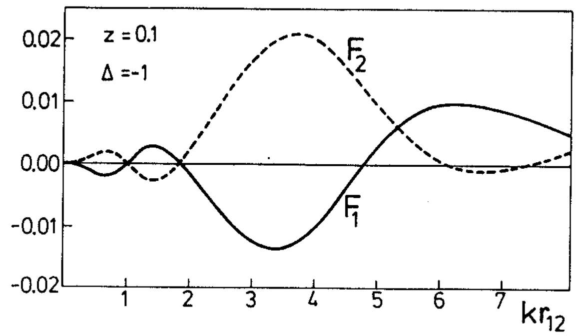

Fig.3. F_1 and F_2 against the interatomic separation r_{12} for z = 0.1 and Δ = -1.

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