Photon antibunching and squeezing in resonance fluorescence of two interacting atoms

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The equal-time intensity correlation function and variances of the in-phase and out-of-phase components of the electric field radiated by a system of two interacting atoms in the process of resonance fluorescence are calculated analytically for arbitrary values of the field strength, interatomic interaction, and detuning and are illustrated graphically. It is shown that not only photon antibunching but also squeezing can occur in two-atom resonance fluorescence. The influence of dipole-dipole interaction between the atoms on both nonclassical effects is discussed. It is shown that a considerable amount of photon antibunching and squeezing can be obtained for finite detuning of the laser frequency from atomic resonance, especially when the detuning and dipole-dipole interaction parameters cancel out mutually. The maximum value of squeezing for interacting atoms is shown to be less, however, than that for noninteracting atoms. Very strong dipole-dipole interaction reduces the squeezing effect to zero.

I. INTRODUCTION

Photon antibunching and squeezing are two effects which reveal the quantum properties of the radiation field and cannot be explained if the field is treated classically. Photon antibunching is characterized by a quantum state of the field in which the variance of the number of photons is less than the mean number of photons, i.e., the photons exhibit sub-Poissonian statistics. Squeezing, on the other hand, is characterized by a field state in which the variance of one of two noncommuting observables is less than one half of the absolute value of their commutator. Such a state of the field is referred to as a squeezed state. The obtaining of squeezed states gives the opportunity to reduce quantum fluctuations in one quadrature component of the field at the expense of increased fluctuations in the other component. In general, there is no direct connection between photon antibunching and squeezing, and states exist that exhibit the former but not the latter effect, and vice versa. However, both these effects have one feature in common—the states of the electromagnetic field exhibiting them have no classical analog in the sense that their diagonal coherent-state representation cannot be non-negative.[1,2] The fundamental importance of photon antibunching and squeezing as well as their potential practical applications have attracted the attention of many researchers in recent years. Several review articles[3–6] covering the subject of photon correlations are now available and the extensive literature on photon antibunching is to be found there.

Quite recently, a number of papers have appeared analyzing the possibilities of generating squeezed electromagnetic field states in various processes offered by nonlinear optics.[7–13] Resonance fluorescence exhibits both photon antibunching and squeezing. Photon antibunching in resonance fluorescence has been predicted theoretically by Carmichael and Walls[14] and Kimble and Mandel,[15] and experimentally observed by Kimble, Dagenais, and Mandel[16] and Leuchs, Rateike, and Walther.[17] Cohen-Tannoudji and Reynaud[18] have proposed the dressed-atom picture of this effect. Agarwal *et al.*[19] have pointed out that the cooperative behavior of a system of two atoms leads to a significant reduction of antibunching as compared to the one-atom case. Some difficulties that can arise in the observation of the effect due to fluctuations in the number of radiating atoms have also been discussed.[20–24] Walls and Zoller[25] have shown that beside antibunching squeezing also can occur in one-atom resonance fluorescence. Mandel[26] has made a comparison of the two effects showing that the detection of squeezed states by phase-sensitive interference with another optical field in a coherent state and measuring the resulting intensity fluctuations, that leads always to sub-Poissonian photon statistics, is at least an order-of-magnitude more difficult than the detection of photon antibunching in this phenomenon.

In this paper we consider the possibility of obtaining both photon antibunching and squeezed states in resonance fluorescence of two interacting atoms. To this aim we adapt Lehmberg's[27] approach to the case of coherent pumping and obtain a closed system of 15 equations describing the time evolution of the atomic variables. We solve this set of equations for the steady state. Apart from the Rabi frequency Ω describing the interaction of an individual atom with the field, this solution is dependent on the collective parameters γ_{12} and Ω_{12} describing collective damping as well as collective shift of energy levels. These two collective parameters, which provide a measure of the magnitude of interatomic interaction, determine the collective properties of the system. Using the steady-state solution we calculate the influence of these collective parameters (dependent on the interatomic separation) on photon antibunching and squeezing.

II. FORMULATION OF THE PROBLEM

To describe two-atom resonance fluorescence, we start with the Lehmberg master equation[27] which, for an arbitrary combination Q of atomic operators, reads

$$\dot{Q}=i\omega_0\sum_i[S_i^+S_i^-,Q]-\frac{i\vec{\mu}}{\hbar}\cdot\{[S_i^++S_i^-,Q]\vec{E}_0^{(+)}(\vec{r},t)-\vec{E}_0^{(-)}(\vec{r},t)[Q,S_i^++S_i^-]\}$$
$$+\sum_{i\neq j}i\Omega_{ij}[S_i^+S_j^-,Q]+\sum_{i,j}\gamma_{ij}[S_i^+QS_j^--\tfrac{1}{2}(S_i^+S_j^-Q+QS_i^+S_j^-)]\,, \tag{1}$$

where all atomic operators are evaluated at time t, ω_0 is the atomic transition frequency (all atoms are assumed to be identical), and $\vec{\mu}$ is the transition electric dipole moment. The operators S_i^+ and $S_i^-=[S_i^+]^\dagger$ raise and lower the energy of the ith atom and satisfy the well-known commutation relations

$$[S_i^+,S_j^-]=2S_i^z\delta_{ij}\,,\quad [S_i^z,S_j^\pm]=\pm S_i^\pm\delta_{ij}\,. \tag{2}$$

The field operator $\vec{E}_0^{(+)}(\vec{r},t)$ in the transverse mode decomposition is given by

$$\vec{E}_0^{(+)}(\vec{r},t)=\sum_q\left[\frac{2\pi\hbar\omega_q}{V}\right]^{1/2}\vec{e}_q a_q(0)e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,, \tag{3}$$

where V is the normalization volume, $\vec{e}_q$ the unit polarization vector ($\vec{k}_q\cdot\vec{e}_q=0$), and a_q the annihilation operator for the qth mode.

The collective parameters Ω_{ij} and γ_{ij}, describing the interatomic coupling, both depend on the interatomic distance r_{ij}, and are defined as[27,28]

$$\Omega_{ij}=\tfrac{3}{2}\gamma\left[-[1-(\hat{\mu}\cdot\hat{r}_{ij})^2]\frac{\cos(kr_{ij})}{kr_{ij}}+[1-3(\hat{\mu}\cdot\hat{r}_{ij})^2]\left[\frac{\sin(kr_{ij})}{(kr_{ij})^2}+\frac{\cos(kr_{ij})}{(kr_{ij})^3}\right]\right], \tag{4}$$

$$\gamma_{ij}=\tfrac{3}{2}\gamma\left[[1-(\hat{\mu}\cdot\hat{r}_{ij})^2]\frac{\sin(kr_{ij})}{kr_{ij}}+[1-3(\hat{\mu}\cdot\hat{r}_{ij})^2]\left[\frac{\cos(kr_{ij})}{(kr_{ij})^2}-\frac{\sin(kr_{ij})}{(kr_{ij})^3}\right]\right], \tag{5}$$

where $\hat{\mu}$ and $\hat{r}_{ij}$ are unit vectors along the transition electric dipole moment and the vector $\vec{r}_{ij}=\vec{r}_j-\vec{r}_i$, respectively; moreover, $r_{ij}=|\vec{r}_{ij}|$, $k=\omega_0/c=2\pi/\lambda$ with λ—the resonance wavelength. In (4) and (5), $2\gamma=\gamma_{ii}=\gamma_{jj}=(4\mu^2k^3/3\hbar)$ is the Einstein A coefficient for spontaneous emission.

We assume the free field to consist of the pump mode of a linearly polarized laser beam of frequency ω, which is in a coherent state with the eigenvalue $\vec{\mathscr{E}}_0^{(+)}(\vec{r},t)$ of the operator (3), all other modes being in their vacuum states. The frequency ω of the driving mode is tuned close to the atomic transition frequency ω_0. To simplify the problem, we moreover assume that both atoms experience the same driving field, including its phase. Therefore, we choose the reference frame such that the atoms are at the positions $\vec{r}_1=(-\frac{1}{2}r_{12},0,0)$ and $\vec{r}_2=(\frac{1}{2}r_{12},0,0)$, and the phase of the field is chosen to have the Rabi frequency $\Omega=2\vec{\mu}\cdot\vec{\mathscr{E}}_0^{(+)}/\hbar$ real and the same for both atoms. In this case, our model is similar to that of one atom in a resonant field in the presence of a conducting metallic surface,[29–31] or to that of two atoms fixed on the surface of a plane glass plate perpendicular to the direction of propagation of the laser beam.[32]

In the one-atom case, the master equation (1) gives nothing but the well-known optical Bloch equations commonly used to describe optical resonance phenomena.[33] In the two-atom case, however, extra terms, related to the collective frequency shift Ω_{ij} and the collective damping γ_{ij} appear.

Introducing the notation

$$\tau=2\gamma t\,,\quad \beta=\frac{\Omega}{4\gamma}\,,\quad a=\frac{\gamma_{ij}}{\gamma}\,,\quad b=\frac{\Omega_{ij}}{\gamma}\,,\quad \Delta=\frac{\omega_0-\omega}{\gamma}\,, \tag{6}$$

and substituting for Q in Eq. (1) the operators $S_i^\pm$ ($i=1,2$), one obtains the following equations (for the slowly varying parts of the operators):

$$\frac{d}{d\tau}S_i^\pm=-\tfrac{1}{2}(1\mp i\Delta)S_i^\pm+[\pm i\beta+\tfrac{1}{2}(a\mp ib)S_j^\pm](S_i^+S_i^--S_i^-S_i^+)\,,\quad i\neq j\,. \tag{7}$$

For two atoms, this system of equations becomes a closed set of fifteen equations describing the evolution of the atomic variables. It splits into nine equations for symmetric and six equations for antisymmetric combinations of the atomic operators. Both subsystems can be written in matrix form

$$\frac{d\vec{X}}{d\tau}=\underline{A}\vec{X}+\vec{\alpha}\,,\quad \frac{d\vec{Y}}{d\tau}=\underline{B}\vec{Y}\,, \tag{8}$$

where $\underline{A}$ is the real 9×9 matrix

$$\underline{A}=\begin{bmatrix}
-\frac{1}{2}(1+a) & 4\beta & 0 & 0 & a & 0 & \frac{1}{2}(\Delta+b) & -b & 0\\
-\beta & -1 & -a & 0 & 0 & 0 & 0 & 0 & 0\\
-\beta & -a & -1 & 0 & 2\beta & 4a & 0 & 0 & 0\\
-\beta & 0 & 0 & -1 & 2\beta & 0 & 0 & 0 & \Delta\\
0 & -2\beta & -2\beta & -2\beta & -\frac{1}{2}(3+a) & 8\beta & 0 & \frac{1}{2}(\Delta-b) & 0\\
0 & 0 & 0 & 0 & -\beta & -2 & 0 & 0 & 0\\
-\frac{1}{2}(\Delta+b) & 0 & 0 & 0 & b & 0 & -\frac{1}{2}(1+a) & a & 0\\
0 & 0 & 0 & 0 & -\frac{1}{2}(\Delta-b) & 0 & 0 & -\frac{1}{2}(3+a) & -2\beta\\
0 & 0 & 0 & -\Delta & 0 & 0 & -\beta & 2\beta & -1
\end{bmatrix}. \tag{8a}$$

In order to remove the imaginary unit i in the terms in β, we have redefined the operators S_i^- and S_i^+ when going over from (7) to (8). What we henceforth refer to as S_i^- (S_i^+) is actually iS_i^- ($-iS_i^+$) in terms of the original operators. The vector $\vec{X}$ has the following components:

$$X_1=S_1^+ +S_2^+ +S_1^- +S_2^- \ , \quad X_2=S_1^+S_1^- +S_2^+S_2^- \ ,$$
$$X_3=S_1^+S_2^- +S_2^+S_1^- \ , \quad X_4=S_1^+S_2^+ +S_1^-S_2^- \ ,$$
$$X_5=S_1^+S_1^-S_2^- +S_1^+S_2^+S_1^- +S_2^+S_1^-S_2^- +S_1^+S_2^+S_2^- \ , \tag{8b}$$
$$X_6=S_1^+S_2^+S_1^-S_2^- \ , \quad X_7=-i(S_1^- +S_2^- -S_1^+ -S_2^+) \ ,$$
$$X_8=-i(S_1^+S_1^-S_2^- -S_1^+S_2^+S_1^- +S_2^+S_1^-S_2^- -S_1^+S_2^+S_2^-) \ ,$$
$$X_9=-i(S_1^-S_2^- -S_1^+S_2^+) \ ,$$

and the vector $\vec{\alpha}$ is given by the components

$$\alpha_i=-4\beta\delta_{1i} \ . \tag{8c}$$

$\underline{B}$ is the following 6×6 real matrix:

$$\underline{B}=\begin{bmatrix} -\frac{1}{2}(1-a) & 4\beta & a & \frac{1}{2}(\Delta-b) & 0 & -b \\ -\beta & -1 & 0 & 0 & -b & 0 \\ 0 & -2\beta & -\frac{1}{2}(3-a) & 0 & 0 & \frac{1}{2}(\Delta+b) \\ -\frac{1}{2}(\Delta-b) & 0 & b & -\frac{1}{2}(1-a) & 0 & a \\ 0 & b & 0 & -\beta & -1 & -2\beta \\ 0 & 0 & -\frac{1}{2}(\Delta+b) & 0 & 2\beta & -\frac{1}{2}(3-a) \end{bmatrix} . \tag{8d}$$

The vector $\vec{Y}$ has the components

$$Y_1=S_1^- -S_2^- +S_1^+ -S_2^+ \ , \quad Y_2=S_1^+S_1^- -S_2^+S_2^- \ ,$$
$$Y_3=S_1^+S_1^-S_2^- +S_1^+S_2^+S_1^- -S_2^+S_1^-S_2^- -S_1^+S_2^+S_2^- \ ,$$
$$Y_4=-i(S_1^- -S_2^- -S_1^+ +S_2^+) \ , \tag{8e}$$
$$Y_5=-i(S_2^+S_1^- -S_1^+S_2^-) \ ,$$
$$Y_6=-i(S_1^+S_1^-S_2^- -S_1^+S_2^+S_1^- -S_2^+S_2^-S_1^- +S_1^+S_2^+S_2^-) \ .$$

It is obvious from (8a) that the matrix $\underline{A}$ forms two separate blocks if $\Delta=0$ and $b=0$, the one of dimension 6×6 and the other of dimension 3×3. Similarly, in this case, the matrix $\underline{B}$ is found to be composed of two 3×3 blocks.[34] The equations (8) for $\Delta=0$ have already been used by us[35] to calculate the spectrum of resonance fluorescence and intensity correlations for two atoms. Here, the steady-state solutions of equations (8) will be used to study photon antibunching and squeezing in such a system.

III. STEADY-STATE SOLUTIONS

By setting the left-hand side of Eqs. (8) equal to zero we obtain the steady-state solutions of these equations. It is obvious that only the $\vec{X}$ vector can have nonzero steady-state solutions whereas the steady-state solutions of the $\vec{Y}$ vector are zero. A straightforward but lengthy algebraic manipulation of Eqs. (8) leads to the steady-state solutions

$$\langle X_1\rangle=-8\beta\frac{[8\beta^2+(1+a)(1+\Delta^2)]}{D} \ ,$$
$$\langle X_2\rangle=\frac{8\beta^2[(1+\Delta^2)+8\beta^2]}{D} \ ,$$
$$\langle X_3\rangle=\frac{8(1+\Delta^2)\beta^2}{D} \ , \quad \langle X_4\rangle=\frac{8\beta^2[(1+a)-\Delta(\Delta+b)]}{D} \ ,$$
$$\langle X_5\rangle=-\frac{32\beta^3}{D} \ , \quad \langle X_6\rangle=\frac{16\beta^4}{D} \ , \tag{9}$$
$$\langle X_7\rangle=8\beta\frac{[8\Delta\beta^2+(\Delta+b)(1+\Delta^2]}{D} \ ,$$
$$\langle X_8\rangle=\frac{32\Delta\beta^3}{D} \ ,$$
$$\langle X_9\rangle=-8\beta^2\frac{[\Delta(1+a)+(\Delta+b)]}{D} \ ,$$

with

$$D=64\beta^4+16(1+\Delta^2)\beta^2+(1+\Delta^2)[(1+a)^2+(\Delta+b)^2] \ . \tag{9a}$$

The above steady-state solutions, which include the collective damping parameter a, the dipole-dipole interaction parameter b, and the detuning Δ, permit the calculation of the steady-state characteristics of the two-atom system and will be used in our calculations of photon antibunching and squeezing in such a system.

IV. INTENSITY CORRELATION FUNCTION AND PHOTON ANTIBUNCHING

In order to determine the photon antibunching it is necessary to calculate the normalized intensity correlation function, defined as

$$g^{(2)}(\vec{R}_1,t;\vec{R}_2,t+t')=\frac{G^{(2)}(\vec{R}_1,t;\vec{R}_2,t+t')}{G^{(1)}(\vec{R}_1,t)G^{(1)}(\vec{R}_2,t+t')} \tag{10}$$

with

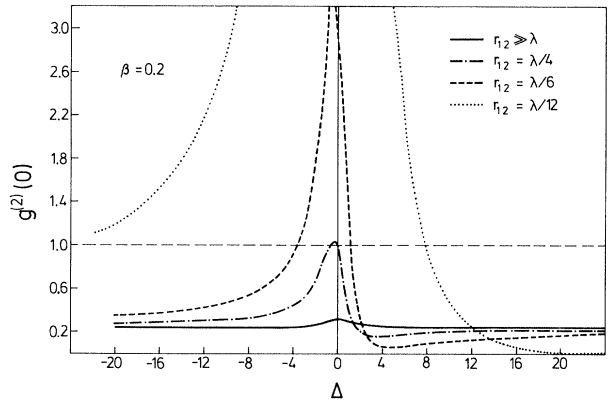

FIG. 1. Intensity correlation function $g^{(2)}(0)$ for $\hat{R}_1=\hat{R}_2\perp\hat{r}_{12}$ vs the detuning Δ for the field strength $\beta=0.2$ and for different interatomic separations r_{12}.

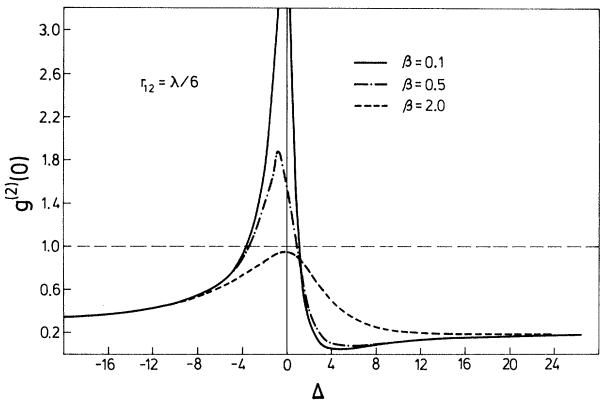

FIG. 2. Intensity correlation function $g^{(2)}(0)$ for $\hat{R}_1=\hat{R}_2\perp\hat{r}_{12}$ vs the detuning Δ for the interatomic separation $r_{12}=\lambda/6$ and for different field strengths β.

$$G^{(2)}(\vec{R}_1,t;\vec{R}_2,t+t')=\langle E^{(-)}(\vec{R}_1,t)E^{(-)}(\vec{R}_2,t+t') \times E^{(+)}(\vec{R}_2,t+t')E^{(+)}(\vec{R}_1,t)\rangle \ , \quad (11)$$

$$G^{(1)}(\vec{R},t)=\langle E^{(-)}(\vec{R},t)E^{(+)}(\vec{R},t)\rangle \ . \quad (12)$$

With the definition (10), the photon antibunching condition can be written simply as $g^{(2)}(\vec{R}_1,t;\vec{R}_2,t+t')<1$. In the far-field limit the positive-frequency ($E^{(+)}$) part of the radiation field can be expressed in terms of the atomic operators[27,36]

$$E^{(+)}(\vec{R},t)=E_0^{(+)}(\vec{R},t)-k^2\sum_{i=1}^{N}\frac{\hat{R}\times(\hat{R}\times\vec{\mu})}{R} \times S_i^-\left[t-\frac{R}{c}\right]e^{-i\vec{k}\cdot\vec{r}_i} \ , \quad (13)$$

where $\hat{R}$ is the unit vector in the direction $\vec{R}=\hat{R}R$ of the observation point, and $\vec{r}_i$ is the position vector of the ith atom. Insertion of (13) into (11) and (12) leads to

$$G^{(2)}(\vec{R}_1,t;\vec{R}_2,t+t')=\psi^2(\vec{R}_1)\psi^2(\vec{R}_2)\sum_{i,j,k,l=1}^{N}\langle S_i^+(t)S_j^+(t+t')S_k^-(t+t')S_l^-(t)\rangle\exp[ik(\vec{r}_{il}\cdot\hat{R}_1+\vec{r}_{jk}\cdot\hat{R}_2)] \ , \quad (14)$$

$$G^{(1)}(\vec{R},t)=\psi^2(\vec{R})\sum_{i,j=1}^{N}\langle S_i^+(t)S_j^-(t)\rangle\exp(ik\vec{r}_{ij}\cdot\hat{R}) \ , \quad (15)$$

with $\psi^2(\vec{R})=(3\hbar k\gamma/2R^2)\sin^2\theta$, where θ is the angle between the observation direction $\vec{R}$ and the atomic transition dipole moment $\vec{\mu}$, and $\vec{r}_{ij}=\vec{r}_j-\vec{r}_i$ is the distance between the two (non-overlapping) atoms i and j.

Having available the scattered field correlation functions expressed by the atomic correlation functions according to Eqs. (14) and (15), we can directly apply our steady-state solutions (9) to calculate the steady-state value of the normalized intensity correlation function (10) for $t'=0$. The time dependence ($t'\neq0$) of this function, even in the steady state, cannot be obtained by the immediate use of (9) and requires considerably more effort (usually numerical calculations or some approximations). We will not discuss this subject here. For $t'=0$, from (14), (15), and (10) with (9), we obtain the analytical formula for the equal-time intensity correlation function in the form

$$g^{(2)}(0)=\lim_{t\to\infty}g^{(2)}(\vec{R}_1,t;\vec{R}_2,t) =\frac{\{32\beta^4+8(1+\Delta^2)\beta^2+\frac{1}{2}(1+\Delta^2)[(1+a)^2+(\Delta+b)^2]\}\{1+\cos[k\vec{r}_{12}\cdot(\hat{R}_1-\hat{R}_2)]\}}{\{8\beta^2+(1+\Delta^2)[1+\cos(k\vec{r}_{12}\cdot\hat{R}_1)]\}\{8\beta^2+(1+\Delta^2)[1+\cos(k\vec{r}_{12}\cdot\hat{R}_2)]\}} \ . \quad (16)$$

Equation (16) is the exact formula describing intensity correlations valid for any values of the field strength and interatomic distances as well as for different configurations of the detectors. This formula is illustrated graphically in Figs. 1 and 2 as a function of the detuning Δ for $\vec{r}_{12}\perp\hat{R}(\hat{R}=\hat{R}_1=\hat{R}_2)$, and for different values of the interatomic separation r_{12} and of the field strength β. These graphs show that $g^{(2)}(0)$ strongly depends on the detuning Δ and a pronounced photon antibunching effect [$g^{(2)}(0)$ close to zero] can be obtained in such a two-atom system for certain values of Δ. This happens for $\Delta=-b$, i.e., when the dipole-dipole interaction b and the detuning

Δ cancel out mutually. In other words, this means that the laser frequency is tuned to resonance with a particular pair of energy levels of the two-atom system that are shifted by the dipole-dipole interaction. In this case, however, the two-atom system behaves like an individual two-level system. The latter situation has been discussed by Richter[32] and our results are in complete agreement with his. For two photodetectors ($\hat{R}_1 \neq \hat{R}_2$) and directions $\hat{R}_1$ and $\hat{R}_2$ for which

$$\cos\theta_1 - \cos\theta_2 = \frac{\lambda}{2r_{12}} , \tag{17}$$

where θ_1 and θ_2 are the angles between $\vec{r}_{12}$ and $\hat{R}_1(\hat{R}_2)$, respectively, we have $g^{(2)}(0)=0$. Thus we obtain anticorrelation between the photons emitted in the directions $\hat{R}_1$ and $\hat{R}_2$. This anticorrelation effect is due to spatial interference causing $\{1+\cos[k\vec{r}_{12}\cdot(\hat{R}_1-\hat{R}_2)]\}=0$. In the weak field limit and for $\Delta=0$ our formula (16) goes over into the result of Wiegand.[37]

V. SQUEEZED QUANTUM STATES

Our steady-state solutions (9) of the two-atom problem give us a good starting point to also consider the problem of generation of squeezed states of the field radiated by such a system. We now proceed to do so. Let E be a real electromagnetic field amplitude with the positive- and negative-frequency parts $E^{(+)}$ and $E^{(-)}$ which, for a quantized field, satisfy the commutation relation

$$[E^{(+)}, E^{(-)}] = C , \tag{18}$$

where C is a positive c number.

Defining the in-phase component E_1 and out-of-phase component E_2 of the field as

$$E_1 = E^{(+)} + E^{(-)} , \quad E_2 = -i(E^{(+)} - E^{(-)}) , \tag{19}$$

we have

$$[E_1, E_2] = 2iC . \tag{20}$$

The variances of E_1 and E_2 can now be written as[26]

$$\begin{aligned} \langle(\Delta E_1)^2\rangle &= C + \langle{:}(\Delta E_1)^2{:}\rangle , \\ \langle(\Delta E_2)^2\rangle &= C + \langle{:}(\Delta E_2)^2{:}\rangle , \end{aligned} \tag{21}$$

where the colon stands for normal ordering of the operators. Since squeezed states have been defined[38] by the requirement that the variance of one of two noncommuting observables shall be less than one-half of the absolute value of the expectation value of their commutator, either $\langle(\Delta E_1)^2\rangle$ or $\langle(\Delta E_2)^2\rangle$ has to be less than C to meet this requirement. In other words, according to (21), a squeezed state of the field is characterized by the condition[25,26] that either $\langle{:}(\Delta E_1)^2{:}\rangle$ or $\langle{:}(\Delta E_2)^2{:}\rangle$ is negative.

From Eq. (13) we find for the scattered field

$$\begin{aligned} \langle{:}(\Delta E_1)^2{:}\rangle &= \psi^2(\vec{R})[\langle(\Delta R_1)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle|] , \\ \langle{:}(\Delta E_2)^2{:}\rangle &= \psi^2(\vec{R})[\langle(\Delta R_2)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle|] , \end{aligned} \tag{22}$$

where R_1, R_2, and R_3 are Dicke's[39] spin variables satisfying the commutation relation

$$[R_1, R_2] = iR_3 . \tag{23}$$

These operators can be expressed in terms of the collective operators S^+ and S^-

$$\begin{aligned} R_1 &= \tfrac{1}{2}(S^+ + S^-) , \\ R_2 &= \frac{1}{2i}(S^+ - S^-) , \\ R_3 &= \tfrac{1}{2}[S^+, S^-] , \end{aligned} \tag{24}$$

where

$$S^{\pm} = \sum_{i=1}^{N} S_i^{\pm} e^{\pm i\vec{k}\cdot\vec{r}_i} .$$

Thus, squeezing in one of the two components E_1 or E_2 of the radiation field may be observed when $\langle(\Delta R_1)^2\rangle < \frac{1}{2}|\langle R_3\rangle|$ or $\langle(\Delta R_2)^2\rangle < \frac{1}{2}|\langle R_3\rangle|$, respectively.

With the help of Eqs. (9) we find that for two atoms

$$F_1 = \langle(\Delta R_1)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle| = 4\beta^2\frac{N_1+N_3}{D^2} , \tag{25}$$

$$F_2 = \langle(\Delta R_2)^2\rangle - \tfrac{1}{2}|\langle R_3\rangle| = 4\beta^2\frac{N_2-N_3}{D^2} , \tag{26}$$

with

$$N_1 = 512\beta^6 + 64[(2+a)+\Delta(2\Delta-b)+(\Delta^2-1)\cos(k\vec{r}_{12}\cdot\hat{R})]\beta^4 + 8(1+\Delta^2)[1+a^2+\Delta^2+b^2+2(\Delta^2-2a-1)\cos(k\vec{r}_{12}\cdot\hat{R})]\beta^2 , \tag{27}$$

$$N_2 = 512\beta^6 + 64[(2-a)+\Delta(2\Delta+b)+(1-\Delta^2)\cos(k\vec{r}_{12}\cdot\hat{R})]\beta^4 + 8(1+\Delta^2)[1+a^2+\Delta^2+b^2+2(1-2\Delta b-\Delta^2)\cos(k\vec{r}_{12}\cdot\hat{R})]\beta^2 , \tag{28}$$

$$N_3 = (1+\Delta^2)\{a(1+a)^2+(2+a)(\Delta+b)^2-\Delta[(b+2\Delta)(1+a)^2+b(\Delta+b)^2] + (1+\Delta^2)[(\Delta+b)^2-(1+a)^2]\cos(k\vec{r}_{12}\cdot\hat{R})\} , \tag{29}$$

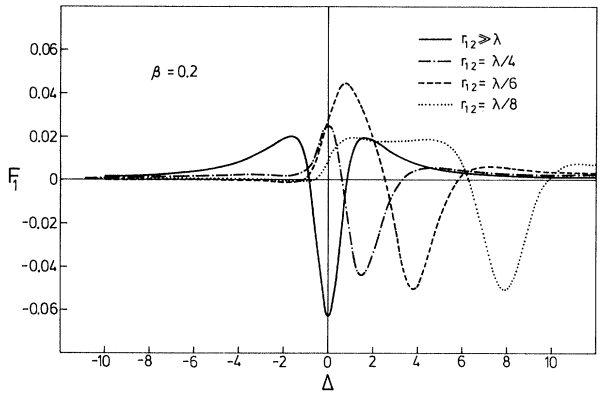

FIG. 3. F_1 vs Δ for $\hat{R}\perp\hat{r}_{12}$, $\hat{\mu}\|\hat{r}_{12}$, the field strength $\beta=0.2$, and for various interatomic separations r_{12}.

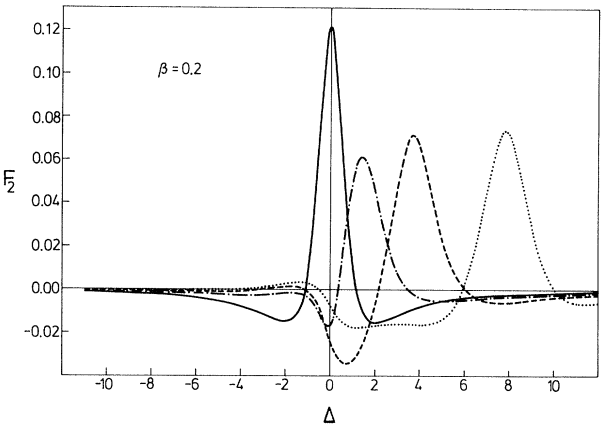

FIG. 5. Same as in Fig. 3, but for F_2.

and D the same as in Eqs. (9).

Similarly as in the case of photon correlation, our formulas (25) and (26) describing the squeezing effect in two-atom resonance fluorescence hold for any values of the parameters they depend on. These formulas are exact expressions describing the field fluctuations in steady-state two-atom resonance fluorescence. In the absence of atomic interactions ($a=b=0$), Eqs. (25) and (26) go over into the equations obtained by Walls and Zoller[25] for one-atom resonance fluorescence except for the factor of 2 standing for two atoms. The factor of 2 expresses the simple fact that our numerical results are twice greater than those of Walls and Zoller[25] due to the presence of two atoms, while the mean value of the field is also two times greater, and the fluctuations scaled to the mean value of the field remain the same. Here, we touch on the very definition of squeezing; we prefer to define squeezing as a relative quantity which, thus defined, would in fact be closely related to the signal-to-noise ratio. To be precise, we should also stress that what we denote as F_1 corresponds to $(\Delta\sigma_2)^2-\frac{1}{2}|\langle\sigma_3\rangle|$ of Walls and Zoller, and vice versa. This is due to the change of phases of our operators S_i^- and S_i^+ when passing from Eq. (7) to (8). F_1, as given by Eq. (25), is plotted in Figs. 3 and 4 versus the detuning Δ for $\vec{r}_{12}\perp\hat{R}$ and for various interatomic separations r_{12} at fixed β as well as at different values of the field strength β and fixed r_{12}. It is evident from Figs. 3 and 4 that, as the interatomic distance r_{12} becomes sufficiently small and the dipole-dipole interaction between the atoms becomes considerable, the squeezing in F_1 which, for independent atoms has its maximum for $\Delta=0$ (the maximum of squeezing actually means the minimum of F_1) shifts to the region of finite Δ. In fact, similarly as in the intensity correlation function $g^{(2)}(0)$, the minimum in F_1 appears for $\Delta=-b$ and can again be attributed to the change in energy-level structure of the two-atom system due to dipole-dipole interaction. Figure 4 shows that this minimum for a given interatomic distance r_{12} (i.e., given dipole-dipole interaction value $-b$) appears for the same value of Δ, even at different values of the field strength β. However, the amount of squeezing which can be obtained in the F_1 component is diminished by interatomic interaction.

The F_2 component given by Eq. (26) is illustrated graphically in Figs. 5 and 6. It is obvious from Figs. 5 and 6 that F_2 is almost the mirror image of F_1. The minima in F_1 correspond to the maxima in F_2, and vice versa. There are regions of Δ where F_2 becomes negative, thus squeezed. Even for noninteracting atoms some small

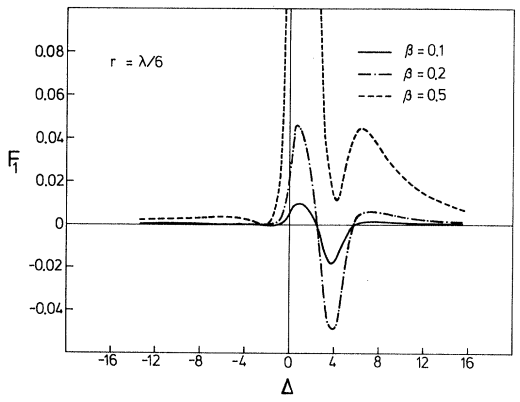

FIG. 4. F_1 vs Δ for $\hat{R}\perp\hat{r}_{12}$, $\hat{\mu}\|\hat{r}_{12}$, the interatomic separation $r_{12}=\lambda/6$, and for different field strengths β.

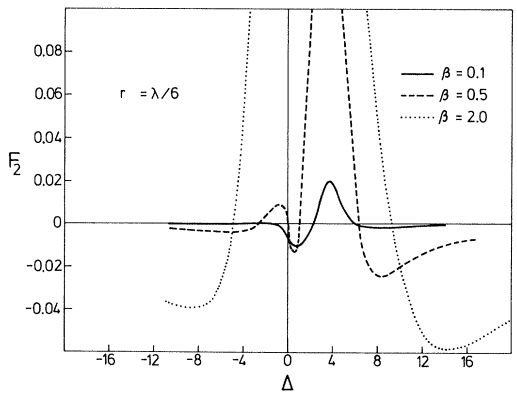

FIG. 6. Same as in Fig. 4, but for F_2.

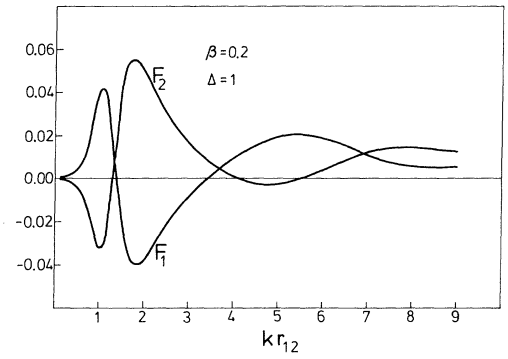

FIG. 7. F_1 and F_2 vs the interatomic separation for $\beta=0.2$ and $\Delta=1$.

squeezing occurs for $\Delta\approx\pm 2$. The dipole-dipole interaction, of course, shifts the region of squeezing in F_2, but in contrast to F_1 the amount of squeezing in F_2 can be greater than that for independent atoms in F_2 (Fig. 5). For a strong field and large detuning (Fig. 6), values of squeezing in F_2 can be obtained that are comparable to the value obtained in F_1 for independent atoms.

According to Eqs. (25) and (26), for a not excessively strong field, F_1 and F_2 tend to zero as $|b|\rightarrow\infty$, i.e., when the interatomic distance tends to zero. This is shown convincingly in Fig. 7, where the functions F_1 and F_2 are plotted versus the distance separating the atoms for a given value of the field strength and detuning.

VI. CONCLUSIONS

We have considered here the problem of two-atom resonance fluorescence with special attention paid to the role of dipole-dipole interatomic interactions and detuning in the intensity correlations and squeezing. The steady-state solutions of the two-atom problem have been calculated. With the use of these solutions, analytical formulas for the equal-time intensity correlation function as well as for the variances of the in-phase and out-of-phase components of the electric field of the fluorescent light have been obtained, making possible a comparison between the two nonclassical effects of photon antibunching and squeezing, both of which can occur in two-atom resonance fluorescence.

Our formula (16) for the intensity correlation function which agrees with the previously obtained results[32] shows explicitly that a sort of counterbalance exists between dipole-dipole interaction and detuning and that a considerable increase in photon antibunching can be achieved when the detuning Δ and the dipole-dipole interaction b cancel out mutually (Fig. 1).

Significantly, too, for special configurations of two photodetectors, we obtain photon anticorrelation $[g^{(2)}(0)=0]$ regardless of interatomic interaction and detuning. This effect, however, is due to spatial interference effects which can be important in the two-atom case and are absent in the one-atom case as well as in the small sample case.

The steady-state values of the quantities $F_1=\langle(\Delta R_1)^2\rangle-\frac{1}{2}|\langle R_3\rangle|$ and $F_2=\langle(\Delta R_2)^2\rangle-\frac{1}{2}|\langle R_3\rangle|$ have been calculated. These quantities, if negative, signify squeezing in either the in- or out-of-phase component of the field. Our results show that for two independent atoms, F_1 is squeezed for small detunings, $|\Delta|<1$, but is not squeezed for $|\Delta|>1$. The situation concerning F_2 is quite the opposite. As the interatomic distance decreases and the dipole-dipole interaction between the atoms becomes appreciable, the same interplay with detuning can be observed as in the case of the intensity correlation function. Squeezing in F_1 shifts to the region of finite Δ and attains its maximum value when Δ and b cancel each other. However, this maximum value of squeezing is less than that for independent atoms, Figs. 3 and 4. With decreasing interatomic separation, the squeezing in F_2 increases, as shown in Figs. 5 and 6. For very short interatomic distances, i.e., for $|b|\gg 1$ both F_1 and F_2 tend to zero, as shown in Fig. 7.

From Figs. 3–7 one sees that the curves for F_1 and F_2 are almost, though not strictly, mirror images. The fact that they are not strictly inverse is a simple consequence of the Heisenberg uncertainty relation for R_1 and R_2. It is easily proved that $F_1+F_2\geq[\langle(\Delta R_1)^2\rangle^{1/2}-\langle(\Delta R_2)^2\rangle^{1/2}]^2\geq 0$ meaning that F_1+F_2 cannot be negative and can be zero only if $\langle(\Delta R_1)^2\rangle=\langle(\Delta R_2)^2\rangle$ and if the minimum uncertainty condition holds simultaneously.

The present analysis of photon antibunching and squeezing in resonance fluorescence of two interacting atoms shows that both these effects are sensitive to interatomic interaction. However, the role of this interaction cannot be unambiguously declared as "destructive" or "constructive." We hope our paper may contribute towards the clarification of this situation and may prove useful in designing future experiments.

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