

SQUEEZED STATES IN RESONANCE FLUORESCENCE OF TWO INTERACTING ATOMS [☆]

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The possible existence of so-called "squeezed" states in two-atom resonance fluorescence is discussed in Lehmberg's master equation approach. It is shown that squeezing strongly depends on interatomic separations r_{12} . For large r_{12} one of the quadrature components is squeezed, and as r_{12} decreases its squeezing decreases in order to appear in the other quadrature component for certain value of r_{12} . For very small r_{12} fluctuations in both components tend to zero.

1. Introduction

An interesting problem, recently much considered, is the generation of special states of the radiation field — so-called squeezed states — in which the variance of one of two non-commuting observables is less than one half of the absolute value of their commutator [1]. As it has been shown by Yuen [2], the two-photon coherent states exhibit this feature. It has moreover been shown that squeezing can occur in degenerate parametric oscillator [3,4], one-atom resonance fluorescence [5,6], degenerate four-wave mixing [7], optical bistability [8], free-electron laser [9], Jaynes-Cummings model [10], and second-harmonic generation [11], Caves [12] has shown that squeezed states can be applied to reduce the effect of fluctuations due to quantum noise in a Michelson interferometer to detect gravity waves.

In this paper we consider the possibility of obtaining squeezed states in resonance fluorescence of two atoms as a function of their distance. To this aim we have adopted the model described in our previous papers [13,14] and obtained a closed system of 15 equations of motion for the atomic correlation functions. We solve this system of equations for the steady state. Using the steady-state solutions we calculate the

influence of interatomic interactions (dependent on the interatomic separations) on the squeezing effect.

2. Equations of motion and steady-state solution

We consider, in our model, two identical two-level atoms, distant by r_{12} , in the field of a linearly polarized, coherent laser beam, the frequency ω of which is assumed as exactly equal to the atomic transition frequency ω_0 i.e. detuning is zero. The atoms are coupled to all other modes of the electromagnetic field which are assumed as being initially in their vacuum state. On these assumptions, application of Lehmberg's [15] master equation approach leads to the following equations of motion describing the time evolution of the pseudo-spin operators of an individual atom:

$$\begin{aligned} \dot{S}_1^- &= -\gamma S_1^- - i\Omega S_1^z + 2(\gamma_{12} + i\Omega_{12}) S_1^z S_2^-, \\ \dot{S}_1^+ &= -\gamma S_1^+ + i\Omega S_1^z + 2(\gamma_{12} - i\Omega_{12}) S_2^+ S_1^z, \\ \dot{S}_1^z &= -2\gamma(S_1^z + \frac{1}{2}) + \frac{1}{2}i\Omega(S_1^+ - S_1^-) \\ &\quad - [(\gamma_{12} + i\Omega_{12}) S_1^+ S_2^- + \text{h.c.}], \end{aligned} \quad (1)$$

where S_1^+ and $S_1^- = [S_1^+]^+$ are operators raising and lowering the energy of atom 1 and S_1^z describes its energy.

The equations for the operators of the other atom are of the same form as (1), albeit with the interchange

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1 ↔ 2 of the indices. These operators fulfil the well known commutation relations:

$$[S_i^+; S_j^-] = 2S_i^z \delta_{ij}, \quad [S_i^z; S_j^\pm] = \pm S_i^\pm \delta_{ij}, \quad i, j = 1, 2. \quad (2)$$

In (1), 2γ is the Einstein A-coefficient for spontaneous emission and Ω the Rabi frequency describing the interaction of an individual atom with the laser beam. The collective damping γ_{12} and frequency shift Ω_{12} , which both are dependent on the interatomic distance r_{12} , are defined by [15]:

$$\gamma_{12} = \frac{3}{2}\gamma \left\{ [1 - (\hat{\boldsymbol{\mu}} \cdot \hat{\mathbf{r}}_{12})^2] \frac{\sin kr_{12}}{kr_{12}} + [1 - 3(\hat{\boldsymbol{\mu}} \cdot \hat{\mathbf{r}}_{12})^2] \left[\frac{\cos kr_{12}}{(kr_{12})^2} - \frac{\sin kr_{12}}{(kr_{12})^3} \right] \right\}, \quad (3)$$

$$\Omega_{12} = \frac{3}{2}\gamma \left\{ -[1 - (\hat{\boldsymbol{\mu}} \cdot \hat{\mathbf{r}}_{12})^2] \frac{\cos kr_{12}}{kr_{12}} + [1 - 3(\hat{\boldsymbol{\mu}} \cdot \hat{\mathbf{r}}_{12})^2] \left[\frac{\sin kr_{12}}{(kr_{12})^2} + \frac{\cos kr_{12}}{(kr_{12})^3} \right] \right\}, \quad (4)$$

where $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{r}}_{12}$ are unit vectors along the transition electric dipole moment and the vector $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$, respectively, and $r_{12} = |\mathbf{r}_{12}|$. $k = \omega_0/c = 2\pi/\lambda$, where λ is the resonance wavelength.

The atomic operators of eq. (1) are slowly varying parts of the operators:

$$\begin{aligned} \tilde{S}_1^+(t) &= S_1^+(t) \exp(i\omega_0 t), \\ \tilde{S}_1^-(t) &= S_1^-(t) \exp(-i\omega_0 t), \quad \tilde{S}_1^z(t) = S_1^z(t). \end{aligned} \quad (5)$$

Introducing the notation:

$$\tau = 2\gamma t, \quad \beta = \Omega/4\gamma, \quad \mathbf{a} = \gamma_{12}/\gamma, \quad \mathbf{b} = \Omega_{12}/\gamma, \quad (6)$$

we rewrite eqs. (1) in the form

$$\begin{aligned} (d/d\tau)S_1^- &= -\frac{1}{2}S_1^- - 2i\beta S_1^z + (\mathbf{a} + i\mathbf{b})S_1^z S_2^-, \\ (d/d\tau)S_1^+ &= -\frac{1}{2}S_1^+ + 2i\beta S_1^z + (\mathbf{a} - i\mathbf{b})S_2^+ S_1^+, \\ (d/d\tau)S_1^z &= -(S_1^z + \frac{1}{2}) + i\beta(S_1^+ - S_1^-) \\ &\quad - \frac{1}{2}[(\mathbf{a} + i\mathbf{b})S_1^+ S_2^- + \text{h.c.}]. \end{aligned} \quad (7)$$

In the case of two atoms this system of equations generates a closed system of 15 equations describing the evolution of the atomic variables. It splits into 9 equations for symmetric modes and 6 equations for

antisymmetric modes which we write in matrix form:

$$d\mathbf{X}/d\tau = \mathbf{A}\mathbf{X} + \boldsymbol{\alpha}, \quad d\mathbf{Y}/d\tau = \mathbf{B}\mathbf{Y}, \quad (8)$$

where \mathbf{A} and \mathbf{B} are a real 9×9 and 6×6 matrix, respectively.

For $\mathbf{b} = 0$ the matrix \mathbf{A} forms two separate blocks, the one of dimension 6×6 and the other of dimension 3×3 . Similarly, in this case, the matrix \mathbf{B} is found to be composed of two 3×3 blocks. On these assumptions our eqs. (8) are identical to those considered by us earlier [13,14].

The steady-state solutions of eqs. (8) are obtained easily, and have the form:

$$\begin{aligned} \langle S_1^+ + S_2^+ + S_1^- + S_2^- \rangle_s &= -[32\beta^3 + 4\beta(1 + \mathbf{a})]/M, \\ \langle S_1^+ + S_2^+ - S_1^- - S_2^- \rangle_s &= 4i\mathbf{b}\beta/M, \\ \langle S_1^+ S_2^- + S_2^+ S_1^- \rangle_s &= 4\beta^2/M, \\ \langle S_1^+ S_1^- + S_2^+ S_2^- \rangle_s &= (32\beta^4 + 4\beta^2)/M, \\ \langle S_1^+ S_2^+ + S_1^- S_2^- \rangle_s &= [4\beta^2(1 + \mathbf{a})]/M, \\ \langle S_1^+ S_2^- - S_1^- S_2^+ \rangle_s &= -4i\mathbf{b}\beta^2/M, \end{aligned} \quad (9)$$

with

$$M = 32\beta^4 + 8\beta^2 + \frac{1}{2}\mathbf{b}^2 + \frac{1}{2}(1 + \mathbf{a})^2.$$

3. Squeezing

We shall now discuss the possibility of squeezed states occurring in two-atom resonance fluorescence. To this aim we make use of spin variables $R_1(t)$, $R_2(t)$ and $R_3(t)$ as proposed by Dicke [16] fulfilling the commutation relation

$$[R_1(t); R_2(t)] = iR_3(t). \quad (10)$$

They can be expressed in terms of the operators $S^+(t)$ and $S^-(t)$

$$\begin{aligned} R_1(t) &= \frac{1}{2}[S^+(t) + S^-(t)], \\ R_2(t) &= (1/2i)[S^+(t) - S^-(t)], \\ R_3(t) &= \frac{1}{2}[S^+(t); S^-(t)]. \end{aligned} \quad (11)$$

where, for two atoms, $S^\pm(t) = S_1^\pm(t) + S_2^\pm(t)$.

A squeezed state is then characterized by the condition that $[5]$ either $\langle (\Delta R_1)^2 \rangle < \frac{1}{2}\langle R_3 \rangle$ or $\langle (\Delta R_2)^2 \rangle$

$< \frac{1}{2} |\langle R_3 \rangle|$. With the help of eqs. (9) we find:

$$F_1 = \langle (\Delta R_1)^2 \rangle - \frac{1}{2} |\langle R_3 \rangle| = z \{ 2z^3 + (a+1)z^2 + \frac{1}{2} [(a^2 - 4a - 1) + b^2]z + \frac{1}{4}(a+3)b^2 - \frac{1}{4}(1-a)(1+a)^2 \} \times [2z^2 + 2z + \frac{1}{2}b^2 + \frac{1}{2}(1+a)^2]^{-2}, \quad (12)$$

$$F_2 = \langle (\Delta R_2)^2 \rangle - \frac{1}{2} |\langle R_3 \rangle| = z \{ [2z + (1-a)] \times [z^2 + z + \frac{1}{4}b^2 + \frac{1}{4}(1+a)^2] - b^2 \} \times [2z^2 + 2z + \frac{1}{2}b^2 + \frac{1}{2}(1+a)^2]^{-2}, \quad (13)$$

with $z = 4\beta^2$.

For $a = b = 0$ eqs. (12) and (13) go over into the equation obtained by Walls and Zoller [5] for one-atom resonance fluorescence except for a factor 2 standing for two atoms.

The spin variables R_1 and R_2 are squeezed when $F_1 < 0$ or $F_2 < 0$ respectively. Analysis performed for the sign of the numerator of (12) leads to the conclusion that $F_1 < 0$ if $z < (\eta_1 + \eta_2) - \frac{1}{6}(a+1)$, where:

$$\eta_{1,2} = [p \pm (q^3 + p^2)^{1/2}]^{1/3}, \\ p = -\frac{1}{12} [\frac{1}{9}(a+1)(5a^2 + 10a - 4) + \frac{1}{2}b^2(a+4)], \\ q = \frac{1}{18}(a^2 - 7a - 2 + \frac{3}{2}b^2). \quad (14)$$

Since $z > 0$, the inequality $(\eta_1 + \eta_2) - \frac{1}{6}(a+1) > 0$ has to be fulfilled. By eqs. (14), this condition is fulfilled if $|b| < 1$. Thus the variable R_1 is not squeezed for $|b| > 1$, see fig. 1.

A similar analysis for the sign of the numerator of

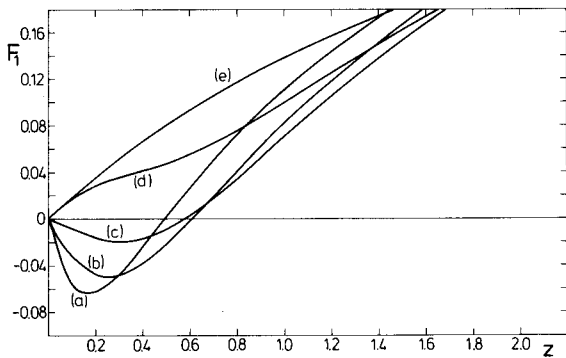


Fig. 1. F_1 versus z for $\hat{\mu} \parallel \hat{r}_{12}$ and various interatomic separations: (a) $r_{12} \gg \lambda$, (b) $r_{12} = \lambda/2$, (c) $r_{12} = \lambda/3$, (d) $r_{12} = \lambda/4$, (e) $r_{12} = \lambda/6$.

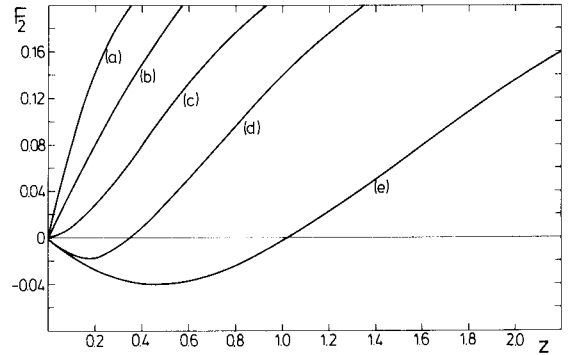


Fig. 2. F_2 versus z for $\hat{\mu} \parallel \hat{r}_{12}$ and various interatomic separations: (a) $r_{12} \gg \lambda$, (b) $r_{12} = \lambda/2$, (c) $r_{12} = \lambda/3$, (d) $r_{12} = \lambda/4$, (e) $r_{12} = \lambda/6$.

eq. (13) shows that $F_2 < 0$ if $z < (\lambda_1 + \lambda_2) - \frac{1}{6}(3-a)$ where:

$$\lambda_{1,2} = [m \pm (n^3 + m^2)^{1/2}]^{1/3}, \\ m = \frac{1}{12} [\frac{1}{9}a^2(5a+9) + \frac{1}{2}b^2(a+6)], \\ n = \frac{1}{18} [a(a+3) + \frac{3}{2}b^2]. \quad (15)$$

Since $z > 0$, the inequality $(\lambda_1 + \lambda_2) - \frac{1}{6}(3-a) > 0$ has to be fulfilled. By eqs. (15), this condition is fulfilled if $|b| > 1$. That is to say, the variable R_2 is not squeezed for $|b| < 1$, see fig. 2.

By eqs. (12) and (13), for a not excessively strong field and at $|b| \rightarrow \infty$, F_1 and F_2 tend to zero. This is shown convincingly in fig. 3 and 4, where the functions F_1 and F_2 are plotted versus the distance separating the atoms for two values of the field strength.

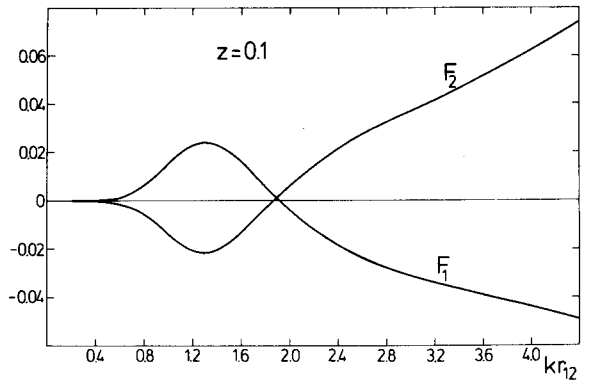


Fig. 3. F_1 and F_2 as functions of the interatomic separations for $z = 0.1$.

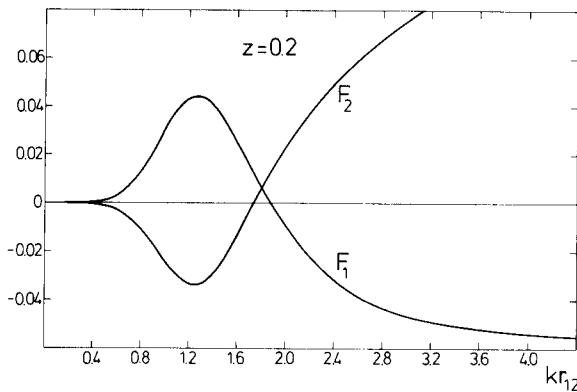


Fig. 4. The same as in fig. 3, albeit for $z = 0.2$.

4. Discussion and conclusions

The formulas (12) and (13) show the functions $F_1 = \langle(\Delta R_1)^2\rangle - \frac{1}{2}\langle R_3\rangle$ and $F_2 = \langle(\Delta R_2)^2\rangle - \frac{1}{2}\langle R_3\rangle$ versus the distance between the atoms and versus the field strength acting on them. With regard to figs. 1 and 2, only the variable R_1 is squeezed when the interatomic distance is great. As the latter decreases so does the squeezing of R_1 , and for distances for which $|b| > 1$ the variable R_1 is no longer squeezed. Inversely, for distances such that $|b| > 1$, the variable R_2 is squeezed. For very short interatomic distances i.e. at $|b| \gg 1$ both R_1 and R_2 are unsqueezed (figs. 3 and 4).

From figs. 3 and 4 one can see that the curves for F_1 and F_2 are almost though not exactly equal and opposite. The fact that they are not exactly opposite is a consequence of the Heisenberg uncertainty relation for the operators R_1 and R_2 . Using the uncertainty relation for R_1 and R_2 , one can easily prove that $F_1 + F_2 \geq \{[\langle(\Delta R_1)^2\rangle]^{1/2} + [\langle(\Delta R_2)^2\rangle]^{1/2}\}^2 \geq 0$, meaning that $F_1 + F_2$ cannot be negative, and can be equal

to zero only if $\langle(\Delta R_1)^2\rangle = \langle(\Delta R_2)^2\rangle$ and if the minimum uncertainty condition holds simultaneously.

It is worth noting that the strongest squeezing of R_1 occurs if the two atoms are mutually independent, $F_{1\min} \simeq -0.061$. Whereas R_2 becomes minimal for $r_{12} = \lambda/6$, when $F_{2\min} \simeq -0.039$. This means that dipole interaction between the atoms has a destructive effect on squeezing.

Significantly, too, the influence of interaction on squeezing resembles that of interactions on photon antibunching: as shown in [14], interactions considerably lower photon antibunching in the case of weak fields whereas in strong fields their role is unimportant.

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