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Effect of interatomic interactions on resonance fluorescence of two atoms coherently driven by a strong resonant laser field†

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Abstract. Resonance fluorescence of two-level atoms pumped by a strong resonant laser field is examined, taking account of cooperative damping γ_{12} and level shifts Ω_{12} due to radiative and dipole–dipole interaction. Applying the Lehmberg master equation for two atoms, we derive a closed set of 15 equations of motion for the time evolution of the atomic variables. The set is solved using the Laplace transform and quantum regression theory for the steady-state spectrum and intensity correlation. The steady-state solutions for $\gamma_{12} \neq \gamma$ (where 2γ is Einstein's coefficient A) are shown to differ by a 'scaling factor' from those for $\gamma_{12} = \gamma$. In the strong field limit $\Omega \gg \gamma$ (where Ω is the Rabi frequency), for the three regions (1) $\Omega \gg \gamma \gg \Omega_{12}$, (2) $\Omega \gg \Omega_{12} \gg \gamma$ and (3) $\Omega \approx \Omega_{12} \gg \gamma$ we derive analytical formulae for the spectra of symmetric and antisymmetric modes. The intensity correlation function is calculated for regions (1)–(3).

1. Introduction

Cooperative effects in resonance fluorescence arising from the interaction of many two-level atoms with an external laser field have been studied extensively during the past few years [1–12]. Strict solutions of the problem, however, can be obtained only for systems of a few (two or three) atoms [13–19]. To solve the problem for the many-atom case some approximations are needed, for example suitable decoupling procedures [7–11]. This is precisely why two-atom resonance fluorescence has become a subject of extensive research in recent years. The resonance fluorescence spectrum and intensity correlations have been considered for the small-sample case (the $\mathbf{S}^2$-conserving system) as well as for an extended system (the $\mathbf{S}^2$ conservation breaking case) in either the master equation [5–10, 15] or Green's function [13–14] approach. It has been shown that the two-atom resonance fluorescence spectrum should exhibit additional side-bands at $\omega - \omega_0 = \pm 2\Omega$, where Ω is the Rabi frequency. As has been shown [10, 15], these additional peaks should be distinguishable from the background only at extremely large Ω, while at moderately large Ω a cooperative system of two atoms has the three-peak structure of resonance fluorescence spectra well known from the one-atom theory [20–23] (see also [24]). In those calculations, the first-order dispersion forces (or dipole–dipole interactions) between the atoms [25] were usually neglected. Freedhoff [26] and Kilin [27], using a dressed atom approach, have included dipole–dipole interaction and obtained analytical formulae for the resonance fluorescence spectrum. They have shown that the spectrum consists of seven lines, with the side-lines symmetrically located with respect to the central line. Their formulae, however, are valid only when the dipole–dipole interaction between the atoms is comparable to the interaction of one atom

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with the field, and the lines are well resolved. Recently, Kuś and Wódkiewicz [28] and Agarwal *et al.* [29] have numerically obtained the two-atom spectrum, taking into account dipole–dipole interaction, and have confirmed the Freedhoff–Kilin results, predicting that for a strong dipole–dipole interaction additional components can appear in the spectrum. Kuś and Wódkiewicz [28] have also pointed out that the spectrum should be asymmetric. Mavroyannis [13, 14], using the Green's functions approach, has also studied the influence of the dipole–dipole interaction on the resonance fluorescence spectrum of a two-atom system, but his spectrum consists of five lines.

In this paper we consider the effect of interatomic interactions on two-atom resonance fluorescence. We have adapted Lehmberg's [30] approach to the case of coherent pumping and have obtained a closed system of 15 equations describing the time evolution of the atomic variables. Apart from the Rabi frequency Ω describing the interaction of an individual atom with the field, this evolution depends on the collective parameters γ_{12} and Ω_{12}, which describe collective damping as well as the collective shift of energy levels. These two collective parameters, which provide a measure of the magnitude of interatomic interaction, determine the collective properties of the system and are the subject of the study in this paper. The system of equations is split into two subsystems, the one of nine equations for symmetric modes, and the other of six equations for antisymmetric modes. The steady-state solutions of these equations are calculated for both an $\mathbf{S}^2$-conserving system and a system without $\mathbf{S}^2$ conservation. We make use of the quantum regression theorem to derive equations for the two-time correlation functions. Applying the Laplace transform method, we obtain an analytical expression for the steady-state spectrum of resonance fluorescence of such a system. In the strong field limit the roots of the denominator defining the positions and widths of the spectral lines are found approximately (up to terms in $\sim\Omega^{-2}$) for various ranges of Ω_{12}. This enables us to derive the approximate analytical formulae for the spectrum in a form explicitly showing its resonant structure. Moreover, we have calculated the time dependence of the intensity correlation function for different values of the dipole–dipole interaction between the atoms.

2. Equations of motion for the atomic operators

In our model we consider two identical two-level atoms, separated by a distance r_{12}, in the field of a linearly polarized, coherent laser beam, the frequency ω of which is tuned close to the atomic transition frequency ω_0. The atoms are coupled to all other modes of the electromagnetic field which are assumed to be initially in their vacuum state. We will assume in our calculations that both atoms experience the same field, including its phase. In practice, a situation like this could be realized, as Richter [31] has proposed, by fixing the atoms on the surface of a plane glass plate placed perpendicular to the direction of the propagation of the laser beam.

To obtain equations of motion describing the time evolution of the atomic variables, we apply Lehmberg's approach [30] which gives us the following equations of motion for the pseudo-spin operators of an individual atom:

$$\left.\begin{aligned}
\dot{S}_1^- &= -(\gamma + i\Delta)S_1^- - i\Omega S_1^z + 2(\gamma_{12} + i\Omega_{12})S_1^z S_2^-, \\
\dot{S}_1^+ &= -(\gamma - i\Delta)S_1^+ + i\Omega S_1^z + 2(\gamma_{12} - i\Omega_{12})S_2^+ S_1^z, \\
\dot{S}_1^z &= -2\gamma(S_1^z + \tfrac{1}{2}) + \tfrac{1}{2}i\Omega(S_1^+ - S_1^-) - [(\gamma_{12} + i\Omega_{12})S_1^+ S_2^- + \text{H.c.}],
\end{aligned}\right\} \tag{1}$$

where S_1^+ and $S_1^- = [S_1^+]^+$ are operators that raise and lower the energy of atom 1, and S_1^z describes its energy; H.c. denotes hermitian conjugate. These operators fulfil the well-known commutation relations

$$[S_i^+, S_i^-] = 2S_i^z, \quad [S_i^z, S_i^\pm] = \pm S_i^\pm, \quad i = 1, 2, \tag{2}$$

and the operators of different atoms are assumed to commute. In equation (1), 2γ is the Einstein A coefficient for spontaneous emission, Ω is the Rabi frequency describing the interaction of an individual atom with the laser beam, and $\Delta = \omega_0 - \omega$ is the detuning of the laser frequency below the atomic resonance. The collective damping γ_{12} and frequency shift Ω_{12}, which both depend on the interatomic distance $\mathbf{r}_{12}$, are defined by [25, 30]

$$\gamma_{12} = \frac{3}{2}\gamma\left\{[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\frac{\sin(kr_{12})}{kr_{12}} + [1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\left[\frac{\cos(kr_{12})}{(kr_{12})^2} - \frac{\sin(kr_{12})}{(kr_{12})^3}\right]\right\}, \tag{3}$$

$$\Omega_{12} = \frac{3}{2}\gamma\left\{-[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\frac{\cos(kr_{12})}{kr_{12}} + [1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\left[\frac{\sin(kr_{12})}{(kr_{12})^2} + \frac{\cos(kr_{12})}{(kr_{12})^3}\right]\right\}, \tag{4}$$

where $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{r}}_{12}$ are unit vectors along the transition electric dipole moment and the vector $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$, respectively, and $r_{12} = |\mathbf{r}_{12}|$, $k = \omega/c$.

The equations for atom 2 have the same form as equations (1), except that the indices 1 and 2 are interchanged. The atomic operators of equation (1) are slowly varying parts of the operators

$$\tilde{S}_1^+(t) = S_1^+(t)\exp(i\omega t), \quad \tilde{S}_1^-(t) = S_1^-(t)\exp(-i\omega t), \quad \tilde{S}_1^z(t) = S_1^z(t). \tag{5}$$

For a small sample, i.e. for $kr_{12} \ll 1$, γ_{12} tends to γ and Ω_{12} represents the dipole–dipole interaction between the atoms. If $\gamma_{12} = \gamma$, and discarding the frequency shift Ω_{12}, equations (1) with $\Delta = 0$ become equivalent to the master equation usually used to describe such a cooperative system [5, 15, 32]. Equations (1) are generalizations of the optical Bloch equations commonly used to describe optical resonance phenomena [33]. The validity of equations (1) hinges on the assumption that the system is markovian. The markovian approximation used in deriving these equations has been discussed by Milonni and Knight [34].

Introducing the notation†

$$\tau = 2\gamma t, \quad \beta = \frac{\Omega}{4\gamma}, \quad \delta = \frac{\Delta}{\gamma}, \quad a = \frac{\gamma_{12}}{\gamma}, \quad b = \frac{\Omega_{12}}{\gamma}, \tag{6}$$

we rewrite equations (1) in the form

$$\left.\begin{aligned}
\frac{d}{d\tau}S_1^- &= -\tfrac{1}{2}(1+i\delta)S_1^- - 2i\beta S_1^z + (a+ib)S_1^z S_2^-, \\
\frac{d}{d\tau}S_1^+ &= -\tfrac{1}{2}(1-i\delta)S_1^+ + 2i\beta S_1^z + (a-ib)S_2^+ S_1^z, \\
\frac{d}{d\tau}S_1^z &= -(S_1^z + \tfrac{1}{2}) + i\beta(S_1^+ - S_1^-) - \tfrac{1}{2}[(a+ib)S_1^+ S_2^- + \text{H.c.}].
\end{aligned}\right\} \tag{7}$$

† In our previous papers [17, 18] the notation $A = \gamma$ was used.

For two atoms, this system of equations generates a closed system of fifteen equations describing the evolution of the atomic variables, nine equations for symmetric modes and six equations for antisymmetric modes. We now write both subsystems in matrix form,

$$\frac{d\mathbf{X}}{d\tau}=\check{A}\mathbf{X}+\boldsymbol{\alpha}, \quad \frac{d\mathbf{Y}}{d\tau}=\check{B}\mathbf{Y}, \tag{8}$$

where $\check{A}$ is the following real 9×9 matrix:

$$\check{A}=\begin{bmatrix}
-\frac{1}{2}(1+a) & 4\beta & 0 & 0 & a & 0 & \frac{1}{2}(\delta+b) & -b & 0 \\
-\beta & -1 & -a & 0 & 0 & 0 & 0 & 0 & 0 \\
-\beta & -a & -1 & 0 & 2\beta & 4a & 0 & 0 & 0 \\
-\beta & 0 & 0 & -1 & 2\beta & 0 & 0 & 0 & \delta \\
0 & -2\beta & -2\beta & -2\beta & -\frac{1}{2}(3+a) & 8\beta & 0 & \frac{1}{2}(\delta-b) & 0 \\
0 & 0 & 0 & 0 & -\beta & -2 & 0 & 0 & 0 \\
-\frac{1}{2}(\delta+b) & 0 & 0 & 0 & b & 0 & -\frac{1}{2}(1+a) & a & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2}(\delta-b) & 0 & 0 & -\frac{1}{2}(3+a) & -2\beta \\
0 & 0 & 0 & -\delta & 0 & 0 & -\beta & 2\beta & -1
\end{bmatrix} \tag{9}$$

The vector $\mathbf{X}$ has the components

$$\left.\begin{aligned}
X_1&=S_1^-+S_2^-+S_1^++S_2^+,\\
X_2&=S_1^+S_1^-+S_2^+S_2^-,\\
X_3&=S_1^+S_2^-+S_2^+S_1^-,\\
X_4&=S_1^+S_2^++S_1^-S_2^-,\\
X_5&=S_1^+S_1^-S_2^-+S_2^+S_1^+S_1^-+S_2^+S_2^-S_1^-+S_1^+S_2^+S_2^-,\\
X_6&=S_1^+S_1^-S_2^+S_2^-,\\
X_7&=-i(S_1^-+S_2^--S_1^+-S_2^+),\\
X_8&=-i(S_1^+S_1^-S_2^--S_2^+S_1^+S_1^-+S_2^+S_2^-S_1^--S_1^+S_2^+S_2^-),\\
X_9&=-i(S_1^-S_2^--S_1^+S_2^+).
\end{aligned}\right\} \tag{10}$$

The components of the vector $\boldsymbol{\alpha}$ are given by

$$\alpha_i=-4\beta\delta_{1i}. \tag{11}$$

$\check{B}$ is the 6×6 real matrix

$$\check{B}=\begin{bmatrix}
-\frac{1}{2}(1-a) & 4\beta & a & \frac{1}{2}(\delta-b) & 0 & -b \\
-\beta & -1 & 0 & 0 & -b & 0 \\
0 & -2\beta & -\frac{1}{2}(3-a) & 0 & 0 & \frac{1}{2}(\delta+b) \\
-\frac{1}{2}(\delta-b) & 0 & b & -\frac{1}{2}(1-a) & 0 & a \\
0 & b & 0 & -\beta & -1 & -2\beta \\
0 & 0 & -\frac{1}{2}(\delta+b) & 0 & 2\beta & -\frac{1}{2}(3-a)
\end{bmatrix}. \tag{12}$$

The vector **Y** has the components

$$\left.\begin{aligned}
Y_1 &= S_1^- - S_2^- + S_1^+ - S_2^+, \\
Y_2 &= S_1^+ S_1^- - S_2^+ S_2^-, \\
Y_3 &= S_1^+ S_1^- S_2^- + S_2^+ S_1^+ S_1^- - S_2^+ S_2^- S_1^- - S_1^+ S_2^+ S_2^-, \\
Y_4 &= -i(S_1^- - S_2^- - S_1^+ + S_2^+), \\
Y_5 &= -i(S_2^+ S_1^- - S_1^+ S_2^-), \\
Y_6 &= -i(S_1^+ S_1^- S_2^- - S_2^+ S_1^+ S_1^- - S_2^+ S_2^- S_1^- + S_1^+ S_2^+ S_2^-).
\end{aligned}\right\} \tag{13}$$

It is obvious from equation (9) that the matrix $\check{A}$ forms two separate blocks if $\delta = 0$ and $b = 0$, one of dimension 6×6 and the other of dimension 3×3. Similarly, in this case the matrix $\check{B}$ is found to be composed of two 3×3 blocks. Under these assumptions equations (8) are identical to those considered by us in an earlier paper [17].

The square of the 'total spin' of the two-atom system can be expressed in terms of the **X** vector components as

$$\mathbf{S}^2 = 2 - X_2 + X_3 + 2X_6 \tag{14}$$

and, in accordance with equations (8) we have that

$$\frac{d}{d\tau}\mathbf{S}^2 = -(1-a)(-X_2 + X_3 + 4X_6). \tag{15}$$

This means that $\mathbf{S}^2$ is conserved in the system for $a = 1$ only, otherwise it decays on a time-scale $\sim[(1-a)2\gamma]^{-1}$. If a is very close to 1, however, this decay time is much longer than $(2\gamma)^{-1}$ and we can ignore this decay in times $\sim(2\gamma)^{-1}$, assuming that $\mathbf{S}^2$ is conserved. This is consistent with the small sample assumption, usually made in describing cooperative systems [5–10, 12, 15], because a tends to 1 as the interatomic distance becomes small compared to the light wavelength. However, for interatomic distances comparable to the wavelength, a differs considerably from 1 and we can no longer ignore the $\mathbf{S}^2$ decay. This is the $\mathbf{S}^2$ conservation breaking case [17].

The system of equations (8) can be transformed using the Laplace transform method into a system of algebraic equations in transformed variables which can be easily solved. To obtain the time dependence, however, we have to know the roots of ninth- and sixth-order polynomials obtained from equations (9) and (12). These roots can be found either numerically, or in an approximate analytic form for the case of a strong laser field; we will proceed along the latter path. However, it is possible to obtain an expression for the resonance fluorescence spectrum in closed form without knowing the roots; this will also be given. To simplify slightly equations (8), we will assume later on that the laser frequency ω is tuned exactly to the atomic transition, i.e. that $\delta = 0$.

3. Steady-state solutions

The steady-state solutions of equations (8) are easily obtained. It is obvious that only the **X**-vector can have non-zero steady-state solutions, while the steady-state solutions of the **Y**-vector are zero. Moreover, it is important to note at this point that there are two different steady-state solutions of **X** depending on whether $a \neq 1$ or $a = 1$. This fact is connected with $\mathbf{S}^2$ conservation for $a = 1$ and the reduction by one

of the number of independent variables. In this case the determinant of the $\check{A}$ matrix vanishes because of the linear dependence of the variables. For $a \neq 1$, the steady-state solutions for the expectation values of the atomic operators have the form

$$\left.\begin{aligned}
\langle X_1\rangle &= -\frac{32\beta^3+4\beta(1+a)}{M'},\\
\langle X_2\rangle &= \frac{32\beta^4+4\beta^2}{M'},\\
\langle X_3\rangle &= \frac{4\beta^2}{M'},\\
\langle X_4\rangle &= \frac{4\beta^2(1+a)}{M'},\\
\langle X_5\rangle &= -\frac{16\beta^3}{M'},\\
\langle X_6\rangle &= \frac{8\beta^4}{M'},\\
\langle X_7\rangle &= \frac{4\beta b}{M'},\\
\langle X_8\rangle &= 0,\\
\langle X_9\rangle &= -\frac{4\beta^2 b}{M'},
\end{aligned}\right\} \tag{16}$$

with

$$M' = 32\beta^4 + 8\beta^2 + \tfrac{1}{2}b^2 + \tfrac{1}{2}(1+a)^2,$$

For an $\mathbf{S}^2$ conserving system, i.e. for $a=1$ when the singlet state is not populated, the steady-state solutions for the same expectation values of the atomic operators are

$$\left.\begin{aligned}
\langle X_1\rangle &= -\frac{16\beta^3+4\beta}{M},\\
\langle X_2\rangle &= \frac{12\beta^4+2\beta^2}{M},\\
\langle X_3\rangle &= \frac{4\beta^4+2\beta^2}{M},\\
\langle X_4\rangle &= \frac{4\beta^2}{M},\\
\langle X_5\rangle &= -\frac{8\beta^3}{M},
\end{aligned}\right\} \tag{17a}$$

$$\left.\begin{aligned}\langle X_6\rangle &= \frac{4\beta^4}{M},\\ \langle X_7\rangle &= \frac{2\beta b}{M},\\ \langle X_8\rangle &= 0,\\ \langle X_9\rangle &= -\frac{2\beta^2 b}{M},\end{aligned}\right\} \qquad (17b)$$

where

$$M = 12\beta^4 + 4\beta^2 + \tfrac{1}{4}b^2 + 1.$$

The above steady-state solutions, apart from the collective damping parameter a which is equal to 1 in an $\mathbf{S}^2$ conserving system, also include the dipole–dipole interaction parameter b, which can play an important role when the interatomic distances are not too great. Using equations (16) or (17), we can calculate any steady-state characteristic of the two-atom system, such as the intensity of fluorescent light or its intensity correlation. The exact steady-state solutions of the master equation for the density matrix of a cooperative system of many atoms have been discussed by Puri and Lawande [12] and Drummond [35]. Recently, Richter [31] has obtained results for the steady state in the two-atom problem, including also the detuning of the laser frequency from resonance.

According to the equations (14) and (16), the square of the total spin of the two-atom system has a steady-state expectation value of

$$\langle \mathbf{S}^2\rangle = 2 - 16\beta^4/M', \qquad (18)$$

giving, in the strong field limit, the value 3/2. This value signifies the same population of all triplet and singlet states of the system. For an $\mathbf{S}^2$-conserving system, the singlet state remains unpopulated and $\langle \mathbf{S}^2\rangle = 2$. In a similar way we can calculate the intensity of fluorescent light, which is proportional to $\langle S^+S^-\rangle = \langle X_2\rangle + \langle X_3\rangle$, and the normalized intensity correlation function, which is proportional to $\langle S^{+2}S^{-2}\rangle/\langle S^+S^-\rangle^2$. For $\mathbf{S}^2$-conservation breaking, this quantity is given (with the use of equations (16)) by

$$g^{(2)}(0) = \frac{16\beta^4 + 4\beta^2 + \frac{1}{4}b^2 + \frac{1}{4}(1+a)^2}{16\beta^4 + 8\beta^2 + 1}, \qquad (19)$$

and for $\mathbf{S}^2$ conservation (from equations (17)) by

$$g^{(2)}(0) = \frac{12\beta^4 + 4\beta^2 + \frac{1}{4}b^2 + 1}{16\beta^4 + 8\beta^2 + 1}. \qquad (20)$$

The results (19) and (20) agree with those obtained previously [5, 18, 35] for $b = 0$. Equation (19) gives the value 1 in the strong field limit, whereas equation (20) gives a value of 0·75. Richter [31] has shown that this value can be reduced considerably if the laser frequency is detuned from resonance. The dipole–dipole interaction b does not considerably affect the initial value of the intensity correlation function $g^{(2)}(0)$ for the strong field. Equations (19) and (20), however, are valid for any values of the parameters. In figure 1 equation (20) is shown, with $g^{(2)}(0)$ plotted against b for different values of β. This shows clearly that, for fields that are not too high, $g^{(2)}(0)$

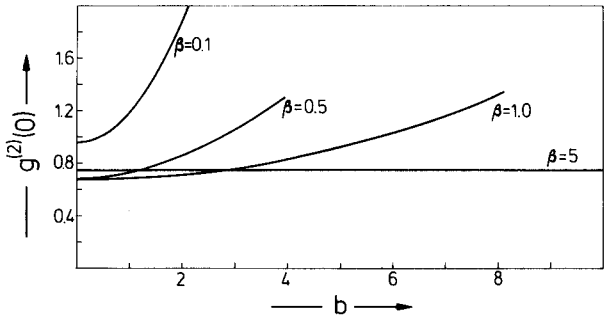

Figure 1. Plots of $g^{(2)}(0)$ against b for different values of β.

depends strongly on b. The possibility of $g^{(2)}(0)$ taking values less than 1 is related to the so-called photon antibunching effect [21, 22, 36].

4. Resonance fluorescence spectrum

Using the Laplace transform method to solve equations (8), and applying the quantum regression theorem, we calculate the resonance fluorescence power spectrum of the two-atom system which is the transform of a suitable combination of atomic correlation functions [30]:

$$P_R(z)=w_R\int_0^\infty d\tau\exp(-z\tau)\lim_{t\to\infty}\sum_{i,j=1}^{2}\exp(ik\hat{\mathbf{R}}\cdot\mathbf{r}_{ij})\langle S_i^+(t)S_j^-(t+\tau)\rangle, \tag{21}$$

where

$$w_R=(3\gamma/4\pi)\sin^2\theta,$$

θ being the angle between the observation direction unit vector $\hat{\mathbf{R}}$ and the atomic transition dipole moment $\boldsymbol{\mu}$.

The resonance fluorescence spectrum can easily be obtained from equation (21) by using the definition

$$P_R(\omega)=\mathrm{Re}\,P_R\left(i\frac{\omega-\omega_0}{2\gamma}\right). \tag{22}$$

The spectrum $P_R(z)$, calculated from equation (21), can be resolved into two parts: the symmetric part $P_+(z)$ and the antisymmetric part $P_-(z)$:

$$P_R(z)=\tfrac{1}{2}w_R\{[1+\cos(k\hat{\mathbf{R}}\cdot\mathbf{r}_{12})]P_+(z)+[1-\cos(k\hat{\mathbf{R}}\cdot\mathbf{r}_{12})]P_-(z)\}. \tag{23}$$

4.1. *Symmetric modes*

Because the main aim in this paper is to discuss the effect of the dipole–dipole interaction on the resonance fluorescence spectrum, we assume here that $a=1$. Under this assumption, the symmetric part of the spectrum $P_+(z)$ is given by

$$P_+(z)=\frac{n\mathscr{L}(z)}{\beta^2 zM(z)}, \tag{24}$$

where

$$\begin{aligned}\mathscr{L}(z)=&[(z+1)^2(z+2)+2\beta^2(2z+1)]\{(z+1+2\beta^2(z+2)]\\&\times[(z+1)(z+2)^3+4\beta^2(4z^2+11z+8)]-\beta^2z[(z+2)(z^2-2)\\&+8\beta^2(5z+7)]\}+\beta^2z[(z+2)(2z+3)+8\beta^2][(z+1)^2(z+2)^3\\&+32\beta^4(2z+3)+4\beta^2z(z+1)^2+2\beta^2(z+2)(8z^2+17z+8)]\\&+b^2(z+1)\left\{\frac{1}{4}(z+1)(z+2)^2[(z+1)^2+(z+2)^2]\right.\\&+\beta^2(z+2)\left[\frac{1}{2}(z+1)(2z+1)\left(z^2+\frac{7}{2}z+4\right)\right.\\&\left.+2(z+1)(z+2)+\frac{3}{2}z(2z+3)\right]\\&\left.+2\beta^4(5z^3+16z^2+15z+8)\right\}+\frac{b^4}{16}(z+1)^2(z+2)^2,\end{aligned}\tag{25}$$

$$\begin{aligned}M(z)=&z^8+12z^7+\left(62+\frac{1}{2}b^2+24\beta^2\right)z^6\\&+\left(180+\frac{9}{2}b^2+188\beta^2\right)z^5+\left[321+\frac{67}{4}b^2+\frac{1}{16}b^4\right.\\&\left.+(600+6b^2)\beta^2+144\beta^4\right]z^4+\left[360+33b^2+\frac{3}{8}b^4\right.\\&\left.+(996+29b^2)\beta^2+668\beta^4\right]z^3+\left[248+\frac{145}{4}b^2\right.\\&\left.+\frac{13}{16}b^4+(904+49b^2)\beta^2+4(301+b^2)\beta^4+256\beta^6\right]z^2\\&+\left[96+21b^2+\frac{3}{4}b^4+(424+34b^2)\beta^2+16(57+b^2)\beta^4+512\beta^6\right]z\\&+16+5b^2+\frac{1}{4}b^4+8(10+b^2)\beta^2+4(64+3b^2)\beta^4+192\beta^6,\end{aligned}\tag{26}$$

and

$$n=\frac{4\beta^4}{M}.\tag{27}$$

Equation (24) is the exact analytical expression for the resonance fluorescence spectrum of a two-atom system, taking the dipole–dipole interaction into account. The $z=0$ pole of equation (24) contributes to the delta-shaped coherent part of the spectrum, which we will drop in the following discussion. The factor n, as given by equation (27) is related to the so-called scaling factor [6]. (This factor has a different value when $a\neq 1$ [17, 18].) Equation (24) is in agreement with the results obtained by

Griffin and Harris [37]. Using equation (24) with the definition (22), the incoherent part of the spectrum can easily be plotted with the help of a computer. Instead of doing this, however, we calculate approximate analytical formulae for a strong laser field and different ranges of values of the dipole–dipole interaction parameter b revealing the explicit resonant structure of the spectrum. In the strong field limit, $\beta \gg 1$, the approximate roots of the expression $M(z)$ given by equation (26) can be found, enabling us to derive the formulae for the spectrum in a familiar form, namely, with resonance denominators. Our approximation includes all terms up to the order of $1/\beta^2$. Three different ranges of values of the parameter b are considered: (1) $\beta \gg 1 \gg |b|$, (2) $\beta \gg |b| \gg 1$ and (3) $\beta \sim |b| \gg 1$.

(1) $\beta \gg 1 \gg |b|$. In this case the dipole–dipole interaction between the two atoms is very weak and the incoherent part of the spectrum given by equation (24) takes the approximate form

$$
\begin{aligned}
P_+(\nu) = n\Bigg\{ & \left(1 - \frac{1+(b^2/8)}{16\beta^2}\right)\frac{1}{\nu^2+1/4} + \frac{1}{24\beta^2}\,\frac{3/2}{\nu^2+(9/4)} \\
& + \frac{1+(b^2/4)}{16\beta^2}\left[\frac{5/2}{(\nu+4\beta)^2+(25/4)} + \beta \to -\beta\right] \\
& - \frac{1}{16}\left(b^2 - \frac{35b^2+64}{64\beta^2}\right)\left[\frac{1/4(7-(b^2/4))}{(\nu+u_1)^2+(7-b^2/4)^2/16} + u_1 \to -u_1\right] \\
& + \frac{b^2}{16\beta}\left[\frac{\nu+u_1}{(\nu+u_1)^2+(7-b^2/4)^2/16} - u_1 \to -u_1\right] \\
& + \left(1 + \frac{35b^2-48}{192\beta^2}\right)\left[\frac{(3+b^2/4)/4}{(\nu+u_2)^2+(3+b^2/4)^2/16} + u_2 \to -u_2\right] \\
& + \frac{1}{16\beta}(10-b^2)\left[\frac{\nu+u_2}{(\nu+u_2)^2+\left(3+\frac{b^2}{4}\right)^2\Big/16} - u_2 \to u_2\right]\Bigg\},
\end{aligned}
\tag{28}
$$

where

$$
\nu = \frac{\omega-\omega_0}{2\gamma} \tag{29}
$$

and

$$
\begin{aligned}
u_1 &= \sqrt{\left(4\beta^2 + \frac{b^2}{4} + \frac{3}{16}\right)}, \\
u_2 &= \sqrt{\left(4\beta^2 - \frac{b^2}{8} - \frac{21}{16}\right)}.
\end{aligned}
\tag{30}
$$

Although equation (28) looks rather complicated, it shows the well-known structure, proposed by Agarwal *et al.* [15] for $b=0$, consisting of a central peak $\nu=0$, side-peaks at the Rabi frequency $\nu = \pm 2\beta$ from the central peak and additional side peaks at $\nu = \pm 4\beta$. In addition to lorentzian-type lines, it also contains dispersion-like terms . As the dipole–dipole interaction comes into play, our equation (28) shows explicitly

the modifications in the spectrum. The amplitudes of the $b=0$ peaks undergo an enhancement, proportional to b^2. The lines $v=\pm 2\beta$ start to split according to equations (30), and their 3/4 and 7/4 widths are also modified. Some additional terms also appear. However, as long as $|b|\ll 1$ these modifications are very small and can be neglected. So equation (28) indicates the trend of possible changes as the dipole–dipole interaction increases.

(2) $\beta\gg|b|\gg 1$. In this case, the dipole–dipole interaction, still much weaker than the interaction with the field, becomes more pronounced. The spectrum is expressed as

$$
\begin{aligned}
P_+(v)=n\Bigg\{&\left(1-\frac{b^2}{128\beta^2}\right)\frac{1}{v^2+(1/4)}+\frac{7}{64\beta^2}\,\frac{1}{v^2+(9/4)}\\
&+\frac{b^2}{64\beta^2}\left[\frac{5/2}{(v+4\beta)^2+(25/4)}+\beta\to-\beta\right]\\
&+\frac{1}{2}\left[\left(1+\frac{b}{8\beta}-\frac{3}{4b\beta}\right)\frac{(5+(3b/8\beta))/4}{(v+2\beta+b/4)^2+(5+(3b/8\beta))^2/16}+\beta\to-\beta\right]\\
&+\frac{1}{2}\left[\left(1-\frac{b}{8\beta}+\frac{3}{4b\beta}\right)\frac{(5-(3b/8\beta))/4}{(v+2\beta-b/4)^2+(5-(3b/8\beta))^2/16}+\beta\to-\beta\right]\\
&+\left[\frac{1}{b}\left(1+\frac{5b}{16\beta}\right)\frac{v+2\beta+(1/4)b}{(v+2\beta+b/4)^2+(5+(3b/8\beta))^2/16}+\beta\to-\beta\right]\\
&-\left[\frac{1}{b}\left(1-\frac{5b}{16\beta}\right)\frac{v+2\beta-b/4}{(v+2\beta-b/4)^2+(5-(3b/8/\beta))^2/16}+\beta\to-\beta\right]\Bigg\}. \qquad (31)
\end{aligned}
$$

Equation (31) exhibits some interesting features. First, the lines at $v=\pm 2\beta$ split into doublets with components separated by $b/2$. Because $|b|\gg 1$, this splitting is larger than the widths of the lines and both components can be resolved. As a result, the spectrum consists of seven lines. The side-lines are located symmetrically either-side of the central line $v=0$, and are given by $v=\pm 2\beta\mp b/4$, $\pm 4\beta$. The amplitude of

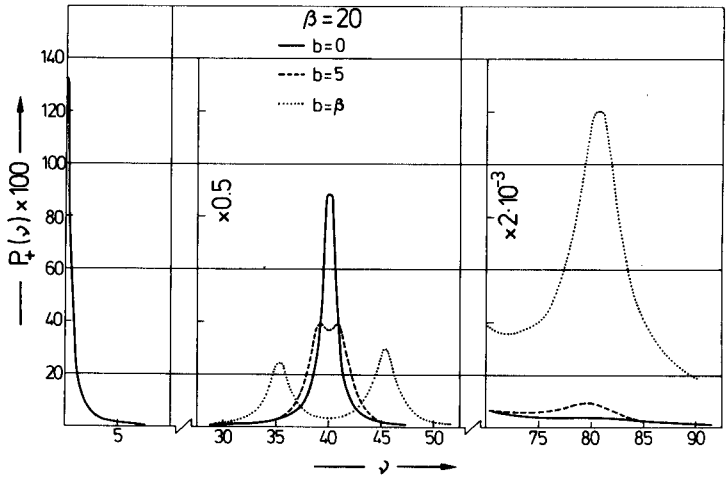

Figure 2. Resonance fluorescence spectrum for symmetric modes, for $\beta=20$ and various values of the dipole–dipole interaction between the atoms. Only one side of the spectrum is shown as it is symmetric.

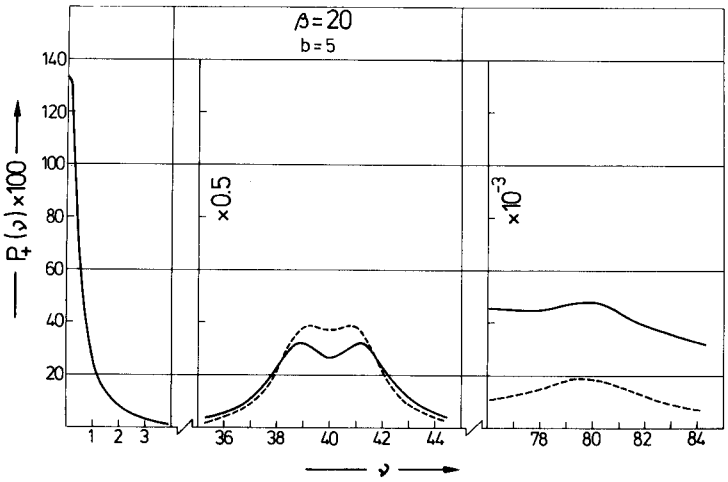

Figure 3. Role of the dispersion-like term in the spectrum. The dashed line shows the full spectrum, while the solid line shows the spectrum without dispersion-like terms.

the additional pair of side-bands at $\pm 4\beta$ is proportional to b^2/β^2 and is much greater than for $b=0$, for which it is proportional to $1/\beta^2$ [10, 15]. This means that the dipole–dipole interaction can play a crucial role in the possible observation of these lines. The central peak amplitude decreases as b^2/β^2. The spectrum calculated using equation (31) for $\beta=20$ and $|b|=5$ is plotted in figure 2. There are a number of dispersion-like terms in equation (31). Their role in the spectrum is illustrated separately in figure 3, where the spectrum is plotted with and without these terms. It is interesting that these terms somewhat worsen the resolution of the doublets at $\nu=\pm 2\beta \mp b/4$, but improve the resolution of the peaks at $\nu=\pm 4\beta$, although generally they do not contribute significantly to the spectrum.

(3) $|b| \sim \beta \gg 1$. As the interatomic distance decreases, the dipole–dipole interaction can approach a value comparable to the Rabi frequency, and the spectrum takes the form

$$
\begin{aligned}
P_+(\nu) = n\Bigg\{ & \left[\frac{4(s+(\alpha/4)-4)}{u^2 s} \, \frac{(8+\alpha^2+s)/2u^2}{\nu^2+((8+\alpha^2+s)/2u^2)^2} + s \to -s \right] \\
& + \frac{\alpha^2}{4u^2}\left[\frac{(3+(8/u^2))/2}{(\nu+2u\beta)^2+(3+(8/u^2))^2/4} + \beta \to -\beta \right] \\
& + \frac{u+\alpha/2}{2u}\left[\frac{(5+(\alpha/2u))/4}{(\nu+u\beta+\alpha\beta/2)^2+(5+(\alpha/2u))^2/16} + \beta \to -\beta \right] \\
& + \frac{u-\alpha/2}{2u}\left[\frac{(5-(\alpha/2u))/4}{(\nu+u\beta-\alpha\beta/2)^2+(5-(\alpha/2u))^2/16} + \beta \to -\beta \right] \\
& + \left[\frac{\alpha^2 w}{32(18+\alpha^2)u^5\beta} \, \frac{\nu+2u\beta}{(\nu+2u\beta)^2+(3+(8/u^2))^2/4} + \beta \to -\beta \right] \\
& + \left[\frac{(u+\alpha/2)(up+\alpha q)}{4\alpha\beta u^3(6u+\alpha)} \, \frac{\nu+u\beta+\alpha\beta/2}{(\nu+u\beta+\alpha\beta/2)^2+(5+(\alpha/2u))^2/16} + \beta \to -\beta \right] \\
& - \left[\frac{(u-\alpha/2)(up-\alpha q)}{4\alpha\beta u^3(6u-\alpha)} \, \frac{\nu+u\beta-\alpha\beta/2}{(\nu+u\beta-\alpha\beta/2)^2+(5-(\alpha/2u))^2/16} + \beta \to -\beta \right] \Bigg\}, \qquad (32)
\end{aligned}
$$

where

$$b=2\alpha\beta, \qquad u^2=4+\frac{\alpha^2}{4}, \qquad s^2=16+\alpha^2+\frac{\alpha^4}{4},$$
$$w=4\left(596+\frac{295}{4}\alpha^2+\frac{27}{8}\alpha^4+\frac{1}{16}\alpha^6\right), \tag{33}$$
$$p=48-12\alpha^2-\alpha^4, q=44+3\alpha^2.$$

The parameter α provides a measure of the strength of the dipole–dipole interaction with respect to the Rabi frequency. For $\alpha=1$, $b=2\beta$, and the dipole–dipole interaction (in units of frequency) is equal to the Rabi frequency. In this case the spectrum again has seven lines, but their positions and widths are different than for the previous case. The positions are: $v=0$, $\pm(u\pm\alpha/2)\beta$, $\pm 2u\beta$. The components of the doublet in the vicinity of the Rabi frequency are very well resolved, with splitting equal to α/β. The amplitude of the peaks at $v=\pm 2u\beta$ increases substantially and they are clearly resolved. This is shown in figure 2 for $\beta=20$ and $|b|=\beta$. The spectrum (32) apart from the dispersion-like terms, agrees with the results obtained by Freedhoff [26] and by Kilin [27].

4.2. *Antisymmetric modes*

The resonance fluorescence spectrum of two atoms consists of two parts: symmetric $P_+(z)$ and antisymmetric $P_-(z)$ as given by equation (23). If the interatomic distance is much smaller than the wavelength of the emitted light, the factor $1-\cos(k\hat{\mathbf{R}}\cdot\mathbf{r}_{12})$ at $P_-(z)$ tends to zero and the antisymmetric modes do not contribute much to the spectrum. For larger distances, however, their contribution can be important. To evaluate the spectrum of antisymmetric modes we assume here that the interatomic distance is large enough for $1-\cos(k\hat{\mathbf{R}}\cdot\mathbf{r}_{12})$ to be not too close to zero, but we set $a=1$ to simplify the calculation of $P_-(z)$. This is of course an approximation, because $a=1$ only for small distances. With this approximation we easily obtain the following formula for the antisymmetric part of the spectrum:

$$P_-(z)=\frac{n}{2}\left\{\frac{4[(z+\frac{1}{2}ib)(z+2+ib)+1+\frac{1}{2}ib+2\beta^2]}{(z+\frac{1}{2}ib)(z+1+\frac{1}{2}ib)(z+1+ib)+4\beta^2(z+\frac{1}{2}+\frac{1}{2}ib)}\right\} \tag{34}$$

In the strong-field limit this spectrum has the following approximate resonant structure:

$$\begin{aligned}P_-(v)=\frac{n}{2}\Bigg\{&\frac{1+(3/16\beta^2)}{(v-(b/2))^2+(1/4)}-\frac{b}{4\beta^2}\,\frac{v-(b/2)}{(v-(b/2))^2+(1/4)}\\&+\frac{3(1+(b/12\beta)-(36+b^2)/192\beta^2)/4}{(v-3b+2\beta)^2+9(1-(b/24\beta))^2/16}\\&+\frac{3(1-(b/12\beta)-(36+b^2)/192\beta^2)/4}{(v-(3b/4)-2\beta)^2+9(1+(b/24\beta))^2/16}\\&+\frac{1}{8\beta}\Bigg[\frac{(5+(b/\beta))(v-((3b/4)+2\beta))}{(v-(3b/4)+2\beta)^2+9(1-(b/24\beta))^2/16}\\&-\frac{(5-(b/\beta)(v-3/4b-2\beta)}{(v-(3b/4)-2\beta)^2+9(1+(b/24\beta))^2/16}\Bigg]\Bigg\}.\end{aligned} \tag{35}$$

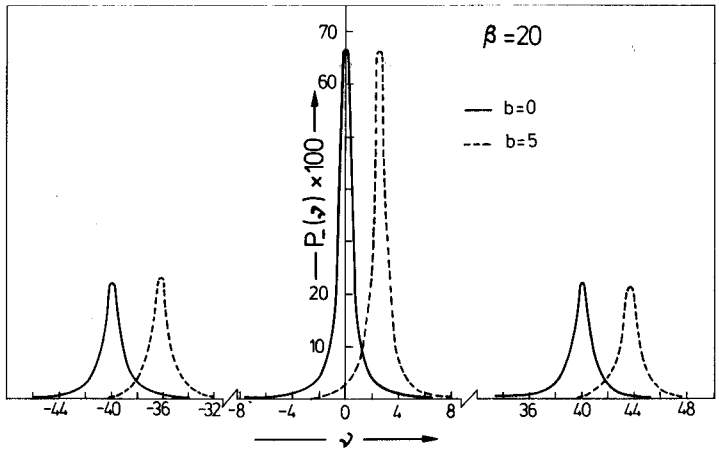

Figure 4. Resonance fluorescence spectrum for antisymmetric modes.

This spectrum is shown in figure 4. It has three lines at $\nu=\frac{1}{2}b$ and $\frac{3}{4}b\pm 2\beta$. Thus, it is shifted with respect to the symmetric part of the spectrum, the shift being proportional to the dipole–dipole interaction between the two atoms. Because of the factor 1/2 in equation (35), all line heights are smaller than the lines of the symmetric part. This spectrum is asymmetric both in the positions of the lines and their amplitudes.

5. Time dependence of the intensity correlation function

To calculate the intensity correlation function of the fluorescent light, we have to calculate the time second-order correlation function (intensity correlation function), which in the far-field zone can be expressed in terms of suitable combinations of atomic operators. With the proper normalization (the same as used in equation (21)), it can be written for the two-atom case we are considering, as [16]

$$G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t+\tau) = w_{\mathbf{R}_1} w_{\mathbf{R}_2} \sum_{i,j,k,l=1}^{2} \langle S_i^+(t) S_j^+(t+\tau) S_k^-(t+\tau) S_l^-(t)\rangle \times \exp\left[ik(\mathbf{r}_{il}\cdot\hat{\mathbf{R}}_1 + \mathbf{r}_{jk}\cdot\hat{\mathbf{R}}_2)\right], \quad (36)$$

where $w_{\mathbf{R}}$ is the same as in equation (21), $\mathbf{r}_{12}$ is the vector connecting the two atoms, and $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_2$ are unit vectors in the two possible directions of observation of the photons. A more convenient quantity to deal with is the normalized intensity correlation function, defined as

$$g^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t+\tau) = \frac{G^{(2)}(\mathbf{R}_1, t; \mathbf{R}_2, t+\tau)}{G^{(1)}(\mathbf{R}_1, t) G^{(1)}(\mathbf{R}_1, t+\tau)}, \quad (37)$$

where

$$G^{(1)}(\mathbf{R}, t) = w_{\mathbf{R}} \sum_{i,j=1} \langle S_i^+(t) S_j^-(t)\rangle \exp(ik\mathbf{r}_{ij}\cdot\hat{\mathbf{R}}) \quad (38)$$

is the first-order correlation function (intensity) of the fluorescent light at the space–time point $(\mathbf{R}, t)$. We assume here that $\mathbf{R}_1$ is equal to $\mathbf{R}_2$, i.e. that both photons are registered at the same point $\mathbf{R}$. Moreover, we assume $\mathbf{R}$ to be perpendicular to $\mathbf{r}_{12}$. Under these assumptions, equations (36)–(38) simplify considerably. In the steady

state the normalized intensity correlation function given by equation (37) depends only on the time difference τ. Using again the quantum regression theorem and the equations of motion (8), together with the steady-state solutions (17), we calculate the Laplace transform of the function $g^{(2)}(\tau)$ to be

$$\tilde{g}^{(2)}(z)=g^{(2)}(0)\frac{N(z)}{zM(z)}, \tag{39}$$

where

$$\begin{aligned}N(z)=&[z+1+4\beta^2]\{8\beta^4(2z+1)(5z+8)\\&+\beta^2(z+1)[4(z+2)(7z^2+18z+10)+b^2(z+4)]\\&+2(z+1)^3(z+2)[(z+2)^2+\tfrac{1}{4}b^2]\}\\&+z\{(z+1)^2(z+2)[(z+1)^2+\tfrac{1}{4}b^2][(z+2)^2+\tfrac{1}{4}b^2]\\&+\beta^2(z+1)(z+2)[4(z+1)(4z^2+9z+4)+b^2(4z+3)]\\&+2\beta^4[2(14z^3+40z^2+35z+10)+b^2(z+1)]+16\beta^6(2z+1)\},\end{aligned} \tag{40}$$

$M(z)$ is given by equation (26) and, from equation (20), the initial value of the normalized intensity correlation function is $g^{(2)}(0)$ (the tilde represents transformed quantities). The expression (39) was calculated under the assumption that $a=1$, i.e. for an $\mathbf{S}^2$-conserving system; none the less, it is valid for any values of the parameters β and b. In the strong field limit the approximate roots of $M(z)$ used in the spectral calculations can be applied here to invert the Laplace transform (39) and obtain the time dependence of the function $g^{(2)}(\tau)$. Three different regions of the values of the dipole–dipole interaction between the atoms can again be distinguished, and we obtain

(1) for $\beta\gg 1\gg|b|$,

$$\begin{aligned}g^{(2)}(\tau)=\frac{3}{16}\Bigg(&\frac{16}{3}+\frac{1}{6}\exp\left(-\frac{3}{2}\tau\right)+\frac{1}{2}\exp\left(-\frac{5}{2}\tau\right)\cos(4\beta\tau)\\&+\left\{\frac{b^2}{8}\exp\left[-\frac{1}{4}\left(7-\frac{b^2}{4}\right)\tau\right]-\left(2+\frac{b^2}{8}\right)\right.\\&\left.\times\exp\left[-\frac{1}{4}\left(3+\frac{b^2}{4}\right)\tau\right]\right\}\cos(2\beta\tau)\Bigg),\end{aligned} \tag{41}$$

(2) $\beta\gg|b|\gg 1$,

$$\begin{aligned}g^{(2)}(\tau)=1+&\frac{1}{32}\exp\left(-\frac{3}{2}\tau\right)+\frac{3}{32}\exp\left(-\frac{5}{2}\tau\right)\cos(4\beta\tau)\\&-\frac{3}{16}\left\{\cos\left(2\beta+\frac{1}{4}b\right)\tau+\cos\left(2\beta-\frac{1}{4}b\right)\tau\right.\\&\left.+\frac{2}{b}\left[\sin\left(2\beta+\frac{1}{4}b\right)\tau-\sin\left(2\beta-\frac{1}{4}b\right)\tau\right]\right\}\exp\left(-\frac{5}{4}\tau\right),\end{aligned} \tag{42}$$

(3) for $|b| \sim \beta \gg 1$ and $b = 2\beta\alpha$,

$$g^{(2)}(\tau) = \frac{3}{4}\left\{\frac{4}{3} + \frac{1}{2u^2}\exp\left[-\frac{1}{2}\left(3+\frac{8}{u^2}\right)\tau\right]\cos(2u\beta)\tau\right.$$
$$-\frac{1}{u(u+\alpha/2)}\exp\left[-\frac{1}{4}\left(5+\frac{\alpha}{2u}\right)\tau\right]\cos\left(u+\frac{1}{2}\alpha\right)\beta\tau$$
$$-\frac{1}{u(u-\alpha/2)}\exp\left[-\frac{1}{4}\left(5-\frac{\alpha}{2u}\right)\tau\right]\cos\left(u-\frac{1}{2}\alpha\right)\beta\tau$$
$$+\frac{64+20\alpha^2+3\alpha^4+\alpha^6/4+(16+5\alpha^2+\alpha^4/2)s}{16u^2s(8+\alpha^2+s)}$$
$$\left.\times\exp\left(\frac{-8+\alpha^2+s}{2u^2}\right)\tau + s \to -s\right\}, \tag{43}$$

where

$$u^2 = 4+\tfrac{1}{4}\alpha^2 \quad \text{and} \quad s^2 = 16+\alpha^2+\tfrac{1}{4}\alpha^4.$$

The time dependence of the normalized intensity correlation function given by equations (41)–(43) is plotted, for different values of the dipole–dipole interaction parameter b, in figures 5 and 6. Obviously, as the dipole–dipole interaction between the atoms becomes comparable to the Rabi frequency, this time dependence is different from that for small b. In the strong field limit, however, the initial value for $\tau = 0$, $g^{(2)}(0)$, is independent of b and is always 0·75, in agreement with the results of

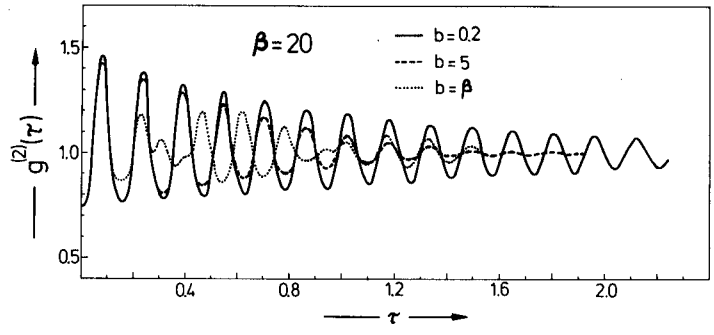

Figure 5. Time dependence of the intensity correlation function $g^{(2)}(\tau)$, for various values of b.

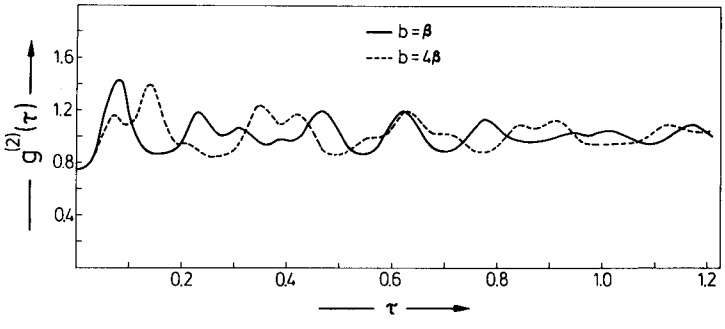

Figure 6. As figure 5, but for a very large dipole–dipole interaction parameter.

Agarwal *et al.* [5] for $b=0$. This means that the possibility of observing the photon antibunching effect in a short time interval is not affected by the dipole–dipole interaction. In the case of an $\mathbf{S}^2$-conservation breaking system, i.e. for $a \neq 1$, these results will change. We have discussed previously [18] the intensity correlations in such a system, neglecting the dipole–dipole interaction. We have shown that, in spite of the fact that $g^{(2)}(0)=1$ in the strong field limit, and if we observe the intensity correlations at the same point $\mathbf{R}$, it is still possible to find a configuration of two photodetectors which allows antibunching of the two photons to be observed, i.e. a configuration for which $g^{(2)}(\mathbf{R}_1, \mathbf{R}_2, 0)<1$. This means, however, that we need two photodetectors to observe this effect.

6. Summary and conclusions

We have considered the problem of two-atom resonance fluorescence, paying particular attention to the role of the interatomic interaction in the spectrum and intensity correlations of the fluorescent light. The closed system of 15 equations describing the system was split into two subsystems—one of dimension 9 for symmetric modes, and one of dimension 6 for antisymmetric modes. These systems of equations have been solved by applying the Laplace transform method. This enabled us to obtain analytical formulae for the power spectrum of resonance fluorescence, both for symmetric and antisymmetric modes. In the strong field limit we obtained approximate analytical formulae that reveal the resonant structure of the spectrum as well as the time dependence of the intensity correlation function. We have considered three ranges of values of the interatomic interaction, which is of the dipole–dipole type. There are two different sets of solutions, depending on whether or not the system is $\mathbf{S}^2$ conserving. The steady-state solutions are discussed for both cases. The role of the dispersion-like terms in the spectrum is also discussed. We have shown that, for small values of the dipole–dipole interaction, the spectrum is slightly different from that obtained by Agarwal *et al.* [15]. Our formula shows explicitly the trend of these modifications. The lines at the Rabi frequency away from the center ($\nu=\pm 2\beta$) start to split, but their splitting $\sim b^2/\beta$ is very small and is masked by the linewidth. There are also small changes in the widths 3/4 and 7/4 of these lines. The amplitudes of the lines are augmented by some extra b-dependent terms. As the dipole–dipole interaction increases, i.e. the interatomic distance decreases, we can have a situation in which $\beta \gg |b| \gg 1$. In this case the spectrum differs significantly from the previous one. The doublets in the vicinity of $\nu=\pm 2\beta$, separated by a distance $b/2$, become clearly resolved and the spectrum consists of seven lines. The amplitude of the additional pair of side-bands at $\nu=\pm 4\beta$, the existence of which several authors have dealt with [8–10, 15, 17], now becomes proportional to b^2/β^2 and is much larger than $1/\beta^2$ when $b=0$. Thus, the existence of these lines can be attributed to the dipole–dipole interaction rather than collective damping in the small sample case. Moreover, we have shown that a number of dispersion-like terms appearing in the spectrum lead to a worsening of the resolution of the doublets at $\pm 2\beta$ yet, surprisingly, improving the resolution of the lines at $\pm 4\beta$. For still smaller interatomic distances the dipole–dipole interaction can become comparable to the interaction of the atoms with the field; the splitting of the doublets is very clearly visible and the amplitude of the peaks at $\pm 4\beta$ drastically increases. Our formula, obtained for this case by discarding the dispersion-like terms, reproduces the spectrum obtained by Freedhoff [26], who used the dressed state formalism, as well as that obtained by Kilin [27]. We should mention at this

point that the way we have calculated the spectrum makes it impossible to recover the spectrum of Agarwal *et al.* [15] simply by setting $b=0$ in the formulae, except for the case of small b.

The antisymmetric modes in the strong field limit show a three-line spectrum, asymmetric both in the positions and amplitudes of the lines. The spectrum for antisymmetric modes has been discussed by Mavroyannis [13]. Our calculations of the intensity correlation function $g^{(2)}(\tau)$ for different ranges of values of the dipole–dipole interaction show clearly that, as this interaction increases, the time behaviour of $g^{(2)}(\tau)$ becomes more complicated, but its initial value $g^{(2)}(0)$ does not depend on b very strongly in the strong field limit. As is clear from our discussion of the steady-state solutions, this value is different for $\mathbf{S}^2$ conserving ($a=1$) and $\mathbf{S}^2$-conservation breaking ($a\neq 1$) systems. This fact can be of importance in the context of the so-called photon antibunching effect observed by Dagenais *et al.* [36] in the one-atom case.

The problem of two interacting atoms in a strong resonant laser field considered in this paper is closely related to the problem of one atom in a resonant field in the presence of a conducting metallic surface [38, 39]†.

La fluorescence à la résonance d'atomes à deux niveaux pompés par un fort champ laser résonnant est étudiée en tenant compte de l'amortissement γ_{12} et des décalages de niveaux Ω_{12} dus aux interactions radiatives et dipôle-dipôle. En appliquant l'équation de Lehmberg pour deux atomes, on obtient un système de 15 équations de mouvement pour l'évolution temporelle des variables atomiques. Le système est résolu en utilisant la transformée de Laplace et la théorie de la régression quantique pour la corrélation du spectre d'état stable et de l'intensité. On montre que les solutions de l'état stable pour $\gamma_{12}\neq\gamma$, 2γ étant le coefficient d'Einstein A, diffèrent par un 'facteur d'échelle' de celles pour $\gamma_{12}=\gamma$. Dans la limite du champ fort $\Omega\gg\gamma$, ou Ω est la fréquence de Rabi, et les trois régions (i) $\Omega\gg\gamma\gg\Omega_{12}$, (ii) $\Omega\gg\Omega_{12}\gg\gamma$ et (iii) $\Omega\simeq\Omega_{12}\gg\gamma$, on trouve des formules analytiques pour les spectres des modes symétriques et antisymétriques. La fonction de corrélation de l'intensité pour (i)–(iii) est calculée.

Die Resonanzfluoreszenz von Zweiniveau-Atomen, die von einem starken resonanten Laserfeld gepumpt werden, wird unter Berücksichtigung von kooperativer Dämpfung γ_{12} und Niveauverschiebungen Ω_{12} durch Strahlungs- und Dipol-Dipol-Wechselwirkung betrachtet. Mit der Lehmberg-Master-Gleichung für zwei Atome leiten wir einen geschlossenen Satz von 15 Bewegungsgleichungen für die Zeitentwicklung der atomaren Variablen her. Dieser Satz wird mit Laplace-Transformation und Quanten-Regressionstheorie für steady-state-Spektrum und Intensitätskorrelation gelöst. Es wird gezeigt, daß die steady-state-Lösungen für $\gamma_{12}\neq\gamma$—2γ ist der Einsteinkoeffizient $\bar{A}$—durch einen Skalierungsfaktor von jenen für $\gamma_{12}=\gamma$ abweichen. Im Grenzfall starker Felder $\Omega\gg\gamma$—worin Ω die Rabifrequenz ist—und in den drei Bereichen (i) $\Omega\gg\gamma\gg\Omega_{12}$, (ii) $\Omega\gg\Omega_{12}\gg\gamma$ und (iii) $\Omega\approx\Omega_{12}\gg\gamma$ leiten wir die analytischen Formeln für die Spektren symmetrischer und antisymmetrischer Moden her. Die Intensitätskorrelation für (i)–(iii) wird berechnet.

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